22W-MATH-32B-LEC-4 Thursday Quiz 5 (Group)

DAN TSAI, STEVE SABIN LEE, JUSTIN ANDREW SHEU, DEVYN MOTOKO CHUN, BRIAN LIHENG YANG, BENSON HEN-YI LIU

TOTAL POINTS

100 / 100

QUESTION 1

1 1 **25** / **25**

✓ **- 0 pts** **Correct: $$\ln(\frac{4}{3})$$ or equivalent**

- 2 pts Minor calculation error

- 3 pts Did not calculate work _against_ F

QUESTION 2

2 2 **25** / **25**

✓ **- 0 pts** Correct

- 2 pts Minor calculation error

- 5 pts Did not plug in point P

QUESTION 3

3 3 **25** / **25**

✓ **- 0 pts** Correct

- 5 pts Missing domain of t.

QUESTION 4

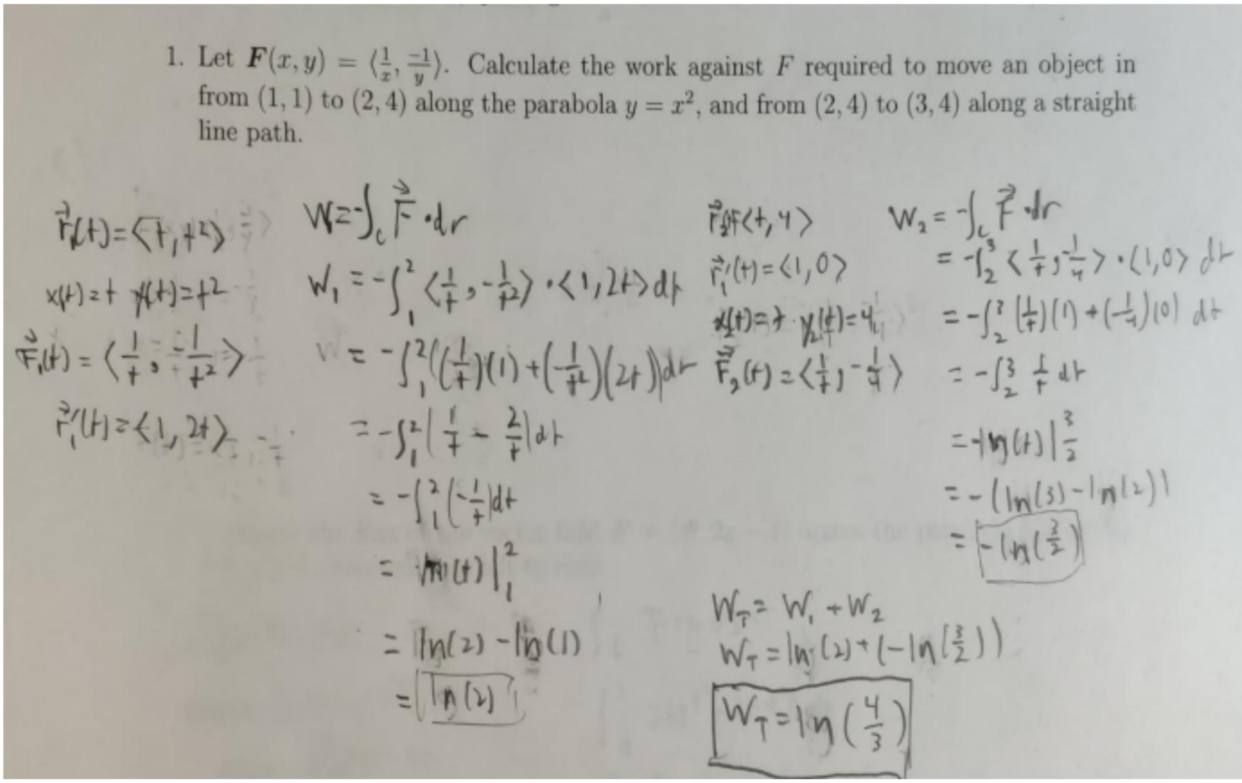
4 4 **25** / **25**

✓ **- 0 pts** Correct

- 3 pts Minor calculation error

Name (UIDs): Benson Liu (405753610), Devyn Chun (305565233), Brian Yang (905747023), Justin Sheu (405773496), Daniel Tsai (305799699), Steve Lee(205827706)
TA/Section: Bertrand Stone/Discussion 4F

1. From (1,1) to (2,4): work=ln(2) and from (2,4) to (3,4) work=ln(⅔). Total work=ln(4/3)



1. Let $\boldsymbol{F}(x, y) = \langle \frac{1}{x}, \frac{-1}{y} \rangle$. Calculate the work against F required to move an object in from (1, 1) to (2, 4) along the parabola $y = x^2$, and from (2, 4) to (3, 4) along a straight line path.

$\vec{r}_1(t) = \langle t, t^2 \rangle$, $x(t) = t$, $y(t) = t^2$, $\vec{F}_1(t) = \langle \frac{1}{t}, -\frac{1}{t^2} \rangle$, $\vec{r}_1{}'(t) = \langle 1, 2t \rangle$

$$W = -\int_C \vec{F} \cdot dr$$

$$W_1 = -\int_1^2 \langle \tfrac{1}{t}, -\tfrac{1}{t^2} \rangle \cdot \langle 1, 2t \rangle \, dt = -\int_1^2 \left((\tfrac{1}{t})(1) + (-\tfrac{1}{t^2})(2t) \right) dt = -\int_1^2 \left(\tfrac{1}{t} - \tfrac{2}{t} \right) dt = -\int_1^2 \left(-\tfrac{1}{t} \right) dt = \ln(t) \Big|_1^2 = \ln(2) - \ln(1) = \boxed{\ln(2)}$$

$\vec{r}_2(t) = \langle t, 4 \rangle$, $\vec{r}_2{}'(t) = \langle 1, 0 \rangle$, $x(t) = t$, $y(t) = 4$, $\vec{F}_2(t) = \langle \frac{1}{t}, -\frac{1}{4} \rangle$

$$W_2 = -\int_C \vec{F} \cdot dr = -\int_2^3 \langle \tfrac{1}{t}, -\tfrac{1}{4} \rangle \cdot \langle 1, 0 \rangle \, dt = -\int_2^3 (\tfrac{1}{t})(1) + (-\tfrac{1}{4})(0) \, dt = -\int_2^3 \tfrac{1}{t} \, dt = -\ln(t) \Big|_2^3 = -(\ln(3) - \ln(2)) = \boxed{-\ln(\tfrac{3}{2})}$$

$$W_T = W_1 + W_2$$
$$W_T = \ln(2) + (-\ln(\tfrac{3}{2}))$$
$$\boxed{W_T = \ln(\tfrac{4}{3})}$$

2. div(G)=37/100 and curl(G)=<⅛, 4/25, -⅕>

2. Compute div($\boldsymbol{G}$) and curl($\boldsymbol{G}$) at $P = (5, 2, 4)$ for

$$\boldsymbol{G} = \langle \frac{y}{x}, \frac{y}{z}, \frac{z}{x} \rangle$$

$$\text{div}(G) = \langle \tfrac{\partial}{\partial x}, \tfrac{\partial}{\partial y}, \tfrac{\partial}{\partial z} \rangle \cdot \langle \tfrac{y}{x}, \tfrac{y}{z}, \tfrac{z}{x} \rangle$$
$$= \frac{-y}{x^2} + \frac{1}{z} + \frac{1}{x} \Big|_{(5,2,4)} = \frac{-2}{25} + \frac{1}{4} + \frac{1}{5} = \boxed{\frac{37}{100}}$$

$$\text{curl}(G) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{y}{x} & \frac{y}{z} & \frac{z}{x} \end{vmatrix} = \hat{i}\left(0 - \frac{-y}{z^2}\right) - \hat{j}\left(\frac{-z}{x^2} - 0\right) + \hat{k}\left(0 - \frac{1}{x}\right) = \langle \frac{y}{z^2}, \frac{z}{x^2}, \frac{-1}{x} \rangle = \boxed{\langle \frac{1}{8}, \frac{4}{25}, \frac{-1}{5} \rangle}$$

1 1 **25 / 25**

✓ **- 0 pts** **Correct: $$\ln(\frac{4}{3})$$ or equivalent**

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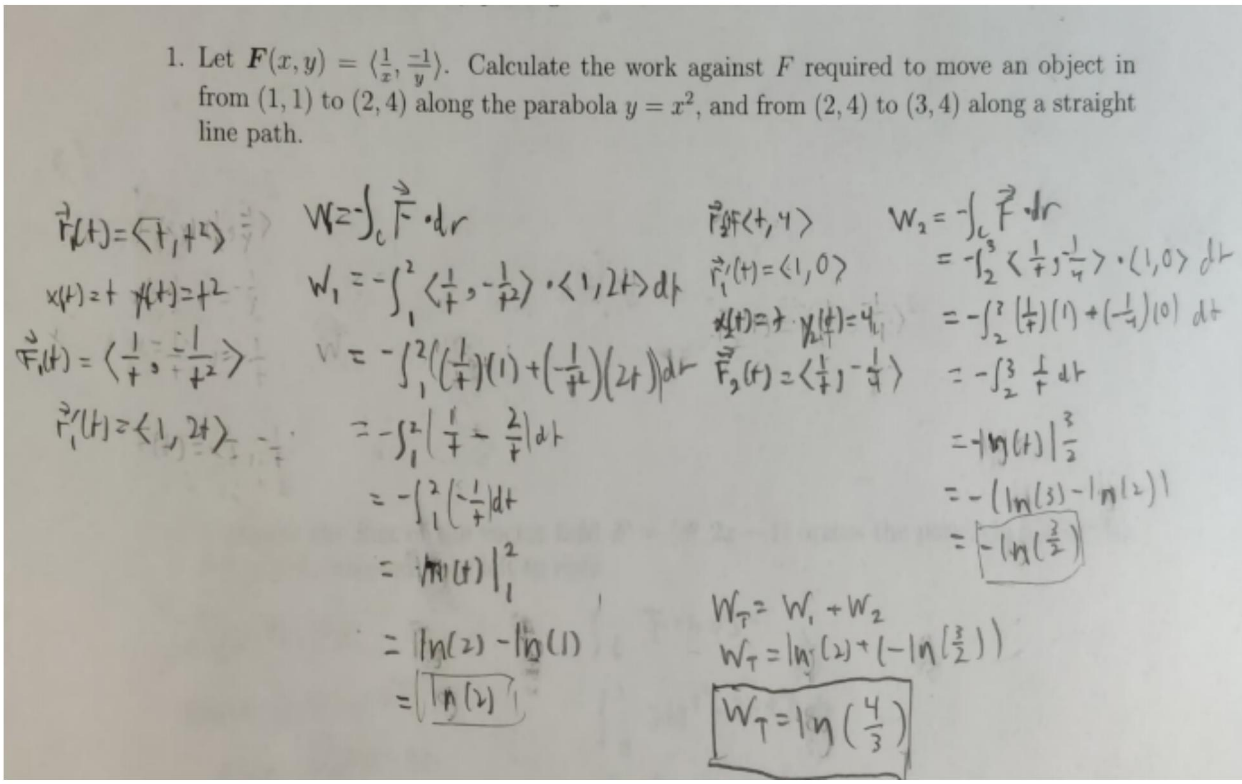

1. Let $\boldsymbol{F}(x,y) = \langle \frac{1}{x}, \frac{-1}{y} \rangle$. Calculate the work against F required to move an object in from $(1,1)$ to $(2,4)$ along the parabola $y = x^2$, and from $(2,4)$ to $(3,4)$ along a straight line path.

$\vec{r}_1(t) = \langle t, t^2 \rangle$

$x(t) = t \quad y(t) = t^2$

$\vec{F}_1(t) = \langle \frac{1}{t}, -\frac{1}{t^2} \rangle$

$\vec{r}_1'(t) = \langle 1, 2t \rangle$

$W = -\int_C \vec{F} \cdot dr$

$W_1 = -\int_1^2 \langle \frac{1}{t}, -\frac{1}{t^2} \rangle \cdot \langle 1, 2t \rangle \, dt$

$= -\int_1^2 \left((\frac{1}{t})(1) + (-\frac{1}{t^2})(2t) \right) dt$

$= -\int_1^2 \left(\frac{1}{t} - \frac{2}{t} \right) dt$

$= -\int_1^2 \left(-\frac{1}{t} \right) dt$

$= \ln(t) \Big|_1^2$

$= \ln(2) - \ln(1)$

$= \boxed{\ln(2)}$

$\vec{r}_2(t) = \langle t, 4 \rangle$

$\vec{r}_2'(t) = \langle 1, 0 \rangle$

$x(t) = t \quad y(t) = 4$

$\vec{F}_2(t) = \langle \frac{1}{t}, -\frac{1}{4} \rangle$

$W_2 = -\int_C \vec{F} \cdot dr$

$= -\int_2^3 \langle \frac{1}{t}, -\frac{1}{4} \rangle \cdot \langle 1, 0 \rangle \, dt$

$= -\int_2^3 (\frac{1}{t})(1) + (-\frac{1}{4})(0) \, dt$

$= -\int_2^3 \frac{1}{t} \, dt$

$= -\ln(t) \Big|_2^3$

$= -(\ln(3) - \ln(2))$

$= \boxed{-\ln(\frac{3}{2})}$

$W_T = W_1 + W_2$

$W_T = \ln(2) + (-\ln(\frac{3}{2}))$

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$$\text{div}(G) = \langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \rangle \cdot \langle \frac{y}{x}, \frac{y}{z}, \frac{z}{x} \rangle$$

$$= \frac{-y}{x^2} + \frac{1}{z} + \frac{1}{x} \Big|_{(5,2,4)} = \frac{-2}{25} + \frac{1}{4} + \frac{1}{5} = \boxed{\frac{37}{100}}$$

$$\text{curl}(G) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{y}{x} & \frac{y}{z} & \frac{z}{x} \end{vmatrix} = \hat{i}\left(0 - \frac{-y}{z^2}\right) - \hat{j}\left(\frac{-z}{x^2} - 0\right) + \hat{k}\left(0 - \frac{1}{x}\right) = \langle \frac{y}{z^2}, \frac{z}{x^2}, -\frac{1}{x} \rangle$$

$$= \boxed{\langle \frac{1}{8}, \frac{4}{25}, \frac{-1}{5} \rangle}$$

2 2 25 / 25

- ✓ **- 0 pts** Correct
- **- 2 pts** Minor calculation error
- **- 5 pts** Did not plug in point P

3. $r(t) = \langle t, t^2 \rangle$, $0 \leq t \leq 1$

3. $y = x^2$ $\quad 0 \leq x \leq 1$ $\quad$ left to right $(0,0)$ to $(1,1)$

Parametrization: $\boxed{\begin{array}{c} \langle t, t^2 \rangle \\ 0 \leq t \leq 1 \end{array}}$

Bounds of t:

$(0,0)$

$0 = t \qquad 0 = t^2$

$\qquad\qquad 0 = t$

$(1,1)$

$1 = t \qquad 1 = t^2$

$\qquad\qquad 1 = t$

$0 \leq t \leq 1$

4. flux = e - 1

4. $$\int_C F \cdot n \, ds = \int_0^1 \langle e^{t^2}, 2t-1 \rangle \cdot N(t) \, dt$$

$$N(t) = \langle y'(t), -x'(t) \rangle$$
$$= \langle 2t, -1 \rangle$$

$$= \int_0^1 \langle e^{t^2}, 2t-1 \rangle \cdot \langle 2t, -1 \rangle \, dt$$

$$= \int_0^1 2te^{t^2} - 2t + 1 \, dt$$

$$= e^{t^2} - t^2 + t \Big|_0^1$$

$$= e - 1 + 1 - 1 + 0 - 0$$

$$= \boxed{e - 1}$$

3 3 25 / 25

- ✓ **- 0 pts** Correct
- **- 5 pts** Missing domain of t.

3. r(t) = <t, t²>, $0 \leq t \leq 1$

3. $y = x^2$ $\quad 0 \leq x \leq 1$ $\quad$ left to right $(0,0)$ to $(1,1)$

Parametrization: $\boxed{\langle t, t^2 \rangle \\ 0 \leq t \leq 1}$

Bounds of t:

$(0,0)$

$0 = t \quad 0 = t^2$

$0 = t$

$(1,1)$

$1 = t \quad 1 = t^2$

$1 = t$

$0 \leq t \leq 1$

4. flux = e - 1

4. $\int_C F \cdot n\, ds = \int_0^1 \langle e^{t^2}, 2t-1 \rangle \cdot N(t)\, dt$

$N(t) = \langle y'(t), -x'(t) \rangle$

$= \langle 2t, -1 \rangle$

$= \int_0^1 \langle e^{t^2}, 2t-1 \rangle \cdot \langle 2t, -1 \rangle\, dt$

$= \int_0^1 2te^{t^2} - 2t + 1\, dt$

$= e^{t^2} - t^2 + t \Big|_0^1$

$= e - 1 + 1 - 1 + 0 - 0$

$= \boxed{e - 1}$

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