Name: ____________________ **UID:** ____________________
TA/Section: ____________________

Instructions:

- Read each problem carefully.
- Show all work clearly, and clearly denote your answer by putting a box around it.
- Justify your answers. **A correct final answer without valid reasoning will not receive credit.**
- You are permitted to use your notes, textbooks, computers, and calculators on this exam.
- **You are not allowed to collaborate or use human resources** (including but not limited to Chegg, Math Stack Exchange, etc.).

Question	Possible Points	Score
1	25	
2	25	
3	25	
4	25	
5	25	
6	25	
7	25	
8	25	
Total:	200	

1. Let S be the diamond in the xy-plane with vertices $A = (\pi, 0)$, $B = (2\pi, \pi)$, $C = (\pi, 2\pi)$, $D = (0, \pi)$, and consider the change of variables

$$G(u, v) = \left\langle \frac{u+v}{2}, \frac{u-v}{2} \right\rangle$$

(a) (7 points) Describe, as a set, the region S_0 in the uv-plane that G **maps to** S.

(b) (8 points) Compute the determinant of the Jacobian matrix, $\text{Jac}(G)$.

(c) (10 points) Use your work from the previous parts to compute

$$\iint_S (x-y)^2 \sin^2(x+y)\ dx\ dy$$

2. Let D be the region in the first quadrant inside the circle of radius 1, and bounded by the polar spiral $r = \theta$ and the y-axis.

   (a) (5 points) Sketch the region D.

   (b) (10 points) Express the region D as a (union of) radially simple region(s).

   (c) (10 points) Use your work from the previous parts to compute

$$\iint_D \sqrt{x^2 + y^2}\, dA$$

3. The velocity vector field of a fluid is given by

$$\boldsymbol{v} = \langle 0, x^2 + y^2, z^2 \rangle$$

(a) (10 points) Find the flow rate of $\boldsymbol{v}$ in the **negative** z-direction through the disk

$$D = \{(x, y, 0) \mid x^2 + y^2 \leq 1\}$$

(b) (15 points) Find the flow rate of $\boldsymbol{v}$ in the **positive** z-direction through the hemisphere

$$S = \{(x, y, z) \mid x^2 + y^2 + z^2 = 1, z \geq 0\}$$

4. (a) (5 points) Let W be a solid region in $\mathbb{R}^3$ bounded by the paraboloids $z = 2x^2 + 2y^2$ and $z = x^2 + y^2 + 4$. Sketch W.

   (b) (10 points) Let W be a solid region in $\mathbb{R}^3$ bounded by the paraboloids $z = 2x^2 + 2y^2$ and $z = x^2 + y^2 + 4$. Find the volume of W.

   (c) (10 points) Let ∂W denote the boundary of W, oriented by outward-pointing normal vectors. Compute

$$\iint_{\partial W} \left\langle x + xy + \sin(z), x^3 + 3y - \frac{y^2}{2}, 4z + \cos(y) \right\rangle \cdot d\boldsymbol{S}$$

5. (a) (10 points) Let D be the region in $\mathbb{R}^2$ given by

$$D = \{(x, y) \mid 0 \leq x \leq 1, x^{2/3} \leq y \leq 1\}$$

Let ∂D denote the boundary of D, with the boundary orientation. Use Green's theorem to rewrite the following circulation integral as a double integral over D.

$$\oint_{\partial D} \left\langle x^3 - 3y\sin(x) - y, 3\cos(x) + 2x + \frac{x^2 e^{y^4}}{2} \right\rangle \cdot d\boldsymbol{r}$$

(b) (15 points) Use your work from the previous part to compute

$$\oint_{\partial D} \left\langle x^3 - 3y\sin(x) - y, 3\cos(x) + 2x + \frac{x^2 e^{y^4}}{2} \right\rangle \cdot d\boldsymbol{r}$$

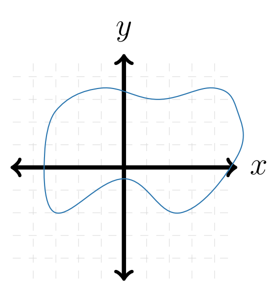
6. Consider the vector field on $\mathbb{R}^2 - \{(0,0)\}$ given by

$$\boldsymbol{F} = \left\langle \frac{x}{x^2+y^2}, \frac{y}{x^2+y^2} \right\rangle$$

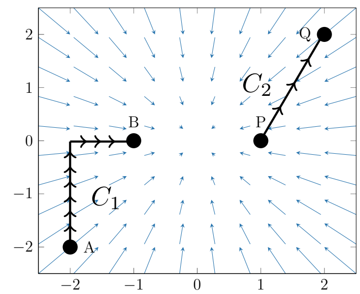
(a) (5 points) Compute $\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}$.

(b) (5 points) Let C be the circle of radius R centered at the origin, oriented counter-clockwise. Compute $\int_C \boldsymbol{F} \cdot d\boldsymbol{r}$.

(c) (15 points) Let Q be the closed curve below, oriented counterclockwise. Compute $\int_Q \boldsymbol{F} \cdot d\boldsymbol{r}$.



7. **You must explain your reasoning.** Consider the following radial vector field $\boldsymbol{F}$:



(a) (9 points) Is $\boldsymbol{F}$ a conservative vector field?

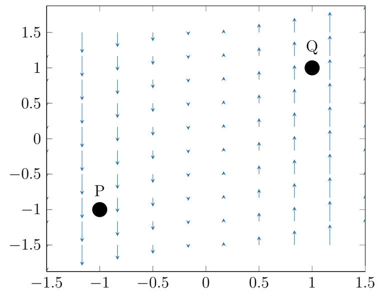
(b) (8 points) Suppose that for the two curves C_1 (left) and C_2 (right), we have

$$\int_{C_1} \boldsymbol{F} \cdot d\boldsymbol{r} = 5$$

What is $\int_{C_2} \boldsymbol{F} \cdot d\boldsymbol{r}$?

(c) (8 points) Which is greater, $\int_{C_1} (\boldsymbol{F} \cdot \boldsymbol{n})\, ds$ or $\int_{C_2} (\boldsymbol{F} \cdot \boldsymbol{n})\, ds$? Or are they equal?

8. **You must explain your reasoning.** Consider the following vector field $\boldsymbol{F}$:



(a) (9 points) Is $\boldsymbol{F}$ a conservative vector field?

(b) (8 points) Is divergence of $\boldsymbol{F}$ at $P = (-1, -1)$ positive, negative, or zero?

(c) (8 points) Is $\text{curl}_z\boldsymbol{F}$ at $Q = (1, 1)$ positive, negative, or zero?