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Midterm I – Mathematics 32B/1

February 14, 2014

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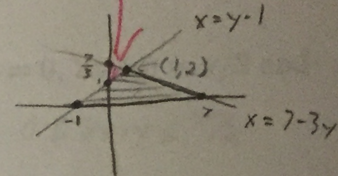
4

1. (10 pts.) Write $\int\int_D f(x,y)dA$ as an iterated integral when D is the plane domain bounded by the lines $x + 3y = 7$, $x - y = -1$ and $x = 0$. Sketching the domain will help here.

$x = 7-3y \quad x = y-1$



$7-3y = y-1$

$4y = 8$

$\left.\begin{array}{l} y=2 \\ x=1 \end{array}\right\}$ intersection

$$\iint_D f(x,y)\,dA = \int_0^2 \int_{y-1}^{7-3y} f(x,y)\,dx\,dy$$

2. (10 pts.) Make an appropriate change of variables and evaluate

$$\int\int_D \cos(2x+y)dA$$

when D is the parallelogram bounded by $2x-y=\pi$, $2x-y=0$, $2x+y=\pi/2$ and $2x+y=0$.

$0 \le 2x-y \le \pi \qquad 0 \le 2x+y \le \pi/2$

$u = 2x-y \qquad v = 2x+y$

$\left.\begin{array}{l} u-v=-2y \\ u+v=4x \end{array}\right\} \quad x=\frac{u+v}{4} \qquad y=\frac{v-u}{2}$

$0 \le u \le \pi \qquad 0 \le v \le \pi/2$

$$\frac{\partial(x,y)}{\partial(u,v)} = \left(\tfrac{1}{4}\right)\left(\tfrac{1}{2}\right)-\left(\tfrac{1}{4}\right)\left(-\tfrac{1}{2}\right) = \frac{1}{4}$$

$$\iint_D \cos(2x+y)dA = \int_0^{\pi}\int_0^{\pi/2} \cos\left(\frac{u+v}{2}+\frac{v-u}{2}\right)\left|\frac{1}{4}\right| dv\,du$$

$$= \frac{1}{4}\int_0^{\pi}\int_0^{\pi/2} \cos(v)\,dv\,du$$

$$= \frac{1}{4}\int_0^{\pi} \sin(v)\Big|_{v=0}^{v=\pi/2} du$$

$$= \frac{1}{4}\int_0^{\pi}(1-0)\,du = \pi/4$$

10

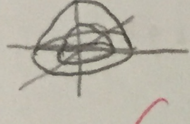
10

3. (10 pts.) Find the centroid of the spherical half shell defined by $c^2 \le x^2+y^2+z^2 \le 1$ and $z \ge 0$, where $0 < c < 1$. [In high school you learned the that volume of this shell is $(2\pi/3)(1-c^3)$, and you can assume that here.]

$c \le \rho \le 1$

$0 \le \phi \le \pi/2$

$0 \le \theta \le 2\pi$



By Symmetry, $\bar{x} = \bar{y} = 0$ ✓

$$\bar{z} = \frac{\iiint_W z\, dV}{\iiint_W 1\, dV} = \frac{\iiint_W z\, dV}{\frac{2\pi}{3}(1-c^3)} \checkmark$$

$$\iiint_W z\, dV = \int_0^{2\pi}\int_0^{\pi/2}\int_c^1 \rho\cos\phi\ \rho^2\sin\phi\ d\rho\, d\phi\, d\theta \checkmark$$

ρ^3

$$= \int_0^{2\pi}\int_0^{\pi/2} \cos\phi\sin\phi\ \tfrac{1}{4}\rho^4\Big|_{\rho=c}^{\rho=1} d\phi\, d\theta \checkmark$$

$$= \frac{2\pi}{4}\int_0^{\pi/2} \cos\phi\sin\phi\,(1^4-c^4)\, d\phi$$

$$= \frac{\pi}{2}(1-c^4)\int_0^{\pi/2} \cos\phi\sin\phi\, d\phi$$

$u = \sin\phi \qquad 0 \to 0$

$du = \cos\phi\, d\phi \qquad \frac{\pi}{2} \to 1$

$$= \frac{\pi}{2}(1-c^4)\int_0^1 u\, du = \frac{\pi}{4}(1-c^4)$$

$$\bar{z} = \frac{\frac{\pi}{4}(1-c^4)}{\frac{2\pi}{3}(1-c^3)} = \frac{3}{8}\frac{(1-c^4)}{(1-c^3)} \checkmark$$

$\frac{\pi}{4}\cdot\frac{3}{2\pi}$

Centroid $= \left(0,\ 0,\ \frac{3}{8}\frac{(1-c^4)}{(1-c^3)}\right)$ ✓

$y=x \quad y=\sqrt{4-x^2} \quad 4=x^2+y^2$

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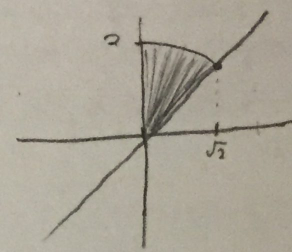
4. (10 pts.) Use polar coordinates to evaluate $\int_0^{\sqrt{2}}\left(\int_x^{\sqrt{4-x^2}} e^{x^2+y^2}\,dy\right)dx$. Sketching the domain will help here.

$\pi/4 \le \theta \le \pi/2$

$0 \le r \le 2$



$$\int_{\pi/4}^{\pi/2}\int_0^2 e^{r^2}\, r\,dr\,d\theta$$ ✓

$u=r^2$ $\qquad$ $\overset{r}{0}\to\overset{u}{0}$

$du=2r\,dr$ $\qquad$ $2\to4$

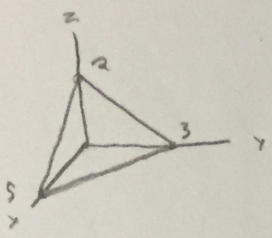
$$=\frac{1}{2}\int_{\pi/4}^{\pi/2}\int_0^4 e^u\,du\,d\theta$$ ✓

$$=\frac{1}{2}\int_{\pi/4}^{\pi/2} e^4-1\,d\theta$$

$$=\frac{1}{2}\left(\frac{\pi}{2}-\frac{\pi}{4}\right)(e^4-1)=\frac{\pi}{8}(e^4-1)$$ ✓

10

5. (10 pts.) Let $\mathcal{W}$ be the region in the first octant ($x \geq 0,\ y \geq 0,\ z \geq 0$) cut off by the plane through $(5,0,0)$, $(0,3,0)$ and $(0,0,2)$. That plane has the equation $x/5+y/3+z/2=1$. Write $\int\int\int_{\mathcal{W}} g(x,y,z)dV$ as an iterated integral in both of the forms $\int_a^b(\int_c^d(\int_e^f g(x,y,z)dz)dy)dx$ and $\int_a^b(\int_c^d(\int_e^f g(x,y,z)dx)dy)dz$ – with different choices of a, b, c, d, e and f, of course.



$\frac{x}{5}+\frac{y}{3}+\frac{z}{2}=1$ $\qquad z=2-\frac{2x}{5}-\frac{2y}{3}$ $\qquad y=3-\frac{3}{5}x$

$x=5-\frac{5}{3}y-\frac{5}{2}z$ $\qquad y=3-\frac{3}{2}z$

$$\iiint_W g\,dV = \int_0^5\int_0^{3-\frac{3}{5}x}\int_0^{2-\frac{2x}{5}-\frac{2y}{3}} g(x,y,z)\,dz\,dy\,dx$$ ✓

$$\iiint_W g\,dV = \int_0^2\int_0^{3-\frac{3}{2}z}\int_0^{5-\frac{5}{3}y-\frac{5}{2}z} g(x,y,z)\,dx\,dy\,dz$$ ✓