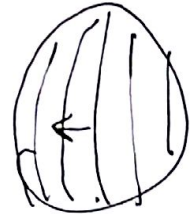


MATH 32B  
FIRST MIDTERM EXAMINATION



Please show your work. You will receive little or no credit for a correct answer to a problem which is not accompanied by sufficient explanations. If you have a question about any particular problem, please raise your hand and one of the proctors will come and talk to you. At the completion of the exam, please hand the exam booklet to your TA. If you have any questions about the grading of the exam, please see the instructor *within 15 calendar days of the examination*.

Name: JAI SRIVASTAV *vi finta* Section: 3C  
604-605-757

#1	#2	#3	#4	#5	Total
10	10	10	10	10	50

*well done!*

10

2

## 32B MIDTERM 1 RADKO

Problem 1. Evaluate the integral

$$\int_0^{\frac{\pi}{2}} \int_0^{\frac{2}{\pi}} xy \sin(x^2 y) dy dx = \int_0^{\frac{2}{\pi}} \int_0^{\frac{\pi}{2}} xy \sin(x^2 y) dx dy$$

$$\frac{d}{dx}(\cos(x^2 y)) = -2xy \sin(x^2 y)$$

using Pabini's thm.

$$\int_0^{\frac{2}{\pi}} \left[ -\frac{1}{2} \cos(x^2 y) \right]_0^{\frac{\pi}{2}} dy = -\frac{1}{2} \int_0^{\frac{2}{\pi}} \cos\left(\frac{\pi^2}{4} y\right) dy$$

$$-\frac{1}{2} \left[ \frac{4}{\pi^2} \sin\left(\frac{\pi^2}{4} y\right) - y \right]_0^{\frac{2}{\pi}} = -\frac{1}{2} \left( \frac{4}{\pi^2} \cdot 1 - \frac{2}{\pi} \right) =$$

$$\boxed{-\frac{2}{\pi^2} + \frac{1}{\pi}}$$

Problem 2. Use a triple integral of the form

$$\int_a^b \int_{g_1(x)}^{g_2(x)} \int_{h_1(x,y)}^{h_2(x,y)} f(x,y,z) dz dy dx$$

to find the volume of the solid bounded by the coordinate planes and the plane going through the points  $(1, 0, 0)$ ,  $(0, 1, 0)$  and  $(0, 0, 2)$ .

To find volume  $f(x,y,z) = 1$

$$0 \leq x \leq 1$$

$$0 \leq y \leq 1-x$$

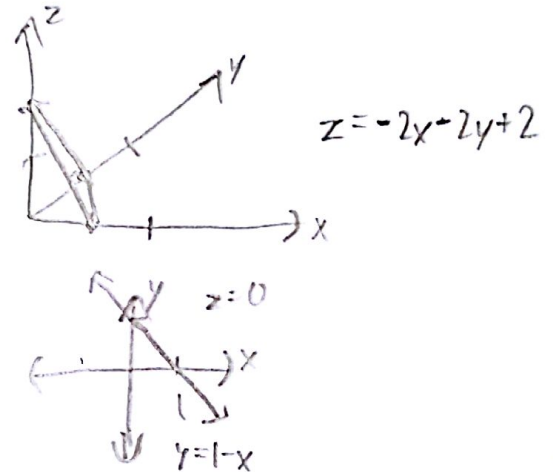
$$0 \leq z \leq 2-2x-2y$$

$$\int_0^1 \int_0^{1-x} \int_0^{2-2x-2y} dz dy dx =$$

$$\int_0^1 \int_0^{1-x} (2-2x-2y) dy dx =$$

$$\int_0^1 [y(2-2x)-y^2]_0^{1-x} dx = \int_0^1 (1-x)(2-2x) - (1-x)^2 dx = \int_0^1 2(1-x)^2 - (1-x)^2 dx =$$

$$\int_0^1 (1-x)^2 dx = \int_0^1 x^2 - 2x + 1 dx = \left[ \frac{x^3}{3} - x^2 + x \right]_0^1 = \frac{1}{3} - 1 + 1 = \boxed{\frac{1}{3}}$$



Problem 3. Use integration in polar coordinates to find the area of the region that lies inside of the circle  $(x-1)^2 + y^2 = 1$  and outside of the circle  $x^2 + y^2 = 1$ .

$$(x-1)^2 + y^2 = 1$$

$$x^2 - 2x + 1 + y^2 = 1$$

$$(x^2 + y^2) = 2x$$

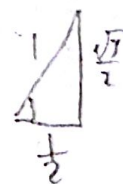
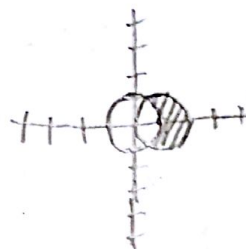
$$r^2 = 2r \cos \theta$$

$$r = 2 \cos \theta$$

$$x^2 + y^2 = 1$$

$$r^2 = 1$$

$$r = 1$$



$$r = 2 \cos \theta = 1$$

$$\cos \theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{3} \text{ or } -\frac{\pi}{3}$$

$$-\frac{\pi}{3} \leq \theta \leq \frac{\pi}{3}, 1 \leq r \leq 2 \cos \theta$$

Area of region is the same as volume of region with height 1

$$f(r, \theta) = 1$$

$$\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \int_1^{2 \cos \theta} r \, dr \, d\theta = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \left[ \frac{r^2}{2} \right]_1^{2 \cos \theta} d\theta = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} 2 \cos^2 \theta - \frac{1}{2} d\theta =$$

$$\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} 2 \left( \frac{1}{2} (1 + \cos(2\theta)) \right) - \frac{1}{2} d\theta = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} 1 + \cos(2\theta) - \frac{1}{2} d\theta = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{1}{2} + \cos(2\theta) d\theta =$$

$$\left[ \frac{1}{2} \theta + \frac{1}{2} \sin(2\theta) \right]_{-\frac{\pi}{3}}^{\frac{\pi}{3}} = \boxed{\frac{\pi}{3} + \frac{\sqrt{3}}{2}}$$

10

**Problem 4.** Set up (but do not evaluate) triple integral in cylindrical coordinates that represents the volume of the region bounded by the  $xy$ -plane and the surfaces

$$z = 1 - (x^2 + y^2) = 1 - r^2$$

$$x = x^2 + y^2$$

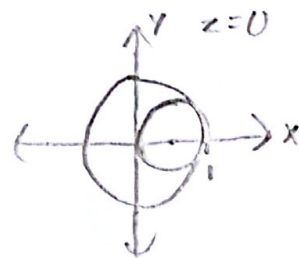
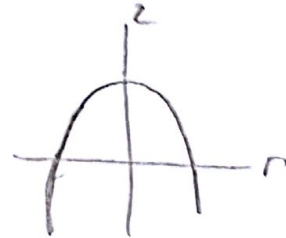
$$x^2 - x + \frac{1}{4} + y^2 = \frac{1}{4}$$

$$\left(x - \frac{1}{2}\right)^2 + y^2 = \frac{1}{4}$$

$$x^2 + y^2 = x$$

$$r^2 = r \cos \theta$$

$$r = \cos \theta$$



$$\int_{-\pi/2}^{\pi/2} \int_0^{\cos \theta} \int_0^{1-r^2} r dz dr d\theta$$

$$-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$$

$$0 \leq r \leq \cos \theta$$

$$0 \leq z \leq 1 - r^2$$



## 32B MIDTERM 1 RADKO

**Problem 5.** Set up (but do not evaluate) triple integral in spherical coordinates representing the volume of the region lying in the first octant (i.e.,  $x \geq 0, y \geq 0, z \geq 0$ ) and

- above the surface
- below the surface
- between the planes

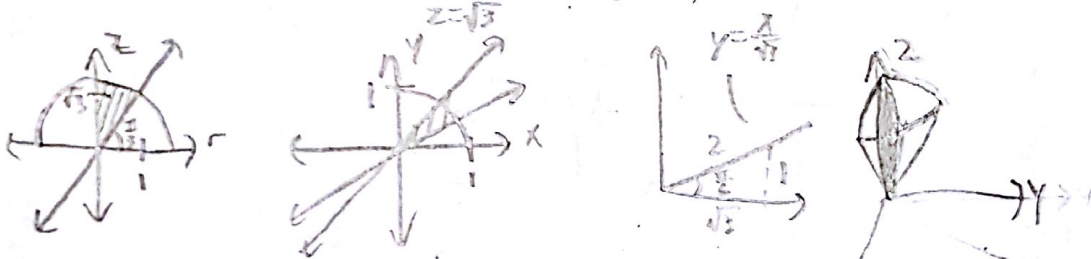
$$z = \sqrt{3(x^2 + y^2)} = \sqrt{3}r = \sqrt{3}r$$

$$z = \sqrt{4 - (x^2 + y^2)} = \sqrt{4 - r^2}$$

$$y = \frac{x}{\sqrt{3}}$$

$$y = x$$

(Hint: Draw pictures on several coordinate planes).



$$\begin{aligned} z &= \sqrt{3}r = \sqrt{4-r^2} \\ 3r^2 &= 4-r^2 \\ 4r^2 &= 4 \\ r &= 1 \end{aligned}$$

$$\begin{aligned} \frac{\pi}{6} &\leq \theta \leq \frac{\pi}{4} \\ 0 &\leq \phi \leq \frac{\pi}{6} \\ 0 &\leq \rho \leq 2 \end{aligned}$$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \int_0^{\frac{\pi}{6}} \int_0^2 \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta$$