Course 32 B Sec. 1 UCLA Department of Mathematics

Spring 2015 (Iza) Instructor: Oleg Gleizer

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Pr 1	Pr 2	Pr 3	Pr 4	Pr 5	Pr 6	Pr 7	Total
0 / 15	20 / 20	15 / 15	20 / 20	15 / 15	15 / 15	6 / 10	85 / 100

Midterm 1

Please print your name and student ID in the designated space at the top of the page. Show your work! Answers unsupported by work yield no credit.

Problem 1

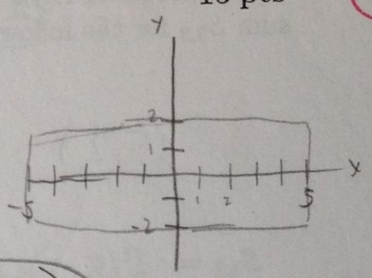
15 pts

(0)

Use the symmetry of the problem to find the integral

$$\iint_D \left(1 + \frac{xy}{x^2+y^2+5}\right) dA$$

over the domain $D = [-5,5] \times [-2,2]$.



$4\left(10 + \frac{1}{4}\int_0^5 \left(\ln|x^2+9| - \ln|x^2+5|\right) dx\right)$

$$\iint_D \left(1 + \frac{xy}{x^2+y^2+5}\right) dA = 4\int_0^5 \int_0^2 \left(1 + \frac{xy}{x^2+y^2+5}\right) dy\, dx$$

$u = x^2+y^2$

$du = 2y\,dy$

1

$\frac{x}{2}\int_0^2 \frac{1}{u+5} du$

$= \frac{x}{2}\ln(u+5)$

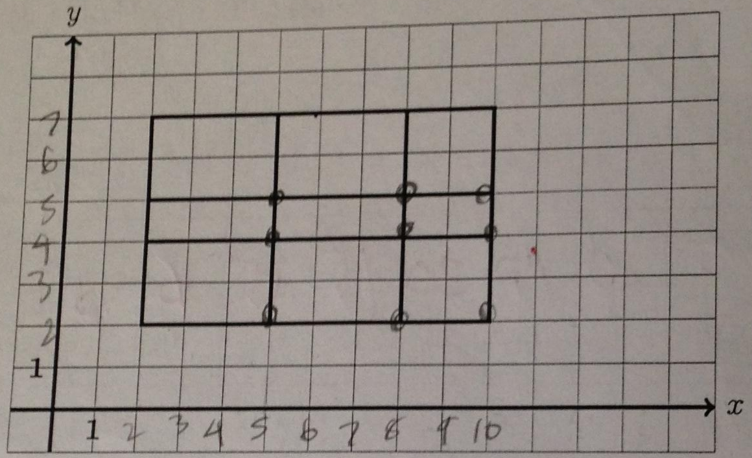
$= 4\int_0^5 \left([y]_0^2 + \int_0^2 \frac{xy}{x^2+y^2+5} dy\right) dx = 4\int_0^5 (2) + \left(\frac{1}{2}\int_0^2 \frac{x}{u+5} du\right) dx$

$= 4\int_0^5 2 + \frac{x}{2}\left[\ln|x^2+y^2+5|\right]_0^2 dx = 4\int_0^5 2 + \frac{x}{2}\left[\ln|x^2+9| - \ln|x^2+5|\right] dx$

20 pts

Problem 2

The domain $D = [2, 10] \times [2, 7]$ is partitioned as shown on the picture below.



- What is the maximal length $||P||$ of the partition? **5 pts**

$||P|| =$ 3

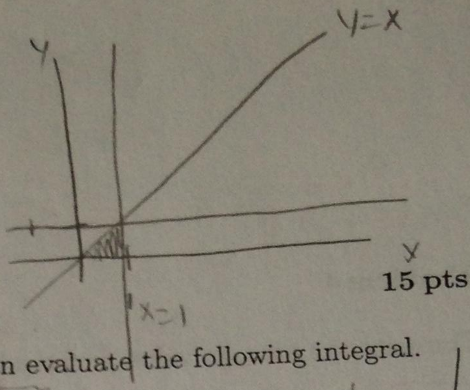
- Use the lower-right vertices of the sub-rectangles of P to find the Riemann sum $S_{3,3}$ for the following integral.

(5,2)

$$\iint_D (2x - 3y)\,dA$$

$S_{3,3} =$ f(5,2)·6 + f(8,2)·6 + f(10,2)·4 + f(5,4)·3 + f(8,4)·3 + f(10,4)·2 + f(5,5)·6 + f(8,5)·6 + f(10,5)·4 **15 pts**

= 24 + 60 + 56 + (−6) + 12 + 16 + (−30) + 6 + 20 = 84 + 56 − 8 + 26 = 158



Problem 3 **15 pts**

Change the order of integration and then evaluate the following integral.

$$\int_0^1 \int_y^1 \frac{\sin x}{x}\, dx dy$$

$y \le x \le 1$, $0 \le y \le 1$ → $0 \le x \le 1$, $0 \le y \le x$

Hint: sketching the domain helps.

$$\int_0^1 \int_0^x \frac{\sin x}{x}\, dy\, dx = \int_0^1 \frac{\sin x}{x} \int_0^x dy\, dx$$

$$= \int_0^1 \frac{\sin x}{x} x\, dx = \int_0^1 \sin x\, dx = -\cos x \Big|_0^1$$

$$= -\cos 1 - (-\cos 0) = -\cos 1 + 1$$

$$= \boxed{1 - \cos 1}$$

$$\int_0^1 \int_y^1 \frac{\sin x}{x}\, dx dy = \boxed{1 - \cos 1}$$

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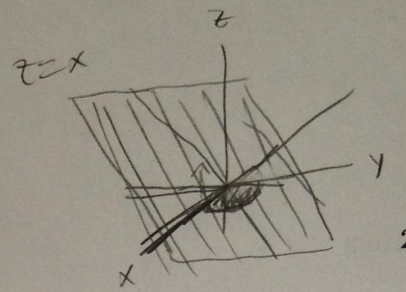
Section 1B
...d Luck! Be sure to justify your a...
...tors, books or notes are allowed.



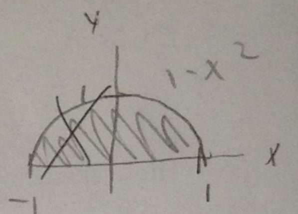
Problem 4 **20 pts**

Compute the average value $\bar{f}$ of the function

$$f(x, y, z) = 6y$$

over the 3D region W described by the following inequalities.

$$W: \ 0 \le y \le 1 - x^2, \ 0 \le z \le x$$



$$0 \le x \le 1$$

$$V = \iiint_W 1 \, dV = \int_0^1 \int_0^{1-x^2} \int_0^x 1 \, dz \, dy \, dx = \int_0^1 \int_0^{1-x^2} x \, dy \, dx$$

$$= \int_0^1 x(1-x^2) \, dx = \int_0^1 (x - x^3) \, dx = \left(\frac{x^2}{2} - \frac{x^4}{4} \right)_0^1$$

$$= \left(\frac{1}{2} - \frac{1}{4} - \left(\frac{0}{2} - \frac{0}{4} \right) \right) = \frac{1}{2} - \frac{1}{4} = \frac{4-2}{8} = \frac{2}{8} = \frac{1}{4}$$

$$\bar{f} = \frac{\iiint_W f(x,y,z) \, dV}{V} = 4 \int_0^1 \int_0^{1-x^2} \int_0^x 6y \, dz \, dy \, dx$$

$$= 24 \int_0^1 \int_0^{1-x^2} y \int_0^x dz \, dy \, dx = 24 \int_0^1 x \int_0^{1-x^2} y \, dy \, dx = 24 \int_0^1 x \left(\frac{y^2}{2} \right)_0^{1-x^2} dx$$

$\bar{f} =$

$$= 24 \int_0^1 x \left(\frac{(1-x^2)^2}{2} \right) dx = 12 \int_0^1 (x - 2x^3 + x^5) \, dx$$

$$\boxed{\bar{f} = 2}$$

$$= 12 \left(\frac{x^2}{2} - \frac{2x^4}{4} + \frac{x^6}{6} \right)_0^1 = 12 \left(\frac{1}{2} - \frac{1}{2} + \frac{1}{6} - (0) \right) = \frac{12}{6} = \boxed{2}$$

20

$x = r\cos\theta$
$x^2 = r^2\cos^2\theta$

15 pts

Problem 5

Sketch the curve given by the following equation.

$$r^2 = \frac{1}{\cos 2\theta}$$

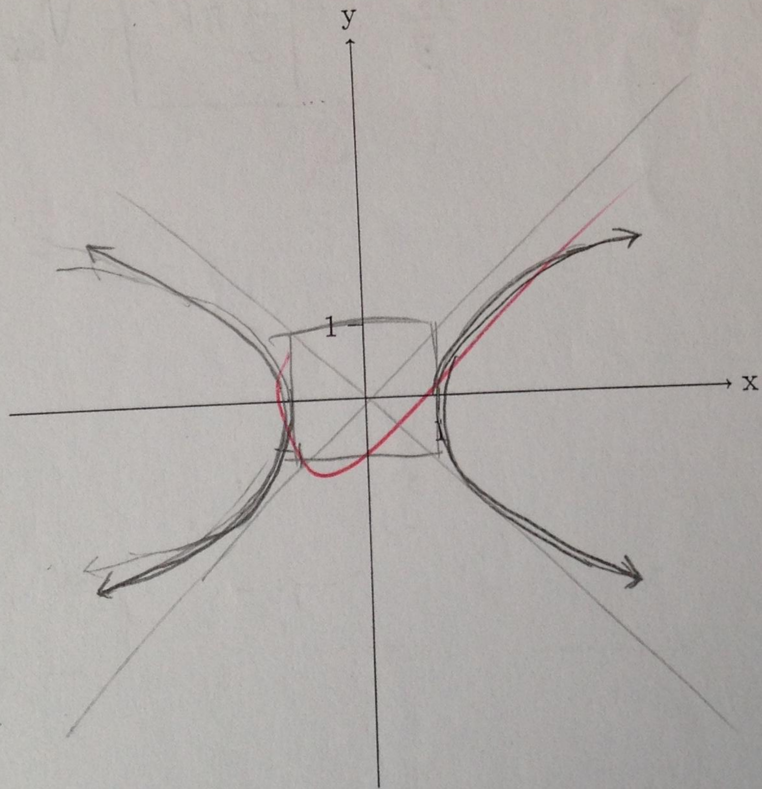
$r^2 = \frac{1}{\cos^2\theta - \sin^2\theta}$

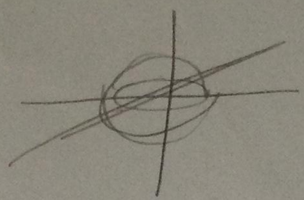
Hint: use the identity $\cos 2\theta = \cos^2\theta - \sin^2\theta$ to switch to rectangular coordinates.

hyperbola

$r^2\cos^2\theta - r^2\sin^2\theta = 1$

$x^2 - y^2 = 1$





$x^2+y^2+z^2=R^2$

$\rho^2=R^2$

$0\le\rho\le R$

15 pts

Problem 6

Use spherical coordinates to find the volume of a 3D sphere of radius R. The answer without the derivation yields no credit.

$dV=\rho^2\sin\varphi$ $\quad 0\le\varphi\le\pi \quad 0\le\rho\le R \quad 0\le\theta\le 2\pi$

$$V=\iiint_W 1\,dV=\int_0^{2\pi}\int_0^{\pi}\int_0^{R}\rho^2\sin\varphi\,d\rho\,d\varphi\,d\theta=\int_0^{2\pi}d\theta\cdot\int_0^{\pi}\sin\varphi\,d\varphi\cdot\int_0^{R}\rho^2\,d\rho$$

$$=2\pi\cdot(-\cos\varphi)\Big|_0^{\pi}\cdot\left(\frac{\rho^3}{3}\right)\Big|_0^{R}=2\pi\left(-\cos\pi-(-\cos 0)\right)\cdot\left(\frac{R^3}{3}\right)$$

$$=2\pi(1+1)\cdot\frac{R^3}{3}=4\pi\cdot\frac{R^3}{3}=\boxed{\frac{4}{3}\pi R^3}=V_{sphere}$$

✓

Problem 7 — Extra credit! **10 pts**

Find the volume V_4 of a 4-dimensional sphere of radius R. The answer without the derivation yields no credit.

$$\iiint_W \cdot \rho^2 \sin\varphi \, d\rho \, d\varphi \, d\theta = \int_0^{2\pi}\int_0^{\pi}\int_0^{R} \rho^4 \sin\varphi \, d\rho \, d\varphi \, d\theta$$

$$= \cdot 2\pi \left(-\cos\varphi\right)_0^{\pi} \cdot \left(\frac{\rho^5}{5}\right)_0^{r} = 4\pi\left(\frac{R^5}{5}\right) = \boxed{\frac{4}{5}\pi R^5}$$

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