Course 32 B Sec. 1

UCLA Department of Mathematics

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Pr 1	Pr 2	Pr 3	Pr 4	Pr 5	Pr 6	Pr 7	Total
3/15	20/20	15/15	20/20	15/15	15/15	10/10	98/100

Midterm 1

Please print your name and student ID in the designated space at the top of the page. Show your work! Answers unsupported by work yield no credit.

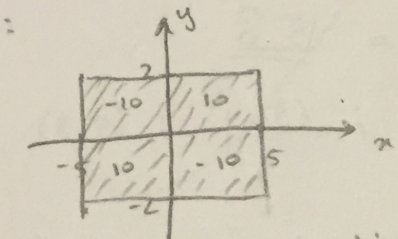
Problem 1

15 pts 3

Use the symmetry of the problem to find the integral

$$\iint_D \left(1 + \frac{xy}{x^2+y^2+5}\right) dA$$

over the domain $D = [-5,5] \times [-2,2]$. D:



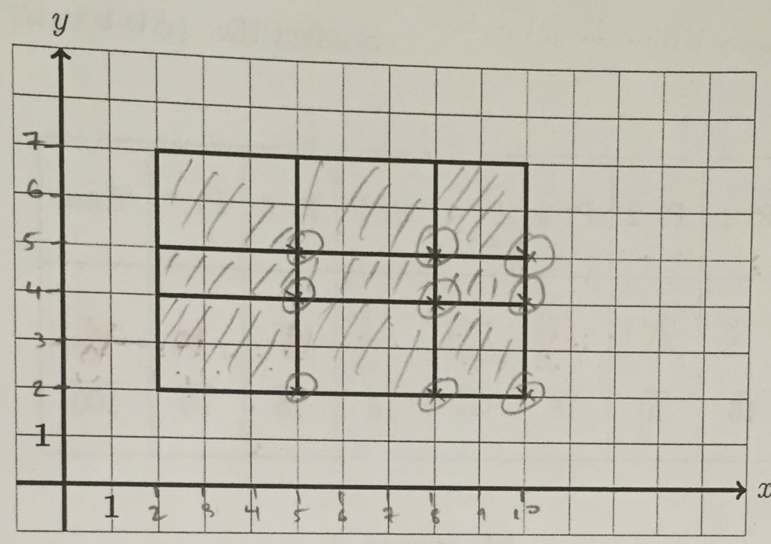
since the domain is symmetric, we have
-10 + 10 - 10 + 1
= 0

$$\iint_D \left(1 + \frac{xy}{x^2+y^2+5}\right) dA = \underline{\underline{0}}$$

Problem 2

20 pts

The domain $D = [2, 10] \times [2, 7]$ is partitioned as shown on the picture below.



- What is the maximal length $||P||$ of the partition? **5 pts**

$||P|| =$ 3

- Use the lower-right vertices of the sub-rectangles of P to find the Riemann sum $S_{3,3}$ for the following integral.

$\sum_{i=1}^{N} \sum_{j=1}^{M} f(i,j) \cdot \Delta A_{ij}$

$$\iint_D (2x - 3y)dA$$

15 pts

$S_{3,3} =$ 158

(5,2) ; (5,4) ; (5,5) ; (8,2) ; (8,4) ; (8,5); (10,2); (10,4), (1

$A_1 = 6$; $A_2 = 3$; $A_3 = 6$; $A_4 = 6$; $A_5 = 3$; $A_6 = 6$; $A_7 = 4$; $A_8 = 2$; A_9

$f(P_{ij}) = 4$; $f(P_{ij}) = -2$; $f(P_{ij}) = -5$; $f(P_{ij}) = 10$; $f(P_{ij}) = 4$; $f(P_{ij}) = 1$; $f(P_{ij}) = 14$; $f(P_{ij}) = 8$; $f(P_{ij})$

$\therefore S_{3,3} = (6\times4) + (3\times-2) + (6\times-5) + (6\times10) + (3\times4) + (6\times1) + (4\times14) + (2\times8) + (4\times5)$

$= 158$

Problem 3

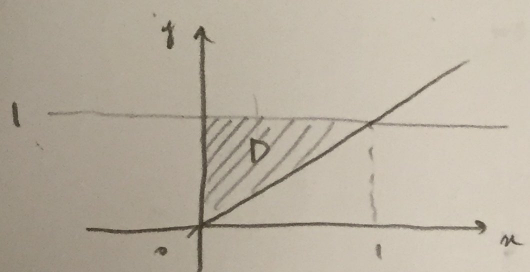
15 pts

Change the order of integration and then evaluate the following integral.

$$\int_0^1 \int_y^1 \frac{\sin x}{x}\, dx dy$$

Hint: sketching the domain helps.

$0 \leq x \leq y \quad ; \quad 0 \leq y \leq 1$



This domain can be expressed as

$0 \leq x \leq 1$

$0 \leq y \leq x$

$$\Rightarrow \int_0^1 \int_0^x \frac{\sin x}{x}\, dy dx$$

$$\Rightarrow \int_0^1 \left(y \cdot \frac{\sin x}{x} \Big|_0^x \right) dx$$

$$\Rightarrow \int_0^1 \sin x \, dx = [-\cos x]_0^1 = -\cos(1) + \cos(0)$$

$$= \underline{\underline{1 - \cos(1)}}$$

$$\int_0^1 \int_y^1 \frac{\sin x}{x}\, dx dy = \underline{\underline{1 - \cos(1)}}$$

Problem 4

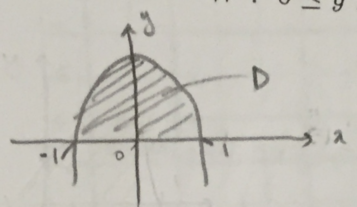
20 pts

Compute the average value $\bar{f}$ of the function

$$f(x, y, z) = 6y$$

over the 3D region W described by the following inequalities.

$$W:\ 0 \leq y \leq 1 - x^2,\ 0 \leq z \leq x$$



$\therefore\ -1 \leq x \leq 1$

$\therefore\ \bar{f} = \frac{1}{V} \iiint_W f(x,y,z)\,dV$

$V = \int_{-1}^{1}\int_{0}^{1-x^2}\int_{0}^{x} dz\,dy\,dx \Rightarrow \int_{-1}^{1}\int_{0}^{1-x^2} x\,dy\,dx \Rightarrow 2\int_0^1 yx\Big|_0^{1-x^2} dx$

$\Rightarrow 2\int_0^1 x - x^3\,dx = 2\left[\frac{x^2}{2} - \frac{x^4}{4}\right]\Big|_0^1$

$= \frac{1}{2}$

$\int_{-1}^{1}\int_{0}^{1-x^2}\int_{0}^{x} 6y\,dz\,dy\,dx$

$\Rightarrow 6\int_{-1}^{1}\int_{0}^{1-x^2} xy\,dy\,dx \Rightarrow 3\int_{-1}^{1} (1-x^2)^2 x\,dx$

$\Rightarrow 6\int_0^1 x - 2x^3 + x^5\,dx$

$\Rightarrow 6\left[\frac{x^2}{2} - \frac{x^4}{2} + \frac{x^6}{6}\right]_0^1 = 1$

$\bar{f} = \frac{1}{1/2}\cdot(1) = \underline{\underline{2}}$

Problem 5

15 pts

Sketch the curve given by the following equation.

$$r^2 = \frac{1}{\cos 2\theta}$$

Hint: use the identity $\cos 2\theta = \cos^2\theta - \sin^2\theta$ to switch to rectangular coordinates.

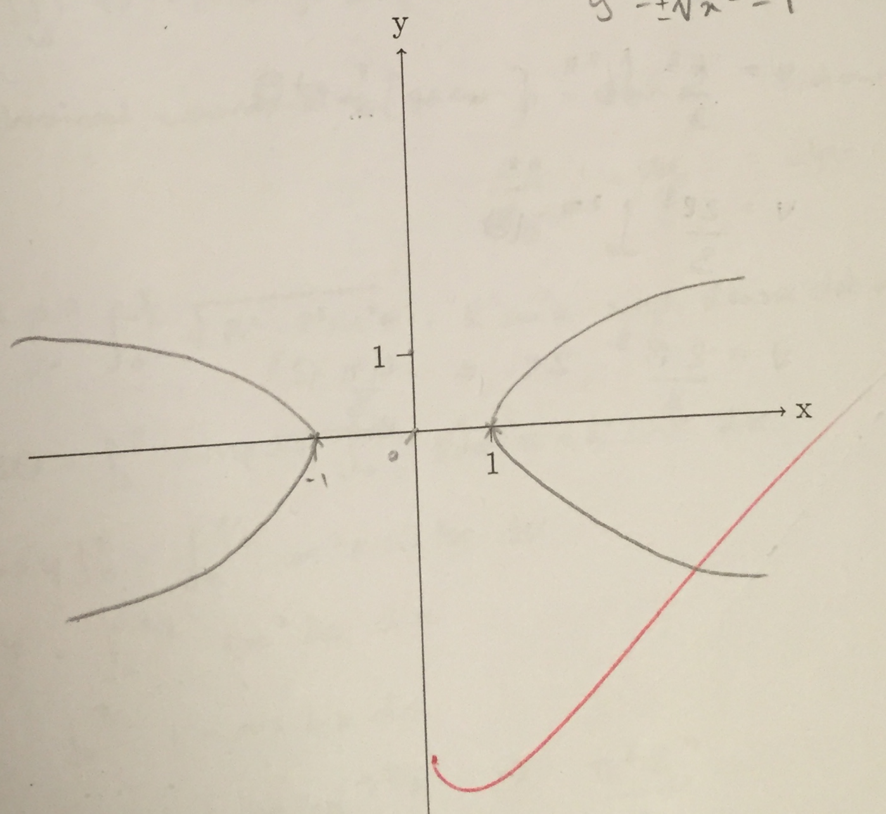
$$r^2 = \frac{1}{\cos^2\theta - \sin^2\theta}$$

$$\Rightarrow \quad r^2\cos^2\theta - r^2\sin^2\theta = 1.$$

$$x^2 - y^2 = 1$$

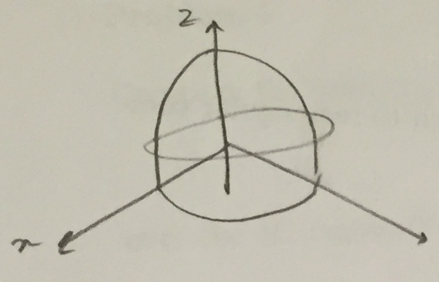
$$\therefore \quad y^2 - x^2 = -1$$

$$y = \pm\sqrt{x^2-1}$$



Problem 6 **15 pts**

Use spherical coordinates to find the volume of a 3D sphere of radius R. The answer without the derivation yields no credit.



for a 3D sphere

$0 \le \rho \le R$

$0 \le \varphi \le \pi$

$0 \le \theta \le 2\pi$

$$\therefore \quad V = \int_0^{2\pi} \int_0^{\pi} \int_0^{R} \rho^2 \, d\rho \, \sin\varphi \, d\varphi \, d\theta$$

$$V = \frac{R^3}{3} \int_0^{2\pi} \int_0^{\pi} \sin\varphi \, d\varphi \, d\theta$$

$$V = \frac{R^3}{3} \int_0^{2\pi} [-\cos\varphi]_0^{\pi} \, d\theta$$

$$V = \frac{2R^3}{3} \int_0^{2\pi} d\theta$$

$$V = \frac{2R^3}{3} \cdot 2\pi = \underline{\frac{4\pi}{3} R^3}$$

✓

Problem 7 — Extra credit! **10 pts**

Find the volume V_4 of a 4-dimensional sphere of radius R. The answer without the derivation yields no credit.

for a 4 dimensional sphere we have

$$x^2 + y^2 + z^2 + w^2 \leq R^2$$

where $w = \pm\sqrt{R^2 - z^2 - x^2 - y^2}$

So we have $\iiint_W \int_{-\sqrt{R^2-z^2-x^2-y^2}}^{\sqrt{R^2-z^2-x^2-y^2}} dw\, dV_3$

$$\Rightarrow 2\iiint_W \sqrt{R^2 - z^2 - x^2 - y^2}\, dV_3$$

we use spherical coordinates so $\rho = R\sin\alpha$ and our bounds become $0 \leq \alpha \leq \frac{\pi}{2}$

$$\frac{d\rho}{R\cos\alpha} = d\alpha \quad = d\rho = R\cos\alpha\, d\alpha$$

$$\Rightarrow 2\int_0^{2\pi}\int_0^{\pi}\int_0^{\frac{\pi}{2}} \sqrt{R^2 - R^2\sin^2\alpha} \cdot R^2\sin^2\alpha \cdot \sin\varphi \cdot R\cos\alpha\, d\alpha\, d\varphi\, d\theta$$

$$\Rightarrow 2\int_0^{2\pi} d\theta \cdot \int_0^{\pi} \sin\varphi\, d\varphi \int_0^{\pi/2} R^2\cos^2\alpha \cdot R^2\sin^2\alpha\, d\alpha$$

Recall: $\sin 2\alpha = 2\cos\alpha \sin\alpha$

$\sin^2 2\alpha = 4\cos^2\alpha\sin^2\alpha$

$$\Rightarrow 4\pi R^4[-\cos\varphi]_0^{\pi} \cdot \int_0^{\pi/2} \cos^2\alpha\sin^2\alpha\, d\alpha$$

Recall: $\cos 4\alpha = 1 - \sin^2 2\alpha$

$\sin^2 2\alpha = \frac{1-\cos 4\alpha}{2}$

$$\Rightarrow 2\pi R^4 \cdot \int_0^{\pi/2} \sin^2 2\alpha\, d\alpha$$

$$\Rightarrow \pi R^4 \cdot \int_0^{\pi/2} 1 - \cos 4\alpha\, d\alpha$$

$$\Rightarrow \pi R^4 \cdot \left[\alpha - \frac{1}{4}\sin 4\alpha\right]_0^{\pi/2} \Rightarrow \frac{\pi^2 R^4}{2}$$