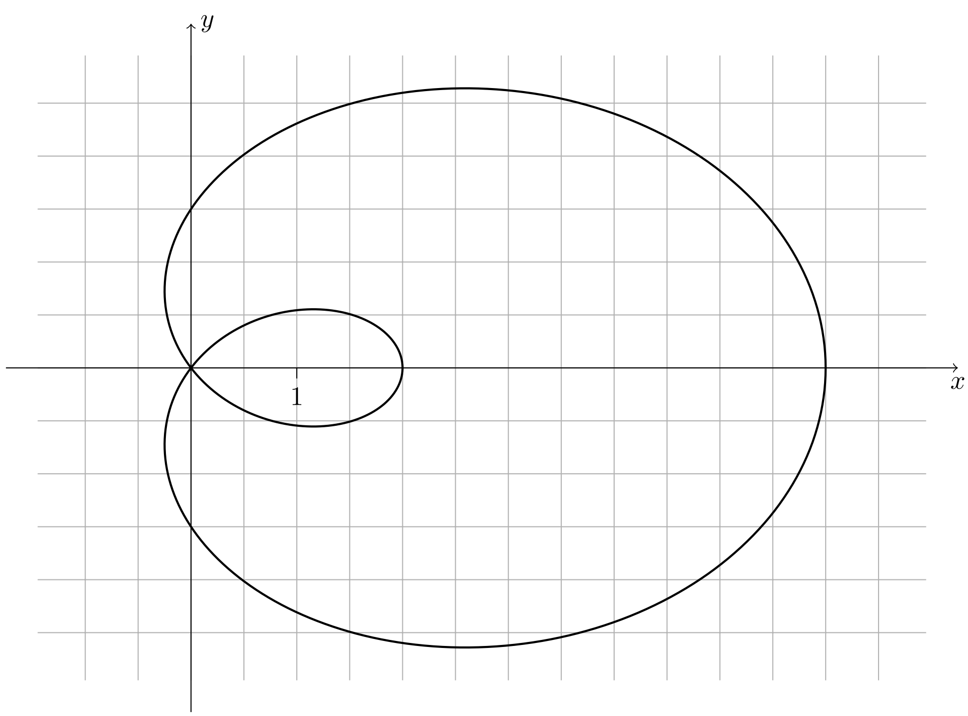
Upload your solutions to gradescope for the following questions by 11:59pm LA time on Sunday 17 April.

- Late exams will not be accepted.
- Your scans must be readable and good quality. Use good lighting and a scanning app.
- Questions 1,2,3 must begin on a new page and questions must be allocated correctly on Gradescope.
- Write your solutions **linearly**. We should be able to easily read your solutions and do not want to hunt around the page for it.

1. The *twisted cubic* is the curve in $\mathbb{R}^3$ that is the intersection of the surfaces $y = x^2$ and $z = x^3$. Let $\mathcal{C}$ be the part of the twisted cubic where $x \in [0, 1]$. Let $f(x, y, z) = (1 + 4y + 9xz)^{-\frac{1}{2}}$.
   (a) (2 points) Find a parametrisation of the curve $\mathcal{C}$, making sure to indicate the range of t. *Hint: If (x, y, z) is a point on the curve where $x = t$, then $y = t^2$ and $z = \ldots$.*
   (b) (3 points) Evaluate the integral
   $$\int_{\mathcal{C}} f(x, y, z)\, ds$$

2. Consider the solid ellipsoid E given by $(x/2)^2 + (y/3)^2 + (z/4)^2 = 1$ measured in meters with density function given by $\delta(x, y, z) = \sqrt{(x/2)^2 + (y/3)^2 + (z/4)^2}\ kg/m^3$.
   (a) (2 points) Find a change of coordinates G so that G applied to the solid ball of radius 1 centered at the origin gives E. *Hint: your map G should change the equation for a sphere into the equation of the ellipsoid.*
   (b) (4 points) What is the mass of E?

3. In this question assume that $\mathbf{E}$ is a vector field defined on the whole of $\mathbb{R}^2$, apart from the point $(1, 0)$. Suppose that $\nabla \times \mathbf{E} = 0$. The function $\mathbf{r}(t) = \big(2\cos t + 4\cos^2 t, 2\sin t + 4\cos t \sin t\big)$ for $t \in [-\frac{2\pi}{3}, \frac{4\pi}{3}]$ traces out the curve $\mathcal{C}$ on the graph below. To give you an idea: it starts at the origin, traces out the large loop, returns to the origin when $t = 2\pi/3$, then traces out the small loop and then returns to the origin once more.



(a) (2 points) Redraw the above graph and indicate the orientation of the curve. *Hint: calculating some tangent vectors might help.*

(b) (4 points) Let $\mathcal{A}$ be the circle radius $\frac{1}{2}$, and centre $(1,0)$ (so it fits entirely within the small loop above) oriented counter clockwise. Suppose that

$$\int_{\mathcal{A}} \mathbf{E} \cdot d\mathbf{r} = 2$$

What is $\int_{\mathcal{C}} \mathbf{E} \cdot d\mathbf{r}$? Justify your answer carefully, the answer itself will only be worth 1 point.

4. (6 points) Consider the vortex field $\mathbf{F} = \left\langle \frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2} \right\rangle$ and the curve $\mathcal{C}$ given by $y = x^4 - 14$ where $-2 \le x \le 2$ and the curve is oriented from left to right. Evaluate

$$\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}.$$