Midterm 1

UCLA: Math 32B, Fall 2019

Instructor: Noah White
Date: 21 October 2019

- This exam has 4 questions, for a total of 20 points.
- Please print your working and answers neatly.
- Write your solutions in the space provided showing working.
- Indicate your final answer clearly.
- You may write on the blank pages found at the back of the booklet however these will not be graded unless very clearly indicated. The reverse of any page will *not* be graded. If you need more space please ask the TA or instructor.
- Non programmable and non graphing calculators are allowed.

Name: Jonathan Chau

ID number: 705 166 732

Question	Points	Score
1	5	
2	5	
3	5	
4	5	
Total:	20	

Jonathan Chau

1. Each of the following questions has exactly one correct answer. Choose from the four options presented in each case. No partial points will be given.

(a) (1 point) If $\mathcal{R} = [-1, 1] \times [0, 2]$, the integral $\iint_{\mathcal{R}} 2\, dA$ is equal to

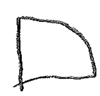
- ○ 2 $\quad 2 \quad 2$
- ○ 4
- ○ 6
- ● 8

(b) (1 point) If $\mathcal{B} = [-1, 1] \times [-2, 2] \times [3, 6]$, the integral $\iiint_{\mathcal{B}} x \ln(y^2 + z^2 + 1)$ ~~dA~~ dV is equal to

- ○ 1
- ○ $e - 1$
- ● 0
- ○ $e^2 + e$

(c) (1 point) The integral of $12\sqrt{x^2 + y^2}$ over the portion of the disc $x^2 + y^2 \leq 1$, in the first quadrant is

- ○ π
- ● 2π
- ○ $\pi\sqrt{2}$
- ○ 0



$\iint 12\sqrt{x^2+y^2}\, dA$

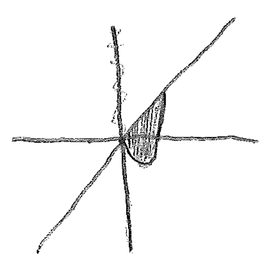
$D: 0 \leq x \leq 1$

$0 \leq y \leq \sqrt{1-x^2}$

$\int_0^{\frac{\pi}{2}} \int_0^1 12r \cdot r\, dr\, d\theta \quad 12\int_0^{\frac{\pi}{2}} \frac{r^3}{3}\Big|_0^1 d\theta = \frac{12}{3}\int_0^{\frac{\pi}{2}} d\theta$

(d) (1 point) If $\mathcal{D}$ is the region between the curves $y = x(x^2 - 1)$ and $y = x$ when $x \geq 0$ then $\mathcal{D}$ has the description

- ● $0 \leq x \leq \sqrt{2},\ \ x(x^2 - 1) \leq y \leq x$
- ○ $0 \leq x \leq \sqrt{2},\ \ x \leq y \leq x(x^2 - 1)$
- ○ $0 \leq x \leq 1,\ \ x(x^2 - 1) \leq y \leq x$
- ○ $0 \leq x \leq 1,\ \ x \leq y \leq x(x^2 - 1)$



(e) (1 point) Let $G(u, v) = \left(\frac{1}{2}(u - v), \frac{1}{2}(u + v)\right)$. What is the Jacobian of G?

- ○ 1
- ● $\frac{1}{2}$
- ○ $\frac{1}{4}uv$
- ○ uv

$\det \begin{vmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{vmatrix} \quad \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

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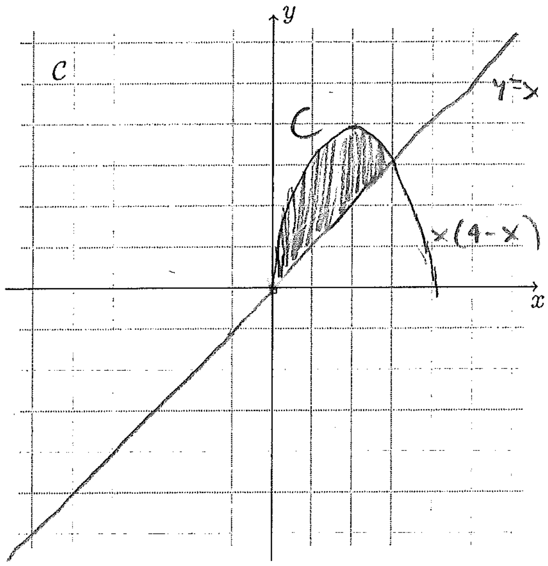
2. In this question we will consider two regions. First, $\mathcal{C}$ which bounded by $y = x(4-x)$ and $y = x$. Second, the region $\mathcal{D}$ given by the inequalities

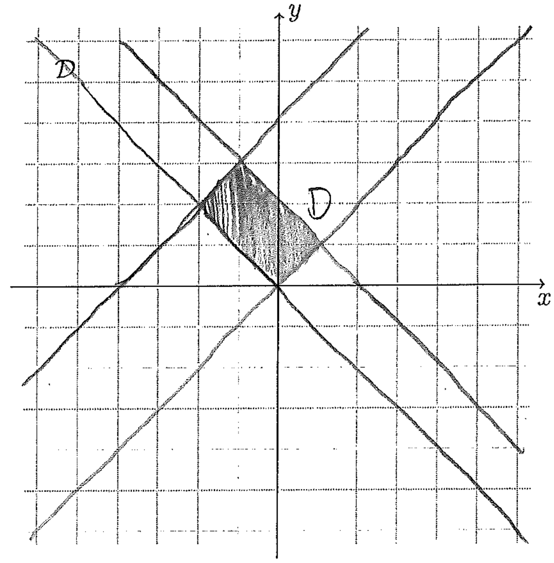
$2 \cdot 2 = 4$

$4x - x^2$

- $0 \leq x + y \leq 2$, and
- $0 \leq y - x \leq 4$.

(a) (2 points) Sketch the two regions on the graphs provided.





(b) (1 point) Write the integral

$$\iint_{\mathcal{C}} x \, dA$$

as an iterated integral (no need to evaluate the integral).

Using Fubini's Theorem

$$\boxed{\int_0^3 \int_x^{x(4-x)} x \, dy \, dx}$$

$\mathcal{C}: 0 \leq x \leq 3$
$x \leq y \leq x(4-x)$

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(c) (2 points) Let $u = x+y$ and $v = y-x$, and use the change of coordinates $G(u,v) = \left(\frac{1}{2}(u-v), \frac{1}{2}(u+v)\right)$ to express the integral

$$\iint_D 2x + 2y\, dA$$

as an iterated integral of u and v (no need to evaluate the integral). *Hint: have a look back at the multiple choice questions.*

$$A = J(G) \cdot \text{Area}$$

If $J(G)$ doesn't need to be evaluated

$$\int_0^4 \int_0^2 uv \left| J(G) \cdot \text{Area}(D) \right| du\, dv$$

If $J(G)$ needs to be evaluated:

$$\boxed{\int_0^4 \int_0^2 uv \left| \frac{1}{2} \cdot \text{Area}(D) \right| du\, dv}$$

$J(G) = \frac{1}{2}$

det: $\begin{vmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{vmatrix} = \frac{1}{4} - \left(-\frac{1}{4}\right) = \frac{1}{2}$

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3. In this question we will consider the region $\mathcal{D}$ which bounded by curve $r = \theta(2\pi - \theta)$.

   (a) (2 points) Sketch the region $\mathcal{D}$ in the xy-plane, on the graph provided (your sketch can be rough, it does not need to be perfect, it just need to show the main features).

$r = 2\pi\theta - \theta^2$

$\frac{\pi}{2}\left(2\pi - \frac{\pi}{2}\right)$

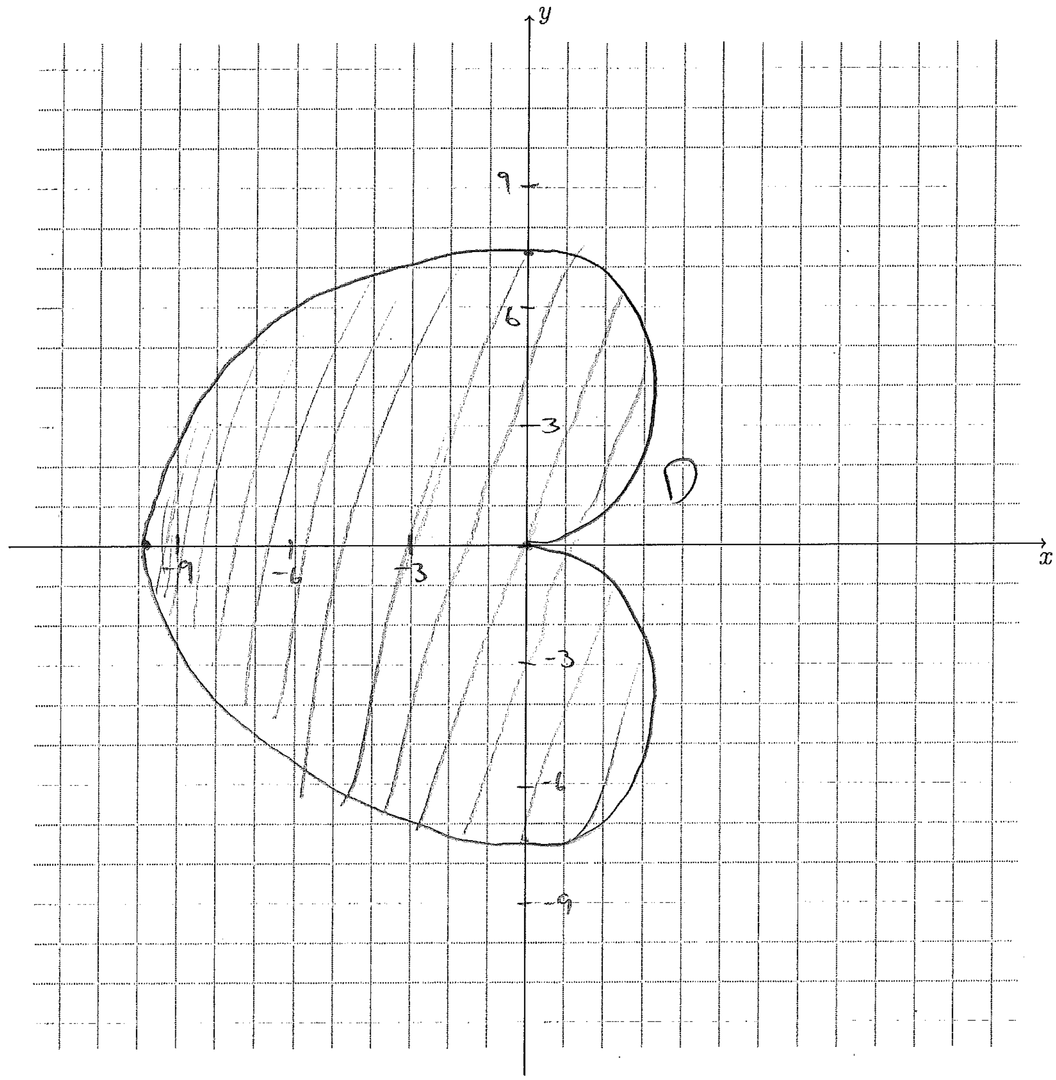
$\pi^2 - \frac{\pi^2}{4}$

$\frac{3\pi}{2}\left(2\pi - \frac{3\pi}{2}\right)$

$3\pi - \frac{9\pi}{4}$

θ	r
0	0
$\frac{\pi}{4}$	
$\frac{\pi}{2}$	$\frac{3\pi^2}{4}$
π	π^2
$\frac{3\pi}{2}$	$\frac{3\pi^2}{4}$
2π	0

$31 \times 31 = 961$



   (b) (3 points) Write the area of $\mathcal{D}$ as an iterated integral (no need to evaluate the integral)

$$\boxed{\int_0^{2\pi}\int_0^{\theta(2\pi-\theta)} r\,dr\,d\theta}$$

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4. Consider the region $\mathcal{E}$ that is bounded by the surface $z = y - x^2$ and the planes $z = 0$ and $y = 1$.

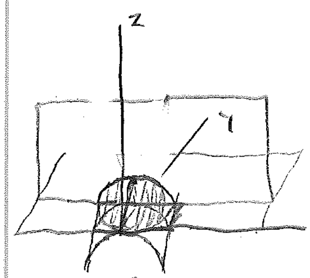
   (a) (2 points) Describe the region in the form

   $y = \frac{1}{2}$

   $z = 0$ $x = \pm\frac{1}{4}$

   $$\mathcal{E} = \{\ (x, y, z) \in \mathbb{R}^3 \mid (x, y) \in \mathcal{D},\ z_1(x, y) \le z \le z_2(x, y)\}$$

   for $\mathcal{D}$ a region in the xy-plane. Your answer should specify what $\mathcal{D}$ is. *Hint: if you are having trouble imagining what $z = y - x^2$ looks like, try setting y to something. Can you stitch together these pictures for different values of y to get a complete picture?*



$$\mathcal{E} = \{(x,y,z) \in \mathbb{R}^3 \mid (x,y) \in D,\ 0 \le z \le y - x^2\}$$

Where D is described by $\boxed{\begin{matrix} -1 \le x \le 1 \\ x^2 \le y \le 1 \end{matrix}}$

$y - x^2$

$1 - 0 = 1$

$y = x^2$

   (b) (3 points) Compute the volume of the region $\mathcal{E}$.

$$\int_{-1}^{1}\int_{x^2}^{1}\int_{0}^{y-x^2} 1\, dz\, dy\, dx$$

$$\int_{-1}^{1}\int_{x^2}^{1} z\Big|_{z=0}^{z=y-x^2} dy\, dx = \int_{-1}^{1}\int_{x^2}^{1} y - x^2\, dy\, dx$$

$\frac{1}{2} - x^2$ $-\frac{x^4}{2}$

$$\int_{-1}^{1} \frac{y^2}{2} - x^2 y\Big|_{y=x^2}^{y=1} dx = \int_{-1}^{1} \frac{1}{2} - x^2 - \left(\frac{x^4}{2} - x^4\right) dx$$

$-\left(\frac{x^4}{2} - x^4\right)$

$$\int_{-1}^{1} \frac{1}{2} - x^2 + \frac{x^4}{2}\, dx = \frac{x}{2} - \frac{x^3}{3} + \frac{x^5}{10}\Big|_{x=-1}^{x=1}$$

$\frac{1}{2} - \frac{1}{3} + \frac{1}{10}$

$-\left(\frac{-1}{2} + \frac{1}{3} - \frac{1}{10}\right)$

$$2\left(\frac{1}{2} - \frac{1}{3} + \frac{1}{10}\right) \to 1 - \frac{2}{3}^{\times 5} + \frac{1}{5}^{\times 3} = \frac{15}{15} - \frac{10}{15} + \frac{3}{15} = \boxed{\frac{8}{15}}$$

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