

# Math 32B Exam 2

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TOTAL POINTS

**50 / 50**

QUESTION 1

Vortex Field 10 pts

1.1 Angle 4 / 4

- ✓ + 2 pts Correct setup (parametrization and integral)
- ✓ + 2 pts Correct evaluation of integral
  - + 4 pts Using potential function  $\arctan(y/x)$
  - 1 pts Minor error
  - + 0 pts Incorrect
  - + 1 pts Attempting to parametrize using sines and cosines

1.2 Ellipse 3 / 3

- ✓ - 0 pts Reasonable attempt
- 3 pts No reasonable attempt

1.3 Conservative 3 / 3

- ✓ + 2 pts No
- ✓ + 2 pts Reasoning
  - + 1 pts Incorrect
  - + 1 pts Partially correct reasoning

QUESTION 2

Integrals 10 pts

2.1 Line integral 5 / 5

- ✓ + 5 pts Correct
  - + 0 pts Click here to replace this description.
  - + 2 pts Set up an integral with a correct parameterization
  - + 2 pts applied fundamental theorem
  - + 2 pts found a potential function
  - + 1 pts Correct answer

2.2 Surface Integral 5 / 5

- ✓ + 5 pts Correct

- + 2 pts Correct | normal vector |
- + 1 pts Correct orientation on normal
- + 2 pts Correct bounds
- + 0 pts Click here to replace this description.

QUESTION 3

Parametrizations 10 pts

3.1 Triangle 5 / 5

- ✓ + 1 pts Correct inequality in D independent of the other variable.
- ✓ + 2 pts Correct inequality in D dependent on the other variable.
- ✓ + 2 pts Correct parameterisation of G.
  - + 0 pts Incorrect

3.2 Cylinder 5 / 5

- ✓ + 1 pts Correct u bounds.
- ✓ + 2 pts Correct v bounds.
- ✓ + 2 pts Correct choice (negative).
  - + 0 pts Incorrect.

QUESTION 4

Simply connected 3 pts

4.1 a 1 / 1

- 1 pts Always True
- ✓ - 0 pts Only if D is simply connected
- 1 pts No answer

4.2 b 0 / 1

- 0 pts Always True
- ✓ - 1 pts Only if D is SC
- 0 pts No answer

4.3 c 1 / 1

- ✓ - 0 pts Always True
- 1 pts Only true if D is SC
- 1 pts No answer

QUESTION 5

Conservative Vector Fields 5 pts

5.1 a 1 / 1

- 1 pts All
- ✓ - 0 pts Conservative

5.2 b 1 / 1

- 1 pts All
- ✓ - 0 pts Conservative

5.3 C 1 / 1

- ✓ - 0 pts All
- 1 pts Conservative

5.4 d 1 / 1

- ✓ - 0 pts All
- 1 pts Conservative
- 1 pts No Answer

5.5 e 1 / 1

- ✓ - 0 pts All
- 1 pts Conservative
- 1 pts No Answer

QUESTION 6

MC 12 pts

6.1 a 3 / 3

- 3 pts Yes
- ✓ - 0 pts No
- 3 pts Not enough info
- 3 pts No answer

6.2 b 3 / 3

- ✓ - 0 pts fifth choice
- 3 pts any other choice

6.3 C 3 / 3

- ✓ + 1 pts first is zero (max of 3 points for this problem)
- ✓ + 1 pts second is negative
- ✓ + 1 pts third is zero
- ✓ + 1 pts fourth is positive
- 1 pts too many zeros, negatives, or positives
- + 0 pts no answer

6.4 d 3 / 3

- ✓ - 0 pts F is tangent to S
- 3 pts F is perpendicular to S
- 3 pts No answer

QUESTION 7

7 Bonus 1 / 0

- ✓ + 1 pts Bonus point

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Section	3	F
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- Do not open this exam packet until you are told that you may begin.
- Turn off all electronic devices and and put away all items except for a pen/pencil and an eraser.
- No phones, calculators, smart-watches or electronic devices of any kind allowed for any reason, including checking the time.
- If you have a question, raise your hand and one of the proctors will come to you. We will not answer any mathematical questions except possibly to clarify the wording of a problem.
- Quit working and close this packet when you are told to stop.

Spherical coordinates:

$$x = \rho \sin \phi \cos \theta$$

$$y = \rho \sin \phi \sin \theta$$

$$z = \rho \cos \phi$$

This derivative might be useful:

$$\frac{d}{dx} \arctan x = \frac{1}{1+x^2}$$

Page:	1	2	3	4	5	Total
Points:	10	10	10	8	12	50
Score:						

You may use this page for scratch work.

1. (10 points) Let  $\mathbf{F}$  denote the vortex field  $\mathbf{F} = \left\langle \frac{-y}{x^2 + y^2}, \frac{x}{x^2 + y^2} \right\rangle$ .

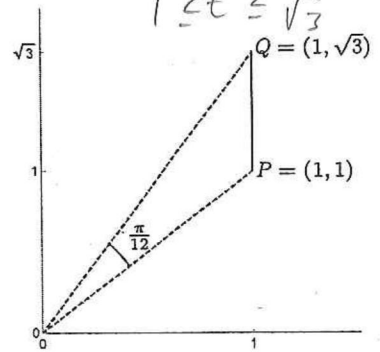
(a) Let  $C$  be the straight line segment from  $P = (1, 1)$  to  $Q = (1, \sqrt{3})$  (see the picture).

Show that  $\int_C \mathbf{F} \cdot d\mathbf{r} = \frac{\pi}{12}$ .

$\mathbf{r}'(t) = \langle 0, 1 \rangle$   $C: \mathbf{r}(t) = \langle 1, t \rangle$   
 $1 \leq t \leq \sqrt{3}$

$$\int_1^{\sqrt{3}} \langle \dots, \frac{1}{1+t^2} \rangle \cdot \langle 0, 1 \rangle dt$$

$$= \int_1^{\sqrt{3}} \frac{1}{1+t^2} dt$$



$\int_{1/2}^{\sqrt{3}/2} \frac{1}{1+t^2} dt = [\arctan(t)]_{1/2}^{\sqrt{3}/2} = \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$

(b) Suppose that  $C$  is the ellipse parametrized by  $\mathbf{r}(t) = \langle 5 \cos(2t), 2 \sin(2t) \rangle$  for  $0 \leq t \leq \pi$ .

Compute  $\int_C \mathbf{F} \cdot d\mathbf{r}$ . Box your answer.

$\mathbf{r}'(t) = \langle -10 \sin(2t), 4 \cos(2t) \rangle$

$$\int_0^{\pi} \left\langle \frac{-25 \sin^2(2t)}{25 \cos^2(2t) + 4 \sin^2(2t)}, \frac{25 \cos(2t)}{25 \cos^2(2t) + 4 \sin^2(2t)} \right\rangle \cdot \left\langle -10 \sin(2t), 4 \cos(2t) \right\rangle dt$$

$$= \int_0^{\pi} \left( \frac{20 \sin^2(2t)}{25 \cos^2(2t) + 4} + \frac{20 \cos^2(2t)}{25 \cos^2(2t) + 4} \right) dt$$

$$= \int_0^{\pi} \frac{20}{25 \cos^2(2t) + 4} dt = \int_0^{\pi} 20 \left( \frac{25}{2} \cos(4t) + \frac{25}{2} + 4 \right) dt$$

~~$$= 20 \left( \frac{25}{8} [\sin(4t)]_0^{\pi} + \frac{25}{2} + 4 \right) = 210 + 80 = 290$$~~

(c) Is the vortex field  $\mathbf{F}$  conservative on the domain  $\mathbb{R}^2 \setminus \{(0,0)\}$ ? Explain your reasoning.

No. The most obvious reason is that the above  $C$  described a closed path, in that domain &  $\int_C \mathbf{F} \cdot d\mathbf{r} \neq 0$ .  $\mathbf{F}$  is not path independent  $\therefore \mathbf{F}$  is not conservative.

2. (10 points) You do not need to simplify your answers.

(a) Compute  $\int_C \mathbf{F} \cdot d\mathbf{r}$  where  $C$  is the straight line from  $P = (1, 2, 3)$  to  $Q = (4, 5, 6)$  and

$$\mathbf{F} = \left\langle \frac{2xy}{x^2+z}, \ln(x^2+z), \frac{y}{x^2+z} \right\rangle.$$

Box your answer

$$f = y \ln(x^2+z), \quad \nabla f = \mathbf{F}$$

$$\int_C \mathbf{F} \cdot d\mathbf{r} = f(4, 5, 6) - f(1, 2, 3)$$

$$= (5 \ln(20) - 2 \ln(4))$$

(b) Suppose that  $\mathbf{F} = \langle y, z, 0 \rangle$  and that  $S$  is the surface parametrized by

$$G(u, v) = (u^3 - v, u + v, v^2), \quad 0 \leq u \leq 2, \quad 0 \leq v \leq 3.$$

with downward-pointing normal.

Fill in the limits and integrand of the integral below so that it equals  $\iint_S \mathbf{F} \cdot d\mathbf{S}$ .

$$\vec{T}_u = \langle 3u^2, 1, 0 \rangle$$

$$\vec{T}_v = \langle -1, 1, 2v \rangle$$

$$\vec{T}_u \times \vec{T}_v = \langle 2v - 0, 0 - 6u^2v, 3u^2 + 1 \rangle$$

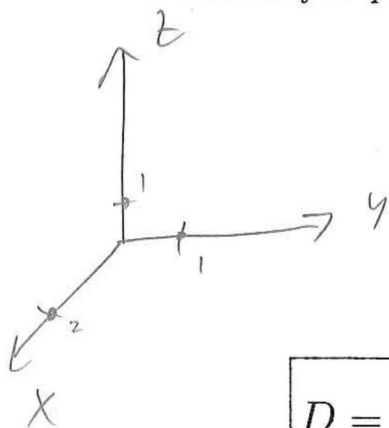
$$-\langle 2v, -6u^2v, 3u^2 + 1 \rangle \cdot \langle u + v, v^2, 0 \rangle$$

$$+ 2uv + 2v^2 - 6u^2v^3 + 0$$

$$\iint_S \mathbf{F} \cdot d\mathbf{S} = - \int_0^3 \int_0^2 (2uv + 2v^2 - 6u^2v^3) du dv$$

You do not need to show work on this page.

3. (10 points) (a) Give a parametrization  $G : D \rightarrow S$ , where  $S$  is the triangle in  $\mathbb{R}^3$  with vertices  $(2, 0, 0)$ ,  $(0, 1, 0)$ ,  $(0, 0, 1)$  in the plane  $\frac{1}{2}x + y + z = 1$ . Be sure to explicitly specify the domain  $D$  and call your parameters  $u$  and  $v$ .



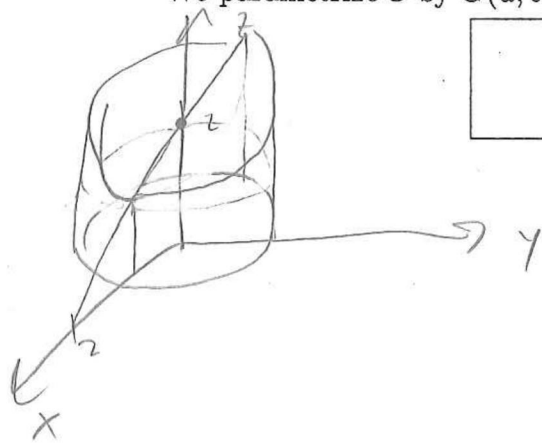
$$\frac{1}{2}x + y = 1$$

$$y = 1 - \frac{1}{2}x$$

$$D = \left\{ 0 \leq u \leq 2, 0 \leq v \leq 1 - \frac{1}{2}u \right\}$$

$$G(u, v) = \left\langle u, v, 1 - \frac{1}{2}u - v \right\rangle$$

- (b) Let  $S$  be the portion of the cylinder  $x^2 + y^2 = 1$  between the  $xy$ -plane and the plane  $x + z = 2$ . We parametrize  $S$  by  $G(u, v) = \langle \cos u, \sin u, v \rangle$ , where (fill these in)



$$0 \leq u \leq 2\pi, 0 \leq v \leq 2 - \cos u$$

$$z = 2 - x$$

The surface integral  $\iint_S x \, dS$  is (circle one) negative zero positive



$$\vec{T}_u = \langle -\sin u, \cos u, 0 \rangle$$

$$\vec{T}_v = \langle 0, 0, 1 \rangle$$

$$\int_0^{2\pi} \int_0^{2-\cos u} \cos u \, dv \, du$$

$$= \int_0^{2\pi} (2\cos u - \cos^2 u) \, du$$

$$= 2(0) - \frac{1}{2} \int_0^{2\pi} (\cos 2u + 1) \, du$$

$$= -\frac{1}{2} (+2\pi) = -\pi$$

Circle the correct answers.

4. (3 points) Let  $\mathbf{F}$  be a vector field on an open connected domain  $\mathcal{D}$  with continuous second order partial derivatives. Which of the following statements are always true, and which are only true if  $\mathcal{D}$  is simply connected?

(a) If  $\text{curl}(\mathbf{F}) = 0$  then  $\mathbf{F}$  is conservative.

Always true

Only true if  $\mathcal{D}$  is simply connected



(b) If  $\mathbf{F}$  has a potential function then  $\mathbf{F}$  is conservative.

~~Always true~~

Only true if  $\mathcal{D}$  is simply connected

(c) If  $\mathbf{F}$  is conservative then  $\text{curl}(\mathbf{F})$  is zero.

Always true

Only true if  $\mathcal{D}$  is simply connected

5. (5 points) Which of the following statements is true for all vector fields, and which is true only for conservative vector fields?

(a) The line integral along a path from  $P$  to  $Q$  does not depend on which path is chosen.

True for all vector fields

Only true for conservative vector fields

(b) The line integral around a closed curve is zero.

True for all vector fields

Only true for conservative vector fields

(c) The line integral over an oriented curve  $\mathcal{C}$  does not depend on how  $\mathcal{C}$  is parametrized as long as each parametrization preserves the orientation of  $\mathcal{C}$ .

True for all vector fields

Only true for conservative vector fields

(d) The (vector) line integral is equal to the (scalar) line integral of the tangential component along the curve.

True for all vector fields

Only true for conservative vector fields

(e) The line integral changes sign if the orientation of the curve is reversed.

True for all vector fields

Only true for conservative vector fields



$$\frac{1}{2}x^2z \quad ye^z - \frac{1}{2}y^2z$$

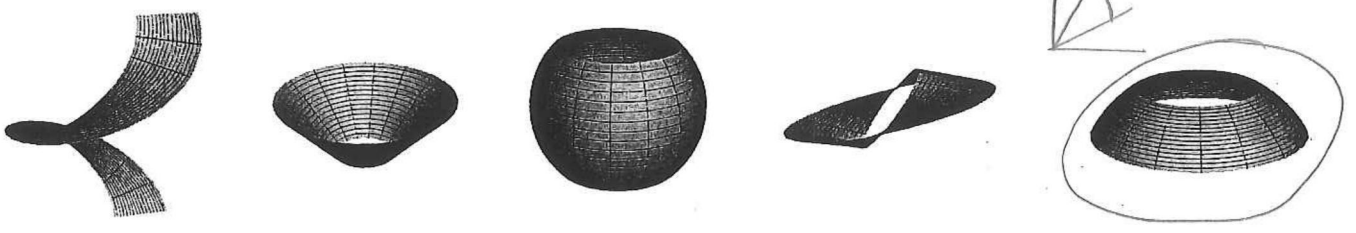
6. (12 points) Multiple choice. Circle the correct answer.  $\rightarrow z \cos x$

(a) Consider the vector field  $\mathbf{F} = \langle xz, e^z - yz, \cos x \rangle$ . Is there a function  $f$  such that  $\mathbf{F} = \nabla f$ ?

Yes      No      We don't have enough information

$$f = z \cos x$$

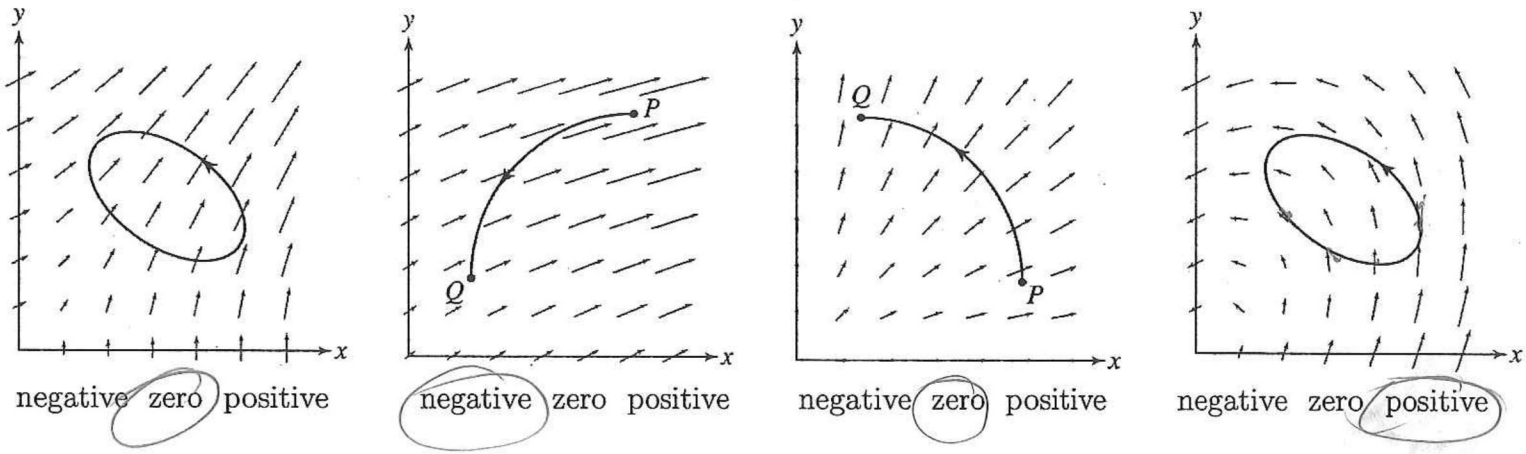
(b)  $G(\theta, \phi) = \langle \cos \theta \sin \phi, \sin \theta \sin \phi, \cos \phi \rangle$  with  $0 \leq \theta \leq 2\pi$  and  $\frac{\pi}{6} \leq \phi \leq \frac{\pi}{3}$  parametrizes which of the surfaces below? (circle one)



(c) Consider the line integrals  $\int_c \mathbf{F} \cdot d\mathbf{r}$  for the vector fields  $\mathbf{F}$  and paths  $\mathbf{r}$  below.

Exactly **two** of the line integrals are zero, **one** is positive, and the remaining **one** is negative. Circle "negative", "zero", or "positive" below each picture to indicate your answers.

Note: the closed curves are oriented counterclockwise and the others are oriented  $P \rightarrow Q$ .



(d)  $\iint_S \mathbf{F} \cdot d\mathbf{S}$  is zero if (circle one)

$\mathbf{F}$  is tangent to  $S$  at every point       $\mathbf{F}$  is perpendicular to  $S$  at every point

