Math 32B Exam 1

Maanek Singh Sehgal

TOTAL POINTS

50 / 50

QUESTION 1

TF 8 pts

1.1 TF 2 / 2

✓ **- 0 pts** True

1.2 Yes/No Integrals 6 / 6

✓ **- 0 pts** no no no no yes yes

QUESTION 2

Worksheet Question 10 pts

2.1 Rectangular coords 4 / 4

✓ **- 0 pts** Correct

2.2 Polar coords 6 / 6

✓ **+ 6 pts** Correct

+ 1 pts Correct bound

+ 2 pts Correct bounds

+ 1 pts Correct integrand (excluding Jacobian)

+ 2 pts Jacobian

+ 1 pts Correct final answer

- 1 pts Minor Miscalculation/Incorrect final answer

+ 0 pts incorrect or nothing shown

QUESTION 3

Non-linear transformation 10 pts

3.1 Picture 4 / 4

✓ **- 0 pts** Correct (third picture)

3.2 Integral 6 / 6

✓ **- 0 pts** Correct

QUESTION 4

Q4 10 pts

4.1 Sphere/Cone 4 / 4

✓ **- 0 pts** Correct

4.2 Volume integrals 6 / 6

✓ **- 0 pts** Correct

QUESTION 5

MC 12 pts

5.1 Spherical Plane 3 / 3

✓ **- 0 pts** theta=pi/4

5.2 Jacobian 3 / 3

✓ **- 0 pts** 2u^2+2v^2

5.3 Cylindrical Plane 3 / 3

✓ **- 0 pts** r=1/cos\theta

5.4 Linear Map 3 / 3

✓ **- 0 pts** (6u+2v, u+4v)

MATH 32B **Exam 1** **Spring 2019**

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- Fill out your name, section letter, and UID above.
- Do not open this exam packet until you are told that you may begin.
- Turn off all electronic devices and and put away all items except for a pen/pencil and an eraser.
- No phones, calculators, smart-watches or electronic devices of any kind allowed for any reason, including checking the time.
- If you have a question, raise your hand and one of the proctors will come to you. We will not answer any mathematical questions except possibly to clarify the wording of a problem.
- Quit working and close this packet when you are told to stop.

Spherical coordinates:

$$\begin{aligned} x &= \rho \sin\phi \cos\theta \\ y &= \rho \sin\phi \sin\theta \\ z &= \rho \cos\phi \end{aligned}$$

$$dxdydz = \rho^2 \sin\phi \, d\rho d\phi d\theta$$

Page:	1	2	3	4	5	Total
Points:	8	10	10	10	12	50
Score:						

You may use this page for scratch work.

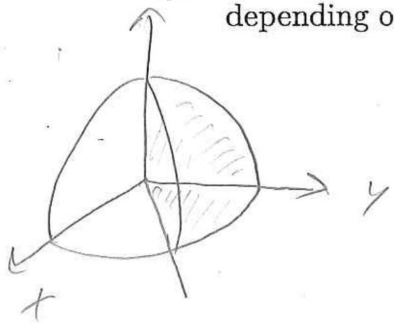
$\overbrace{\sqrt{y}}^{2}\,\overbrace{\sin(x^2y^2)}^{1} \le 2$

1. (8 points) (a) True or False? (circle one) $\int_1^4 \int_0^1 \sqrt{y}\,\sin(x^2y^2)\,dx dy \le 6$

[✓ True] False

$\int_1^4 \int_0^1 2\, dxdy = (1)(3)(2) = 6$

(b) Let D be the region in the positive octant $(x, y, z \ge 0)$ enclosed by the sphere $x^2+y^2+z^2=4$ and the planes $z=0$, $x=0$, and $x=y$. For **each** integral below, circle "yes" or "no" depending on whether or not it equals $\iiint_D x\,dV$.



yes [✓ no] $\int_0^{\pi/2}\int_0^{\overset{\times}{\pi/4}}\int_0^{\overset{\checkmark}{2}} \rho^3 \cos\theta \sin^2\phi \,d\rho\, d\theta\, d\phi$

yes [✓ no] $\int_0^{\sqrt{2}}\int_0^{\overset{\times}{\sqrt{4-x^2}}}\int_0^{\overset{\checkmark}{\sqrt{4-x^2-y^2}}} x\,dz\,dy\,dx$

yes [✓ no] $\int_0^{\overset{\checkmark}{\pi/2}}\int_{\pi/4}^{\overset{\checkmark}{\pi/2}}\int_0^{\overset{\checkmark}{2}} \rho^2 \overset{\times}{\cos\theta} \sin\phi \,d\rho\, d\theta\, d\phi$ $\rho^2 \sin\phi\, \rho \sin\phi \cos\theta$

yes [✓ no] $\int_0^{\overset{\times}{\pi/2}}\int_{\pi/4}^{\overset{\times}{\pi/2}}\int_0^{\overset{\checkmark}{2}} \rho^2 \overset{\times}{\cos\theta} \sin\phi \,d\rho\, d\phi\, d\theta$

[✓ yes] no $\int_0^{\overset{\checkmark}{2}}\int_{\pi/4}^{\overset{\checkmark}{\pi/2}}\int_0^{\overset{\checkmark}{\sqrt{4-r^2}}} r^2 \cos\theta\, dz\, d\theta\, dr$

[✓ yes] no $\int_0^{\overset{\checkmark}{\pi/2}}\int_0^{\overset{\checkmark}{2}}\int_{\pi/4}^{\overset{\checkmark}{\pi/2}} \overset{\checkmark}{\rho^3 \cos\theta \sin^2\phi}\, d\theta\, d\rho\, d\phi$

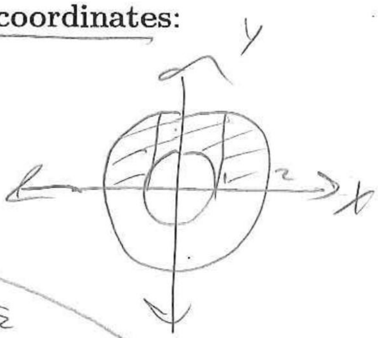
2. (10 points) Let R be the region in $\boxed{\mathbb{R}^2}$ which lies above the x-axis and between the circles of radius 1 and 2 centered at $(0,0)$.

(a) Write the following integral as a sum of integrals in **rectangular coordinates**:

$x^2+y^2=1$

$x^2+y^2=4$

$$\iint_R 3y\,dA.$$



Do not evaluate these integrals. Box your answers.

$$\boxed{\int_{-2}^{-1}\int_{0}^{\sqrt{4-x^2}} 3y\,dy\,dx + \int_{-1}^{1}\int_{\sqrt{1-x^2}}^{\sqrt{4-x^2}} 3y\,dy\,dx + \int_{1}^{2}\int_{0}^{\sqrt{4-x^2}} 3y\,dy\,dx}$$

(b) Evaluate the integral in part (a) using **polar coordinates**. Box your answer.

$1 \le r \le 2$

$0 \le \theta \le \pi$

$$\int_0^{\pi}\int_1^2 3r\sin\theta\, r\,dr\,d\theta$$

$$= \int_0^{\pi} 3\sin\theta\,\left[\tfrac{1}{3}r^3\right]_1^2\,d\theta$$

$$= \cancel{3}\left[-\cos\theta\right]_0^{\pi}\left(\tfrac{1}{\cancel{3}}\right)(8-1)$$

$$= (+(+1)+(+(1)))\,7 = \boxed{14}$$

$G(u=1, uv= \ )$

$G(0,0) \to (0,0)$

3. (10 points) Let $G : \mathbb{R}^2 \to \mathbb{R}^2$ be the non-linear transformation

$y = 4v - 1$

$x = 2v + 1$

$$G(u, v) = (u + v + uv, -u + 2v + 2uv).$$

$G(1,0) \to (1,-1)$

$G(0,1) \to (1,2)$

Let R be the unit square $[0, 1] \times [0, 1]$ in the uv-plane and let $D = G(R)$ in the xy-plane.

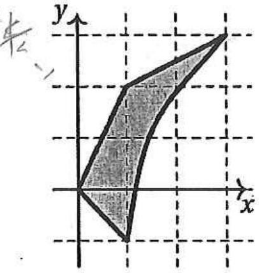
$y + 2x = -4v - 2 + 4v - 1$

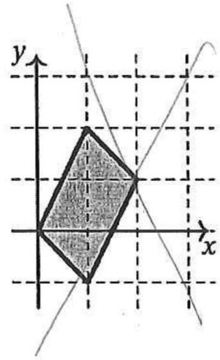
$y - 2x = -3$

$y = 2x - 3$

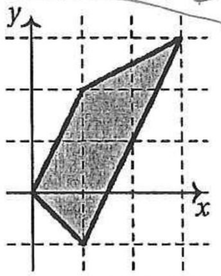
(a) Circle the picture of D below. The dashed grid consists of unit squares.

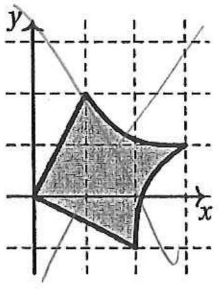
$G(1,1) \to (3,3)$





$y = 2x - 3$





(b) Find the limits and integrand of the integral below so that it equals

* 0 Simplify

$$\iint_D \sqrt{x}\, dA$$

as an integral over the square R. Do not evaluate the integral. Show your work.

$$\underbrace{\int_0^1 \int_0^1}_{R \text{ as given}} \sqrt{\underbrace{u+v+uv}_{x, \text{ as given}}}\, du\, dv$$

$$\frac{\partial(x,y)}{\partial(u,v)} = \det \begin{bmatrix} 1+v & 1+u \\ -1+2v & 2+2u \end{bmatrix}$$

$$= (1+v)(2+2u) - (1+u)(2v-1)$$

$$= 2 + \cancel{2v} + 2u + \cancel{2uv}$$

$$- \cancel{2v} - \cancel{2uv} + 1 + u$$

$$= 3 + 3u$$

$$\boxed{\iint_D \sqrt{x}\, dA = 3\int_0^1 \int_0^1 \sqrt{u+v+uv}\,(u+1) \quad du\, dv}$$

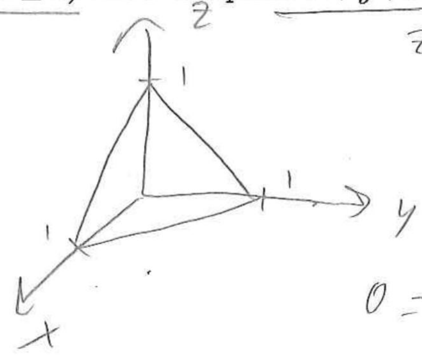
4. (10 points) (a) In **spherical coordinates**, describe the region outside the cone $x^2 + y^2 = z^2$ and inside the sphere $x^2 + y^2 + z^2 = 2$ (shown below – the sphere is translucent so you can see the cone inside).

$0 \leq \theta \leq 2\pi$

$\frac{\pi}{4} \leq \phi \leq \frac{3\pi}{4}$

$0 \leq \rho \leq \sqrt{2}$

(b) Fill in the limits and integrand of the double and triple integrals below so that they both equal the volume of the region in the first octant ($x, y, z \geq 0$) below the plane $x + y + z = 1$. Be sure to follow the provided order of integration.



$z = 1 - x - y$

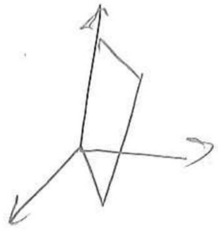
$0 = 1 - x - y$

$x = 1 - y$

$$\text{Vol} = \int_0^1 \int_0^{1-y} (1 - x - y) \, dx \, dy$$

$y = 1 - x - z$

$$\text{Vol} = \int_0^1 \int_0^{1-z} \int_0^{1-z-x} 1 \, dy \, dx \, dz$$



5. (12 points) Multiple choice. Circle the correct answer.

(a) In spherical coordinates the plane $y = x$ can be written as

$\rho = \frac{1}{\cos\phi}$ $\quad$ $\phi = \frac{\pi}{3}$ $\quad$ $\rho = 1$ $\quad$ [✓ $\theta = \frac{\pi}{4}$] $\quad$ $\rho = \frac{1}{\sin\phi}$.

$$\rho \cancel{\sin\phi} \cos\theta = \rho \cancel{\sin\phi} \sin\theta$$

$$\tan\theta = 1$$

(b) The Jacobian of the map $G(u, v) = (u^2 - v^2, uv)$ is

[✓ $2u^2 + 2v^2$] $\quad$ $2u^2 - 2v^2$ $\quad$ $4uv$ $\quad$ $2u + 2v$ $\quad$ $-4uv$

$$\det \begin{bmatrix} 2u & -2v \\ v & u \end{bmatrix} = 2u^2 + 2v^2$$

(c) In cylindrical coordinates the plane $x = 1$ can be written as

[✓ $r = \frac{1}{\cos\theta}$] $\quad$ $\theta = \frac{\pi}{3}$ $\quad$ $r = 1$ $\quad$ $\theta = \frac{\pi}{4}$ $\quad$ $r = \frac{1}{\sin\theta}$

$$x = r\cos\theta = 1$$

$$r = \frac{1}{\cos\theta}$$

(d) The linear map which sends the unit square $[0, 1] \times [0, 1]$ to the parallelogram with vertices $(0, 0)$, $(6, 1)$, $(8, 5)$, and $(2, 4)$ is $G(u, v) =$

~~$(6u + v, 2u + 4v)$~~ $\quad$ [✓ $(6u + 2v, u + 4v)$] $\quad$ ~~$(6u + v, 4u + 2v)$~~

~~$(6u + 2v, 4u + v)$~~ $\quad$ ~~$(6u + 4v, u + 2v)$~~

$(0,0) \to (0,0)$

$(1,0) \to (6,1)$

$(1,1) \to (10,3)$ ✗