Math 32B Final Exam

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TOTAL POINTS

98 / 100

QUESTION 1

1 Change order of integration **4 / 4**

✓ **- 0 pts answer = 2 (limits are x=0,pi y=0,x)**

- **- 1 pts** minor error
- **- 2 pts** incorrect integration bounds
- **- 1 pts** integration error
- **- 4 pts** swapping the order of integration without changing the bounds
- **- 4 pts** incorrect
- **- 2 pts** major integration error

QUESTION 2

2 Spherical coords **8 / 8**

✓ **- 0 pts (1 pt) theta 0 to 2pi**
(2 pts) phi pi/6 to 5pi/6
(2 pts) rho lower bound 1/sin phi
(1 pt) rho upper bound 2
(2 pts) integrand rho^2 sin phi

- **- 1 pts** 1 error
- **- 2 pts** 2 errors
- **- 3 pts** 3 errors
- **- 4 pts** 4 errors
- **- 5 pts** 5 errors
- **- 6 pts** 6 errors
- **- 7 pts** 7 errors
- **- 8 pts** 8 errors

QUESTION 3

Vortex field 12 pts

3.1 line integral **4 / 4**

✓ **- 0 pts 2pi**

- **- 2 pts** Incorrect integral setup
- **- 2 pts** Integration error
- **- 1 pts** Minor error
- **- 4 pts** Incorrect

3.2 curlz(F) **3 / 3**

✓ **- 0 pts 0**

- **- 1 pts** minor error
- **- 2 pts** major error
- **- 3 pts** completely incorrect
- **- 1 pts** should be a scalar, not a vector

3.3 Fill in the blanks **2 / 2**

✓ **- 0 pts 0, simply connected**

- **- 1 pts** one wrong
- **- 2 pts** both wrong

3.4 Conservative? **3 / 3**

✓ **- 0 pts No, because the integral in (a) is nonzero**

- **- 1 pts** No (but partially correct reason)
- **- 2 pts** No (but incorrect reason)
- **- 3 pts** Yes

QUESTION 4

Surface integral w/ vector potential 12 pts

4.1 vector potential **2 / 2**

✓ **- 0 pts Correct.**

- **- 1 pts** Incorrect, but knew that they needed calculate the curl of A.
- **- 2 pts** Incorrect.

4.2 Stokes' theorem **8 / 8**

✓ **+ 2 pts Applying Stoke's Theorem**
✓ **+ 2 pts Correctly parameterising the boundary.**
✓ **+ 1 pts Correct Orientation on boundary**
✓ **+ 2 pts Correctly setting the boundary integral up.**
✓ **+ 1 pts Correct final answer. (-24\pi or 24\pi if orientation wrong.)**

- **+ 0 pts** Incorrect.

4.3 Other orientation **2 / 2**

- ✓ **- 0 pts Correct. (negative of answer in b)**
- **- 1 pts** Almost Correct (same as answer in b)
- **- 2 pts** Incorrect

QUESTION 5

5 Worksheet problem: line integral in a plane **10 / 12**

- **+ 12 pts** Correct
- ✓ **+ 2 pts Stokes' Theorem**
- ✓ **+ 2 pts Correct curl**
- ✓ **+ 2 pts Correct normal vector / orientation**
- **+ 1 pts** normalized
- ✓ **+ 2 pts dot with curl**
- ✓ **+ 2 pts Recognizing the surface area as the integral of 1**
- **+ 1 pts** Correct answer (15*sqrt(3) or 45/sqrt(3))
- **+ 0 pts** Click here to replace this description.

QUESTION 6

Divergence theorem 12 pts

6.1 integral of bottom cap **4 / 4**

- ✓ **- 0 pts Correct**
- **- 1 pts** Incorrect integrand (r^2 instead of r^3)
- **- 1 pts** Incorrect integrand (r^4 instead of r^3)
- **- 1 pts** Sign error
- **- 1 pts** Integration error
- **- 1 pts** Incorrect integrand (r instead of r^3)
- **- 1 pts** Incorrect integrand (should have r^3)

6.2 integral of hemisphere **8 / 8**

- ✓ **- 0 pts Correct**
- **- 0 pts** Correct, given your answer to (a)
- **- 8 pts** Incorrect
- **- 2 pts** Incorrect divergence
- **- 0.5 pts** Minor calculation error
- **- 2 pts** Forgot to solve for flux at end
- **- 1 pts** Incorrect integrand
- **- 1 pts** Sign error
- **- 1 pts** Small calculation error
- **- 2 pts** Incorrect integral
- **- 7 pts** 1 point for attempting to write out the integral in terms of a parametrization

QUESTION 7

MC 15 pts

7.1 (a) **3 / 3**

- **- 3 pts** Incorrect
- ✓ **- 0 pts Correct (positive)**

7.2 (b) **3 / 3**

- **- 3 pts** Incorrect
- ✓ **- 0 pts Correct (positive)**

7.3 (c) **3 / 3**

- ✓ **- 0 pts Correct (zero)**
- **- 3 pts** Incorrect

7.4 (d) **3 / 3**

- **- 3 pts** Incorrect
- ✓ **- 0 pts Correct (0.2)**

7.5 (e) **3 / 3**

- ✓ **- 0 pts Correct (div(curl F)=0)**
- **- 3 pts** Incorrect
- **- 1.5 pts** Click here to replace this description.
- **- 2 pts** Click here to replace this description.

QUESTION 8

MC 15 pts

8.1 (a) **3 / 3**

- ✓ **- 0 pts Correct**
- **- 3 pts** Incorrect

8.2 (b) **3 / 3**

- ✓ **- 0 pts Correct**
- **- 3 pts** Incorrect

8.3 (c) **3 / 3**

- ✓ **- 0 pts Correct**

- **3 pts** Incorrect similar)

8.4 (d) **3 / 3**

✓ - **0 pts Correct**

- **3 pts** Incorrect

8.5 (e) **3 / 3**

✓ - **0 pts Correct**

- **3 pts** Incorrect

QUESTION 9

Fill in the blanks 10 pts

9.1 counterclockwise **1 / 1**

✓ - **0 pts Correct**

- **1 pts** Incorrect.

9.2 curlz(F) **1 / 1**

✓ - **0 pts Correct**

- **1 pts** Incorrect.

- **0.5 pts** Wrote curl(F) instead of curl_z(F), or got the order of derivatives wrong way.

9.3 boundary of D **1 / 1**

✓ - **0 pts Correct**

- **1 pts** Incorrect

9.4 RHS of Stokes' thm **3 / 3**

✓ - **0 pts Correct**

- **1 pts** Incorrect integral bounds (\partial S)

- **1 pts** Did not put single integral.

- **1 pts** Incorrect integrand

9.5 outward **1 / 1**

✓ - **0 pts Correct**

- **1 pts** Incorrect

9.6 LHS of Div thm **3 / 3**

✓ - **0 pts Correct**

- **1 pts** Not triple integral

- **1 pts** Bounds wrong (W)

- **1 pts** Integrand wrong. (div(F)dV, div(F)dxdydz or

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3D		R	MS 6221
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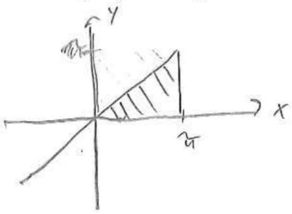
Spherical coordinates:

$$x = \rho \sin\phi \cos\theta$$
$$y = \rho \sin\phi \sin\theta$$
$$z = \rho \cos\phi$$

$$dxdydz = \rho^2 \sin\phi \, d\rho d\phi d\theta$$

Page:	1	2	3	4	5	6	7	8	Total
Points:	12	12	12	12	12	15	15	10	100
Score:									

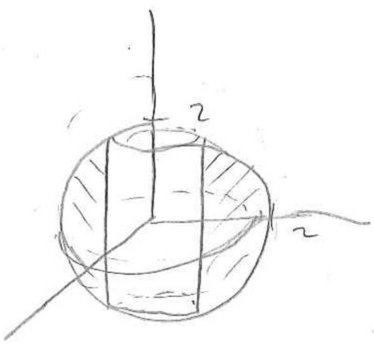
1. (4 points) Evaluate the integral $\displaystyle\int_0^\pi \int_y^\pi \frac{\sin x}{x}\, dx\, dy$ by changing the order of integration.



$$\int_0^\pi \int_0^x \frac{\sin x}{x}\, dy\, dx = \int_0^\pi \sin x\, dx$$

$$= [-\cos x]_0^\pi = (+(+1) + (+(1))) = \boxed{2}$$

2. (8 points) Using spherical coordinates, set up *but do not evaluate* a triple integral that computes the volume of a sphere of radius 2 from which a central cylinder of radius 1 has been removed.

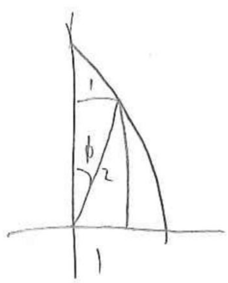


$$\boxed{\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \int_0^{2\pi} \int_{\frac{1}{\sin\phi}}^{2} \rho^2 \sin\phi \, d\rho\, d\theta\, d\phi}$$

$0 \le \theta \le 2\pi$

$a\sin \frac{\pi}{6} \le \phi \le \frac{5\pi}{6}$

$\frac{1}{\sin\phi} \le \rho \le 2$



$\phi = a\sin\left(\frac{1}{2}\right)$

$= \frac{\pi}{6}$

$r = 1$

$\pi - \frac{\pi}{6} = \frac{5\pi}{6}$

$\rho \sin\phi = 1$ $\quad \rho = \frac{1}{\sin\phi}$

3. (12 points) Let $\boldsymbol{F}$ denote the vortex field $\boldsymbol{F} = \left\langle \frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2} \right\rangle$.

(a) Suppose that C_R is the circle of radius R centered at $(0,0)$ oriented counterclockwise. **By parametrizing** C_R, compute $\oint_{C_R} \boldsymbol{F} \cdot d\boldsymbol{r}$. $\boxed{\text{Box your answer}}$

Note: you may not use the fundamental theorem of line integrals or anything about the winding number in this problem. Also, the answer is not zero.

$C_R: \vec{r} = \langle R\cos t, R\sin t \rangle \quad 0 \le t \le 2\pi$

$\vec{r}'(t) = \langle -R\sin t, R\cos t \rangle$

$\int_0^{2\pi} \left\langle \frac{-\cancel{R}\sin t}{\cancel{R^2}}, \frac{\cancel{R}\cos t}{\cancel{R^2}} \right\rangle \cdot \langle -R\sin t, R\cos t \rangle \, dt$

$\int_0^{2\pi} 1 \, dt = \boxed{2\pi}$

(b) Compute $\text{curl}_z(\boldsymbol{F})$. Show your work. $\boxed{\text{Box your answer}}$

$\text{curl}_z(\vec{F}) = \frac{\partial y}{\partial x} - \frac{\partial x}{\partial y}$

$= \frac{-x^2+y^2}{(x^2+y^2)^2} + \frac{x^2-y^2}{(x^2+y^2)^2} = \boxed{0}$

$\frac{-(x^2+y^2)+(+y)(2y)}{(x^2+y^2)^2} = \frac{x^2-y^2}{(x^2+y^2)^2}$

$x(x^2+y^2)^{-1}$

$(x^2+y^2)^{-1} - x(2x)(x^2+y^2)^{-2}$

$= \frac{1}{x^2+y^2} - \frac{2x^2}{(x^2+y^2)^2}$

$= \frac{x^2+y^2-2x^2}{(x^2+y^2)^2}$

(c) Fill in the blanks:

(i) If $\boldsymbol{F} = \nabla f$ on a domain D then $\oint_C \boldsymbol{F} \cdot d\boldsymbol{r} =$ 0 for every closed curve C in D.

(ii) If $\text{curl}_z(\boldsymbol{F}) = 0$ on a Simple conected domain D then $\boldsymbol{F}$ is conservative.

(d) Is the vortex field $\boldsymbol{F}$ conservative on the domain $\mathbb{R}^2 \setminus \{(0,0)\}$? Explain your reasoning.

No, C_R from part a) existed on $\mathbb{R}^2 \setminus \{(0,0)\}$, (assuming $R \neq 0$), but $\oint_{C_R} \vec{F} \cdot d\vec{r} \neq 0$ $\therefore$ $\vec{F}$ is not path independent

$\therefore$ $\vec{F}$ is not conservative on $\mathbb{R}^2 \setminus \{(0,0)\}$

4. (12 points) Let $\boldsymbol{F} = \langle 2x, 0, -2z \rangle$.

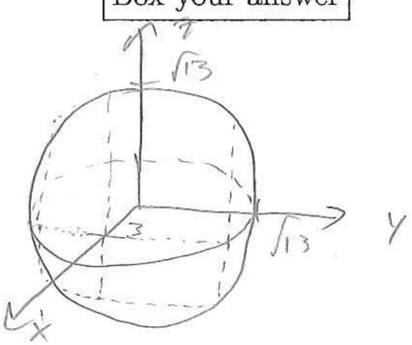
(a) Verify that $\boldsymbol{A} = \langle yz, -xz, yx \rangle$ is a vector potential for $\boldsymbol{F}$.

$\nabla \times \vec{A} = \langle x-(-x), y-y, -z-z \rangle$

$= \langle 2x, 0, -2z \rangle = \vec{F}$

$\therefore \boxed{\nabla \times \vec{A} = \vec{F}}$

(b) Let S be the portion of the sphere $x^2 + y^2 + z^2 = 13$ where $x \le 3$, oriented with outward-pointing normal vector. Find the flux of $\boldsymbol{F}$ through S. *Hint:* use the result of part (a). Box your answer



if $\nabla \times \vec{A} = \vec{F}$, that means that, for a given boundary curve ∂S, $\iint_S \vec{F} \cdot d\vec{S}$ for any S, will be the same. (with the same orientation)

$\iint_S \vec{F} \cdot d\vec{S} = \iint_S \nabla \times \vec{A} \cdot d\vec{S}$

$= \oint_{\partial S} \vec{A} \cdot d\vec{r}$ (Stokes' thm)

$= \boxed{-24\pi}$

$9 + y^2 + z^2 = 13$

$C: y^2 + z^2 = 4, \; x = 3$

$\vec{r} = \langle 3, 2\cos t, 2\sin t \rangle$

$t: 2\pi, 0$

$\vec{r}\,'(t) = \langle 0, -2\sin t, 2\cos t \rangle$ $\int_{2\pi}^{0} \langle 0, -2\sin t, 2\cos t \rangle \cdot \langle \ldots, -6\sin t, 6\cos t \rangle \, dt$

$= \int_{2\pi}^{0} (12\sin^2 t + 12\cos^2 t) dt = \int_{2\pi}^{0} 12 \, dt$

(c) Let S' be the portion of the sphere $x^2 + y^2 + z^2 = 13$ where $x \ge 3$, oriented with outward-pointing normal vector. Find the flux of $\boldsymbol{F}$ through S'. Box your answer
You don't need to show your work for this part of the problem.

$\iint_{S'} \vec{F} \cdot d\vec{S} = \iint_{S'} \nabla \times \vec{A} \cdot d\vec{S} = \oint_{\partial S'} \vec{A} \cdot d\vec{r}$

$= -\oint_{\partial S} \vec{A} \cdot d\vec{r} = \boxed{24\pi}$

5. (12 points) Given that C is a simple closed curve in the plane $x+y+z=1$ (oriented counterclockwise when viewed from above) that encloses a surface area of 5, compute $\int_C \boldsymbol{F} \cdot d\boldsymbol{r}$ for $\boldsymbol{F} = \langle 3z, 2x, 4y \rangle$.

Box your answer *Hint:* it may be helpful to remember that $\iint_S \boldsymbol{G} \cdot d\boldsymbol{S} = \iint_S (\boldsymbol{G} \cdot \boldsymbol{n}) dS$.

$$\nabla \times \vec{F} = \langle \underset{\frac{\partial z}{\partial y}}{4} - \underset{\frac{\partial y}{\partial z}}{0}, \underset{\frac{\partial x}{\partial z}}{3} - \underset{\frac{\partial z}{\partial x}}{0}, \underset{\frac{\partial y}{\partial x}}{2} - \underset{\frac{\partial x}{\partial y}}{0} \rangle = \langle 4, 3, 2 \rangle$$

let $\partial S = C$, for S = enclosed area in the plane $x+y+z=1$, $\vec{n}$ is up

Stokes' thm

$$\oint_C \vec{F} \cdot d\vec{r} = \iint_S \nabla \times \vec{F} \cdot d\vec{S} = \iint_S \langle 4,3,2 \rangle \cdot \langle 1,1,1 \rangle dS$$

$$= (4+3+2) \iint_S dS = 9\,(\text{area}(5)) = 9(5) = \boxed{45}$$

$$\langle 0, \tfrac{1}{3}y^3, y^2 \rangle$$

$$\vec{F}\cdot d\vec{S} = (\vec{n}_z)\, y^2$$

6. (12 points) Let $\boldsymbol{F} = \langle z^2x, \frac{1}{3}y^3 + \sin^2 z, x^2z + y^2\rangle$.

(a) Let $\mathcal{D}$ be the unit disk $x^2+y^2 \le 1$ in the xy-plane, oriented downward. Compute $\iint_{\mathcal{D}} \boldsymbol{F}\cdot d\boldsymbol{S}$.

It may be helpful to know that $\int_0^{2\pi} \sin^2\theta\, d\theta = \pi$. Box your answer

Hint: if $\mathcal{D}$ is parametrized via $G(r,\theta) = \langle r\cos\theta, r\sin\theta, 0\rangle$ then $\boldsymbol{N} = \pm\langle 0,0,r\rangle$.

$G(r,\theta) = \langle r\cos\theta, r\sin\theta, 0\rangle \qquad \vec{N} = -\langle 0, 0, r\rangle$

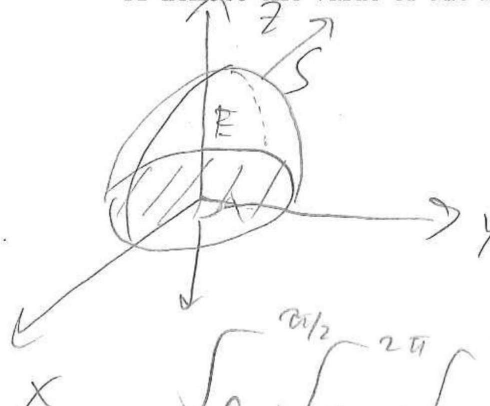
$$\iint_D -\langle 0,0,r\rangle \cdot \langle \ldots, \ldots, 0 + r^2\sin^2\theta\rangle\, dS$$

$$= \int_0^{2\pi}\int_0^1 -r^3 \sin^2\theta\, dr\, d\theta$$

$$= -\underbrace{\left[\tfrac{1}{4}r^4\right]_0^1}_{1/4} \underbrace{\int_0^{2\pi} \sin^2\theta\, d\theta}_{\pi} = \boxed{-\frac{\pi}{4}}$$

(b) Let $\mathcal{S}$ be the top half of the sphere $x^2+y^2+z^2 = 1$, oriented upward. Compute $\iint_{\mathcal{S}} \boldsymbol{F}\cdot d\boldsymbol{S}$.

Box your answer *Hint:* you should use your answer to part (a). If you cannot do part (a), let A denote the value of the integral in part (a) and give your answer in terms of A.



(Divergence Thm)

$$\iint_S \vec{F}\cdot d\vec{S} + \iint_D \vec{F}\cdot d\vec{S} = \iiint_E \nabla\cdot\vec{F}\, dV$$

$$\iint_S \vec{F}\cdot d\vec{S} = \iiint_E \nabla\cdot\vec{F}\, dV - \iint_D \vec{F}\cdot d\vec{S}$$

$$\int_0^{\pi/2}\int_0^{2\pi}\int_0^1 \rho^2\sin\phi\, \rho^2\, d\rho\, d\theta\, d\phi = 2\pi\int_0^{\pi/2}\int_0^1 \rho^4 \sin\phi\, d\rho\, d\phi$$

$\nabla\cdot\vec{F} = z^2 + y^2 + x^2 = \rho^2 \qquad 2\pi\left[\tfrac{1}{5}\rho^5\right]_0^1 \int_0^{\pi/2} \sin\phi\, d\phi$

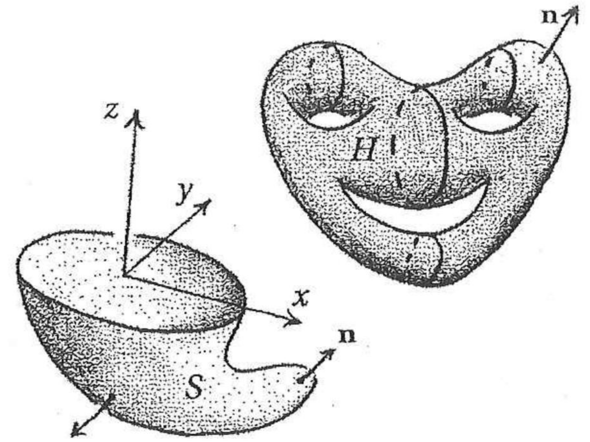
$$= \frac{2\pi}{5}\left[-\cos\phi\right]_0^{\pi/2} = \frac{1}{5}\left(-(0) + (+(1))\right) = \frac{2\pi}{5}$$

$$\iint_S \vec{F}\cdot d\vec{S} = \frac{2\pi}{5} + \frac{\pi}{4} = \frac{8\pi + 5\pi}{20} = \boxed{\frac{13\pi}{20}}$$

7. (15 points) Multiple choice. Circle the correct answer.

Let S and H be the surfaces shown to the right. The boundary of S is the unit circle in the xy-plane, while H has no boundary. Let $\boldsymbol{G} = \langle x, y, z \rangle$.

$\nabla \cdot \vec{G} = 3$



(a) The flux $\iint_H \boldsymbol{G} \cdot d\boldsymbol{S}$ is

negative zero positive

(b) The flux $\iint_S \boldsymbol{G} \cdot d\boldsymbol{S}$ is

negative zero positive

$\iint \vec{G} \cdot d\vec{S} = 0$ unit circle

Hint for (b): use the divergence theorem.

(c) The flux $\iint_S \underbrace{\operatorname{curl} \boldsymbol{G}}_{0} \cdot d\boldsymbol{S}$ is

negative zero positive

$G = \nabla g, \quad g = \frac{1}{2}x^2 + \frac{1}{2}y^2 + \frac{1}{2}z^2$

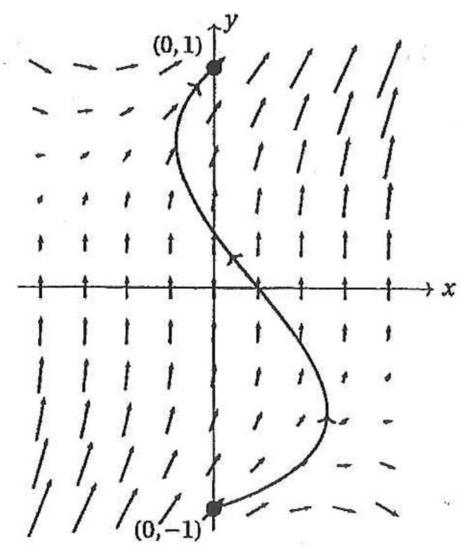
(d) A vector field is shown to the right. For scale, $\boldsymbol{F}(0,0) = \langle 0, 0.1 \rangle$.

path indep.

Given that $\boldsymbol{F}$ is conservative, estimate $\int_C \boldsymbol{F} \cdot d\boldsymbol{r}$, where C is the curve shown from $(0,-1)$ to $(0,1)$.

$-0.5 \qquad -0.2 \qquad 0 \qquad 0.2 \qquad 0.5$

$0.1(2) = 0.2$



(e) Which of the following statements makes sense and is true for any vector field $\boldsymbol{F}$ in $\mathbb{R}^3$ whose components have continuous second-order partial derivatives?

$\nabla(\operatorname{curl} \boldsymbol{F}) = 0 \qquad \operatorname{div}(\operatorname{curl} \boldsymbol{F}) = 0 \qquad \operatorname{div}(\nabla \boldsymbol{F}) = 0 \qquad \operatorname{curl}(\operatorname{curl} \boldsymbol{F}) = 0$

8. (15 points) Multiple choice. Circle the correct answer.

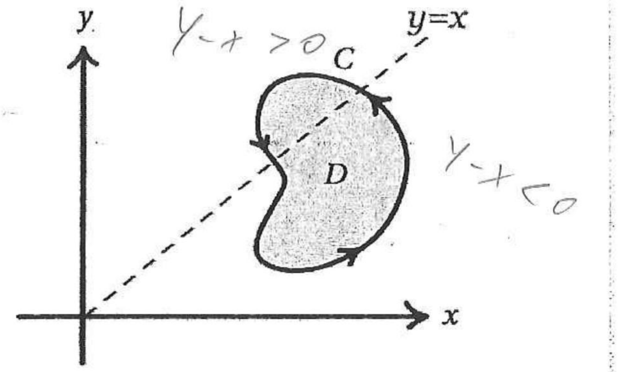
Consider the region D in the plane bounded by the curve C as shown to the right. For parts (a)–(c), circle the best answer.

(a) For $\boldsymbol{F}(x,y) = \langle x+1, y^2 \rangle$, the integral $\int_C \boldsymbol{F} \cdot d\boldsymbol{r}$ is $= \iint_D \text{curl}_z \vec{F} \cdot d\vec{S} = 0$

$\vec{F} = \nabla f$

$f = \frac{1}{2}x^2 + x + \frac{1}{3}y^3$

negative   zero   positive



$y - x > 0$

$y - x < 0$

(b) The integral $\int_C (-y\,dx + 2dy)$ is $\langle -y, 2 \rangle$

$= \iint 1\,dA$

negative   zero   positive

$(\nabla \times \vec{F})_z = 0 + 1$

(c) The integral $\iint_D (y-x)\,dA$ is

negative   zero   positive

Hint for (c): look at the location of D in the plane.

$\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 \quad A = \frac{1}{2}$

$\begin{vmatrix} 1 & 2 \\ -1 & 3 \end{vmatrix} = 3 + 2 = 5 \quad A = \frac{1}{2}(5)$

(d) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation of the plane sending the triangle with vertices $(0,0)$, $(1,0)$, $(0,1)$ to the triangle with vertices $(0,0)$, $(1,2)$, $(-1,3)$, respectively. Find the Jacobian of T.

1   2   3   4   5

$A = \frac{1}{2}(1)(1) = \frac{1}{2}$

$6 - 1 - 1 - \frac{3}{2} = 4 - \frac{3}{2} = 3 - \frac{1}{2} = \frac{5}{2}$

$6 - \frac{1}{2}(1)(2) - \frac{1}{2}(2)(1) - \frac{1}{2}(1)(3)$

$\dfrac{\frac{5}{2}}{\frac{1}{2}}$

(e) Let $\mathcal{R} = [1,2] \times [1,2]$ and let $\mathcal{D} = G(\mathcal{R})$, where G is the map $G(u,v) = (u^2/v, v^2/u)$. Compute the area of $\mathcal{D}$.

1   2   3   4   5

$\dfrac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{2u}{v} & -\frac{u^2}{v^2} \\ -\frac{v^2}{u^2} & \frac{2v}{u} \end{vmatrix} = 4 - 1 = 3$

$\int_1^2 \int_1^2 3\,du\,dv = 3$

9. (10 points) Fill in the blanks in the big theorems of vector calculus.

The fundamental theorem of line integrals. If $\mathcal{C}$ is an oriented curve from P to Q in $\mathcal{D}$ then

$$\int_{\mathcal{C}} \nabla f \cdot d\boldsymbol{r} = f(Q) - f(P).$$

Green's theorem. Let $\mathcal{D}$ be a domain whose boundary ∂D is a simple closed curve, oriented $\boxed{\text{counter-clockwise}}$. Then

$$\iint_{\mathcal{D}} \boxed{\mathrm{curl}_z(\vec{F})}\, dA = \oint_{\boxed{\partial D}} \boldsymbol{F} \cdot d\boldsymbol{r}.$$

Stokes' theorem. Let $\mathcal{S}$ be a "sufficiently nice" surface, and let $\boldsymbol{F}$ be a vector field whose components have continuous partial derivatives on an open region containing $\mathcal{S}$. Then

$$\iint_{\mathcal{S}} \mathrm{curl}(\boldsymbol{F}) \cdot d\boldsymbol{S} = \boxed{\oint_{\partial S} \vec{F} \cdot d\vec{r}}.$$

The integral on the right-hand side is defined relative to the boundary orientation of ∂S.

The divergence theorem. Let $\mathcal{S}$ be a closed surface that encloses a region $\mathcal{W}$ in $\mathbb{R}^3$. Assume that $\mathcal{S}$ is piecewise smooth and is oriented by normal vectors pointing $\boxed{\text{outward}}$. Let $\boldsymbol{F}$ be a vector field whose domain contains $\mathcal{W}$. Then

$$\boxed{\iiint_W \nabla \cdot \vec{F}\, dV} = \iint_{\partial \mathcal{W}} \boldsymbol{F} \cdot d\boldsymbol{S}.$$

You may use this page for scratch work.