Quiz 2 Answer

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1.

$$\begin{aligned}\int_W 1\,d(x,y,z) &= \int_0^\infty \int_0^\infty \int_{[2x+4y,x+y+1]} 1\,dz\,dx\,dy \\ &= \int_D \int_{2x+4y}^{x+y+1} 1\,dz\,d(x,y) = \int_D 1-x-3y\,d(x,y),\end{aligned}$$

where

$$\begin{aligned}D &= \{(x,y)|\,x,y\geq 0,\ 2x+4y\leq x+y+1\} \\ &= \{(x,y)|\,x,y\geq 0,\ x+3y\leq 1\} = \{(x,y)|\,y\geq 0,\ 0\leq x\leq 1-3y\}.\end{aligned}$$

So

$$\begin{aligned}\int_W 1\,d(x,y,z) &= \int_D 1-x-3y\,d(x,y) \\ &= \int_0^\infty \int_{[0,1-3y]} 1-x-3y\,dx\,dy = \int_0^{\frac{1}{3}} \int_0^{1-3y} 1-x-3y\,dx\,dy,\end{aligned}$$

since $0\leq 1-3y$ precisely when $y\leq \dfrac{1}{3}$. Therefore,

$$\int_W 1\,d(x,y,z) = \int_0^{\frac{1}{3}} \frac{1}{2}(1-3y)^2\,dy = \frac{1}{18}.$$

If setting up in $dy\,dx$ instead, you would get

$$\int_W 1\,d(x,y,z) = \int_0^1 \int_0^{\frac{1-x}{3}} 1-x-3y\,dy\,dx.$$

2.

$$\begin{aligned}D &= \{(x,y)|\,x,y\geq 0,\ x^2\leq 2-y^3\} \\ &= \{(x,y)|\,x,y\geq 0,\ 2-y^3\geq 0,\ x^2\leq 2-y^3\} \\ &= \{(x,y)|\,x,y\geq 0,\ 2-y^3\geq 0,\ x\leq \sqrt{2-y^3}\} \\ &= \{(x,y)|\,0\leq y\leq 2^{\frac{1}{3}},\ 0\leq x\leq \sqrt{2-y^3}\}.\end{aligned}$$

So

$$\int_D x\,d(x,y) = \int_0^{2^{\frac{1}{3}}} \int_0^{\sqrt{2-y^3}} x\,dx\,dy = \int_0^{2^{\frac{1}{3}}} \frac{1}{2}(2-y^3)\,dy = 2^{\frac{1}{3}} - \frac{2^{\frac{4}{3}}}{8} = 0.944941.$$

3(i) $(5\cos\frac{\pi}{5}, 5\sin\frac{\pi}{5})$

3(ii) Calculator gives $\tan^{-1}(\frac{-3}{-1}) = 1.249046$. Since $x,y\leq 0$, we have $\theta\in(\pi,\frac{3\pi}{2})$. So $\theta = 4.390638$. Also $r=\sqrt{1^2+3^2}=\sqrt{10}$. Answer: $(\sqrt{10}, 4.390638)$.

3(iii) The curve includes the point $r=0$.
For $r>0$, we have $r=2\cos^3\theta \Leftrightarrow r^4 = 2(r\cos\theta)^3 = 2x^3 \Leftrightarrow (x^2+y^2)^2 = 2x^3$.
Answer: $(x^2+y^2)^2 = 2x^3$.