Quiz 1 Answer

1(i) The nth Riemann sum of f on $[0,1]$ using right endpoints is

$$
\begin{aligned}
&\frac{1}{n}\left(f\left(\frac{1}{n}\right)+f\left(\frac{2}{n}\right)+\ldots+f\left(\frac{n}{n}\right)\right)\\
=&\frac{1}{n}(e^{\frac{3}{n}}+e^{\frac{6}{n}}+\ldots+e^{\frac{3n}{n}})\\
=&\frac{1}{n}e^{\frac{3}{n}}(1+e^{\frac{3}{n}}+\ldots+e^{\frac{3(n-1)}{n}})=\frac{1}{n}e^{\frac{3}{n}}\frac{1-(e^{\frac{3}{n}})^{(n-1)+1}}{1-e^{\frac{3}{n}}}=\frac{1}{n}e^{\frac{3}{n}}\frac{1-e^3}{1-e^{\frac{3}{n}}}.
\end{aligned}
$$

1(ii)

$$
\begin{aligned}
&\lim_{n\to\infty}\frac{1}{n}\left(f\left(\frac{1}{n}\right)+f\left(\frac{2}{n}\right)+\ldots+f\left(\frac{n}{n}\right)\right)\\
=&\lim_{n\to\infty}\frac{1}{n}e^{\frac{3}{n}}\frac{1-e^3}{1-e^{\frac{3}{n}}}\\
=&\lim_{n\to\infty}\frac{1}{n}\frac{1-e^3}{1-e^{\frac{3}{n}}}\\
=&\lim_{h\to 0}h\frac{1-e^3}{1-e^{3h}}=(1-e^3)\lim_{h\to 0}\frac{h}{1-e^{3h}}\overset{\text{L'hôpital}}{=}(1-e^3)\lim_{h\to 0}\frac{1}{-3e^{3h}}=\frac{e^3-1}{3}.
\end{aligned}
$$

2. Let $f(x,y)=x+y^2$. Then $S_{4,2}$ for $\int_{[0,2]\times[0,3]}f(x,y)\,d(x,y)$ using upper-right vertices is

$$
\begin{aligned}
&\frac{2}{4}\cdot\frac{3}{2}[f(0.5,1.5)+f(1,1.5)+f(1.5,1.5)+f(2,1.5)+f(0.5,3)+f(1,3)+f(1.5,3)+f(2,3)]\\
=&\frac{3}{4}(2.75+3.25+3.75+4.25+9.5+10+10.5+11)=41.25
\end{aligned}
$$

3.

$$
\begin{aligned}
\int_{[0,2]\times[0,1]}(x+y)^9\,d(x,y) &= \int_0^1\int_0^2(x+y)^9\,dx\,dy\\
&= \int_0^1 \frac{1}{10}(x+y)^{10}\Big|_{x=0}^{x=2}\,dy\\
&= \int_0^1 \frac{1}{10}(y+2)^{10}-\frac{1}{10}y^{10}\,dy\\
&= \frac{1}{110}(y+2)^{11}-\frac{1}{110}y^{11}\Big|_{y=0}^{y=1}\\
&= \frac{3^{11}-1}{110}-\frac{2^{11}}{110}.
\end{aligned}
$$