Total score: 30 points

- Write your solutions on some papers. Show all your work. Scan as a pdf/jpg file(s). Upload the pdf/jpg file(s) as CCLE Assignment Midterm 2 before the end time.
- Open book. Calculators are not prohibited. But you cannot get any help from other people.
- You may compute any integral using Fundamental Theorem of Calculus without using the definition involving Riemann sum.
- If your final answer is a number, have it in 6 decimal places.
- You may use calculator to compute an integral like $\int_0^1 \int_0^{2x} \int_x^{x+y} x+y+z\,dz\,dy\,dx$ without showing your work for computing such integral. But you cannot use calculator to compute an integral like $\int_D 1\,d(x,y)$ or $\int_0^1 \int_0^1 \int_{[x,y]} x+y+z\,dz\,dy\,dx$ or an integral involving min/max without showing your work. If your set up (that can be put into calculator) is correct, you use calculator without showing your work and your answer happens to be wrong, 1 point deducted, even if you carelessly made the wrong input into the calculator.
- Recommended: After submission, logout and log in CCLE. See if your file is there; download the file you submitted and check if it is the file you intended to submit.

1. (6 points) Consider the 2D domain $D = \left\{(x,y)|\, y>0,\ \frac{1}{2} < x^2+y^2 < x\right\}$.
   Calculate $\int_D 1\,d(x,y)$ using $\int\int dr\,d\theta$ set up.

2. (7 points) Consider the 3D domain $W = \left\{(x,y,z)|\, 2 < x^2+y^2+z^2 < 3,\ \sqrt{x^2+y^2} > 2z > 0\right\}$.
   Calculate $\int_W x^2+y^2+z^2\,d(x,y,z)$ using $\int\int\int d\rho\,d\phi\,d\theta$ set up.

3. (6 points) Consider the 2D domain $D = \{(x,y)|\, 1 \le xy \le 2,\ 3 \le xy^2 \le 4\}$.
   Compute the integral $\int_D x\,d(x,y)$ using the change of variables $x = \frac{u^2}{w}$ and $y = \frac{w}{u}$.

4. (5 points) Consider the vector field
$$F(x,y,z) = \begin{bmatrix} x-z \\ yz \\ xy^2z \end{bmatrix}.$$
   (i) Compute $\text{div}(F)$
   (ii) Compute $\text{curl}(F)$
   (iii) Suppose that $\mathcal{C}$ is an oriented curve in $\mathbb{R}^3$ and that $r(t) = (t^2-3,\ t,\ 2t)$, for $0 \le t \le 2$, is a positively oriented parametrization of $\mathcal{C}$. Compute the line integral $\int_{\mathcal{C}} F \cdot dr$.

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5. (6 points) Consider the curve $\mathcal{C} = \{(t,\, t^2 + 2) |\, 0 \leq t \leq 3\}$.

   (i) Compute the line integral $\displaystyle\int_{\mathcal{C}} x\, d(x, y)$.

   (ii) Suppose that $\mathcal{C}$ is an oriented curve and that $r(t) = (t,\, t^2 + 2)$, for $0 \leq t \leq 3$, is a positively oriented parametrization of $\mathcal{C}$. Let $F(x, y) = \begin{bmatrix} y + x^2 \\ x^3 \end{bmatrix}$. Find the flux of F across $\mathcal{C}$.

End of exam