Total score: 30 points

- Write your solutions on some papers. Scan as pdf/jpg file(s). Upload the **pdf/jpg** file(s) as CCLE Assignment Midterm 2 before the end time.
- Open book. Calculators are not prohibited. But you cannot get any help from other people. Open internet. Except for online calculator, using any of the online forum or simply google something (and then find something useful and copy down to your answer) could easily lead to plagiarism.
- Unless specified otherwise, you may compute any integral using Fundamental Theorem of Calculus without using the definition involving Riemann sum.
- If your final answer is a number, have it in 6 decimal places or in fraction.
- You may use calculator to compute an integral like $\int_0^1 \int_0^{2x} \int_x^{x+y} x+y+z\,dz\,dy\,dx$ without showing your work for computing such integral. But you cannot use calculator to compute an integral like $\int_D 1\,d(x,y)$ or $\int_0^1 \int_0^1 \int_{[x,y]} x+y+z\,dz\,dy\,dx$ or an integral involving min/max without showing your work.
- **Recommended: After submission, logout and log in CCLE. See if your file is there; download the file you submitted and check if it is the file you intended to submit.**
  Avoid the following: Upload wrong file or upload only the first page (unless it is intended to be only 1 page; in this case, it's ok).

1. (9 points) (i) Is the vector field $F(x,y) = \begin{bmatrix} x^2y \\ y^2x \end{bmatrix}$ conservative where the domain is $\mathbb{R}^2$? If yes, find the potential function. If no, justify your answer.
   (ii) Is the vector field $F(x,y) = \begin{bmatrix} y^2+1 \\ 2xy \end{bmatrix}$ conservative where the domain is $\mathbb{R}^2$? If yes, find the potential function. If no, justify your answer.
   (iii) Find the center of mass of $D = \{(x,y)|\, 0 \le x \le 1,\ 0 \le y \le x^2\}$.
   (iv) Consider the curve $\mathcal{C} = \{(4t,\, \cos(2t),\, \sin(2t))|\, 0 \le t \le \pi\}$ in $\mathbb{R}^3$. Compute the line integral $\int_{\mathcal{C}} y^2\,d(x,y,z)$.

2. (4 points) Consider the vector field $F(x,y,z) = \begin{bmatrix} xyz \\ z^2 \\ y^2+z \end{bmatrix}$.
   (i) Compute $\text{div}(F)$
   (ii) Compute $\text{curl}(F)$.

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3. (4 points) Consider the vector field $F(x,y) = \begin{bmatrix} x^2 y \\ 2x \end{bmatrix}$. Suppose that $\mathcal{C}$ is an oriented curve in $\mathbb{R}^2$ and that $r(t) = (t, t^2)$, for $0 \leq t \leq 1$, is a positively oriented parametrization of $\mathcal{C}$.

   (i) Compute the line integral $\displaystyle\int_{\mathcal{C}} F \cdot dr$.

   (ii) Find the flux of F across $\mathcal{C}$.

4. (7 points) Consider the 3D domain $W = \{(x,y,z) |\, z < x^2 + y^2 + z^2 < 4,\ 0 < y < 3x\}$. Calculate $\displaystyle\int_W \frac{1}{x^2+y^2+z^2}\, d(x,y,z)$ using $\displaystyle\int\int\int d\rho\, d\phi\, d\theta$ set up.

5. (6 points) Consider the 2D domain $D = \{(x,y) |\, 2 < x + y < 4,\ 0 < x^2 - y^2 < 1\}$. Compute the integral $\displaystyle\int_D x - y\, d(x,y)$ using the change of variables $x = u + w$ and $y = u - w$.

   (Note: You will get nonconstant bounds. Easier to set up if you use $\int\int dw\, du$. But you may still set up in $\int\int du\, dw$.)

End of exam