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|---|---|
| 1 | 3 |
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MATH 32B Midterm I, Fall 2016

Name:

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**Justify All Your Answers. No Points Will Be Given Without Sufficient Reasoning/Calculations.**

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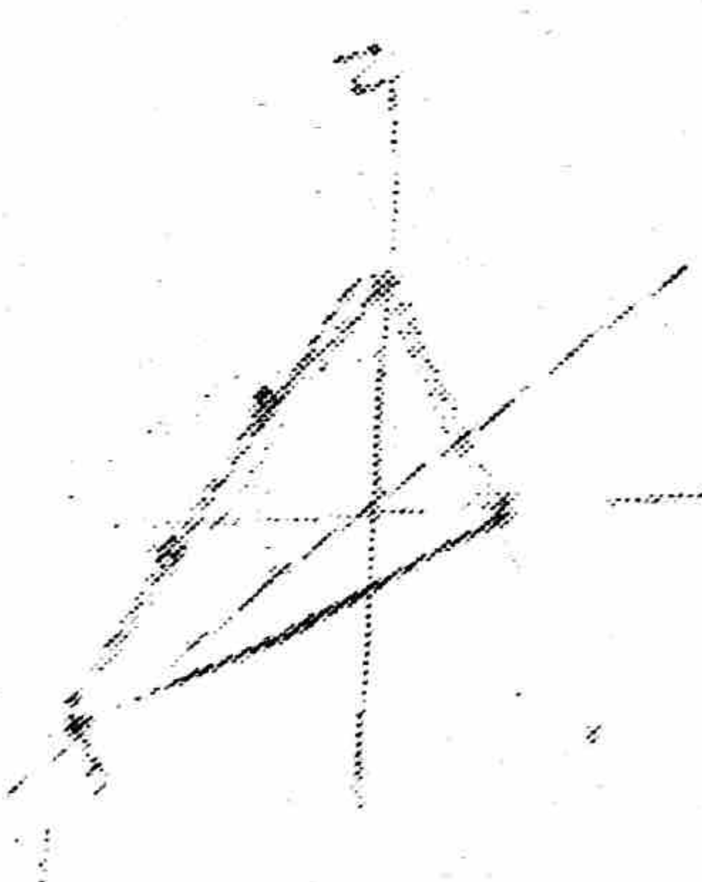
**Problem 1.** (4)

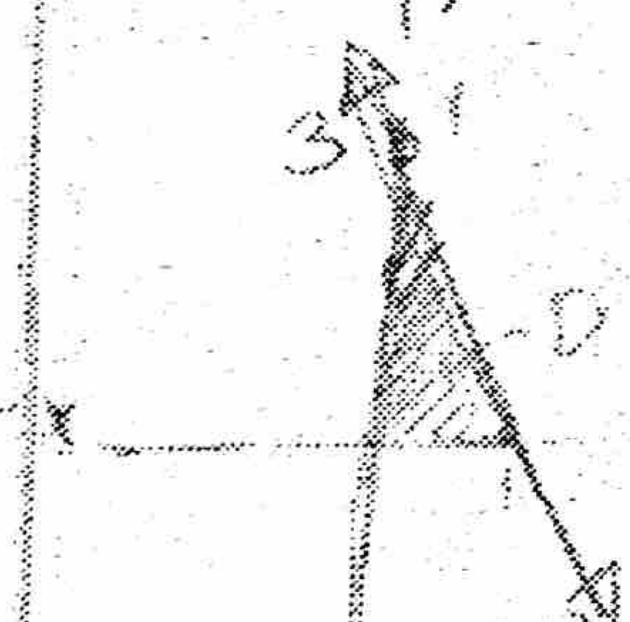
Find the triple integral $\int\int\int_E x\,dx\,dy\,dz$. Here $E$ is the solid bounded by the four planes $x = 0$, $y = 0$, $z = 0$ and $3x + y + z = 3$.

$z = 3 - 3x - y$, $x = 1 - \frac{1}{3}y - \frac{1}{3}z$ $\quad 0 = 3 - 3x - y$

$y = 3 - 3x$, $\frac{y-3}{-3} = \frac{-3x}{-3}$

$1 - \frac{1}{3}y = x$





$D = \begin{matrix} 0 < y < 3 \\ 0 < x < 1 - \frac{1}{3}y \end{matrix}$

$\int_0^1 \int_0^{3-3x} \int_0^{3-y-3x} x \, dz\, dy\, dx = xz\Big|_0^{3-y-3x} = x(3 - y - 3x) = 3x - yx - 3x^2$

$\int_0^{3-3x} 3x - yx - 3x^2 = 3xy - \frac{1}{2}y^2x - 3x^2y\Big|_0^{3-3x} =$

$3x(3-3x) - \frac{1}{2}(3-3x)^2 x - 3x^2(3-3x)$

$9x - 9x^2 - \frac{1}{2}x(9x^2 - 18x + 9) - 9x^2 + 9x^3$

$9x - 18x^2 + 9x^3 - \frac{9}{2}x^3 - 27x^2 - \frac{9}{2}x$

$\int_0^1 \frac{45}{2}x - 45x^2 - \frac{9}{2}x^3\, dx = \frac{45}{4}x^2 - \frac{45}{3}x^3 - \frac{9}{8}x^4\Big|_0^1$

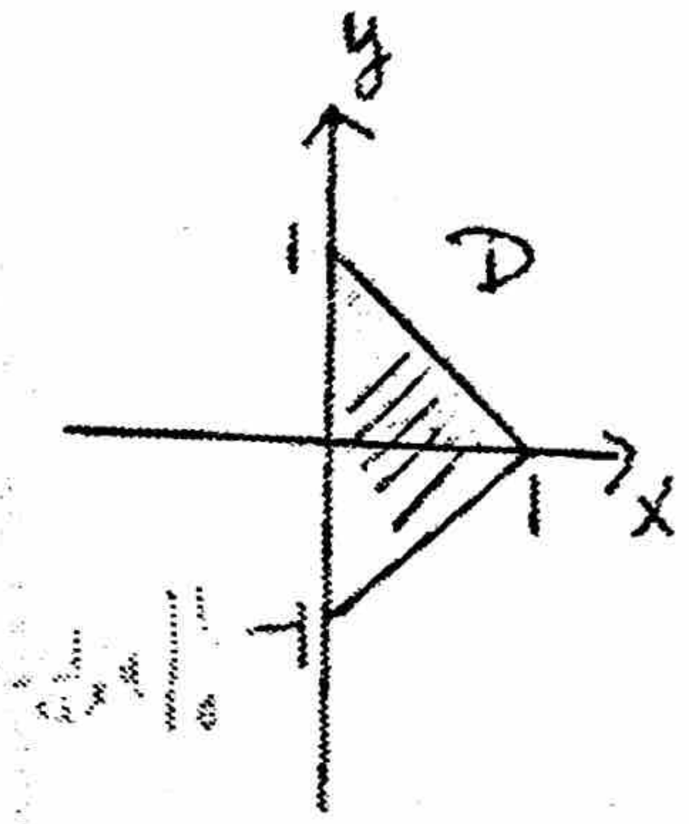
$\frac{45}{4} - \frac{45}{3} + \frac{9}{8} \quad \frac{270}{24} - \frac{360}{24} - \frac{27}{24} = \frac{-117}{24}$

$\begin{array}{r} 3-3x \\ 3\overline{)\,9 - 9x} \\ 9x \;\; 9x^2 \end{array}$

$45 \times 6 = 270$ $\quad 45 \times 8 = 360$ $\quad 360 - 270 = 90$, $90 + 27 = 117$

3.5 **Problem 2.** (4)

Compute the center of of mass of D given by the following picture. Here the density fuction $\rho(x,y) = x^2$.




Total mass: $\iint_D x^2\,dA = \int_0^1 \int_0^{1-x} x^2\,dy\,dx = x^2 y|^{1-x} = x^2(1-x)$

$\int_0^1 x^2 - x^3\,dx = \frac{1}{3}x^3 - \frac{1}{4}x^4\Big|_0^1 = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$

$\int_0^1 \int_{x-1}^{0} x^2\,dy\,dx = x^2 y|^{0} = -(x^2)(x-1) = x^2 - x^3 = \frac{1}{3}x^3 - \frac{1}{4}x^4\Big|_0^1$

$\frac{1}{3} - \frac{1}{4} = \frac{1}{12}$

Total Mass: $\frac{1}{12} \cdot 2 = \underline{\underline{\frac{1}{6}}}$.

$y_{cm} = 0$ b/c it is symmetric about x-axis.

$x_{cm} = \frac{1}{mass}\iint_D$ $\int_0^1\int_0^{1-x} x x^2\,dy\,dx = x^3 y\Big|_0^{1-x}$ $\int_0^1 x^3 - x^4 = \frac{1}{4}x^4 - \frac{1}{5}x^5\Big|_0^1 = \frac{1}{4} - \frac{1}{5} = \frac{1}{20}$

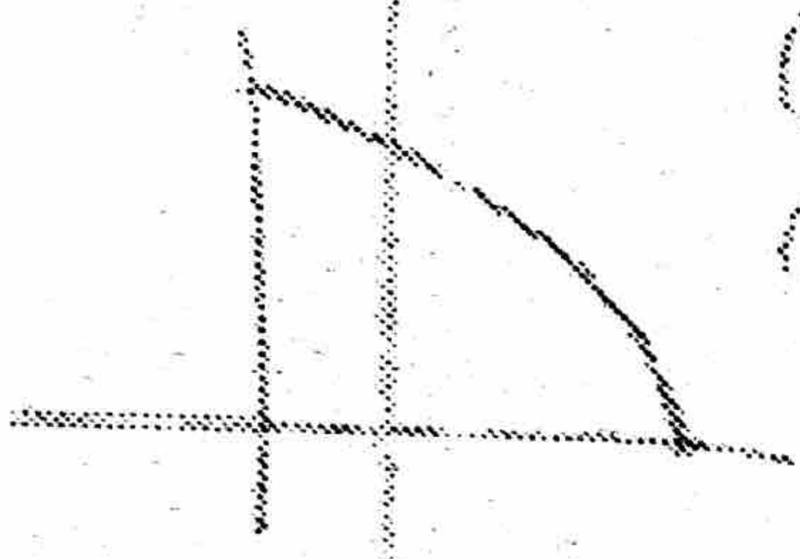
$\int_0^1 \int_{x-1}^{0} x x^2\,dy\,dx = x^3 y\Big|_{x-1}^{0} = \int x^3 - x^4 = \frac{1}{4} - \frac{1}{5} = \frac{1}{20}$

$\boxed{P_{cm} = \left(\frac{1}{10}, 0\right)}$

**Problem 3.** (4)

Find the iterated integral $\int_0^R \int_0^{\sqrt{R^2-x^2}} x \cdot \cos(x^2+y^2)\,dy\,dx$. Here $R$ is a positive constant.

**Hint:** Convert the iterated integral into a double integral and evaluate the double integral.



$0 < \theta < \frac{\pi}{2}$, $x \cdot \cos(x^2+y^2) = r\cos\theta \cdot \cos(r^2)$

$0 < r < R$

$\int_0^{\pi/2} \int_0^R r \cdot \cos\theta \cdot \cos(r^2) \cdot r\,dr\,d\theta =$

$r^2 \cdot \cos\theta \cdot \cos r^2$

$\int_0^{\pi/2} \cos\theta\,d\theta \cdot \int_0^R \cos(r^2) \cdot r^2\,dr$

$u = r^2$

$du = 2r$ $\quad dv = \cos r^2$

$1$

$\sin(r^2) = \sin R^2$

$\int_0^R \int_0^{\sqrt{R^2-x^2}} x \cdot \cos(x^2+y^2)\,dy\,dx =$

$\phi = -\frac{1}{2}(\cos\theta)^2$ $\qquad \int_0^{2\pi}\int_0^{\pi}\int_0^{\rho}\rho^2\sin\phi\, d\rho\, d\phi\, d\theta$ $\quad 3/4$

$d\phi = -\frac{2}{2}(\sin)(\cos\theta)'$ $\qquad (2\pi)\left(-\cos\phi\Big|_0^{\pi}\right)\left(\frac{1}{3}\rho^3\right)$

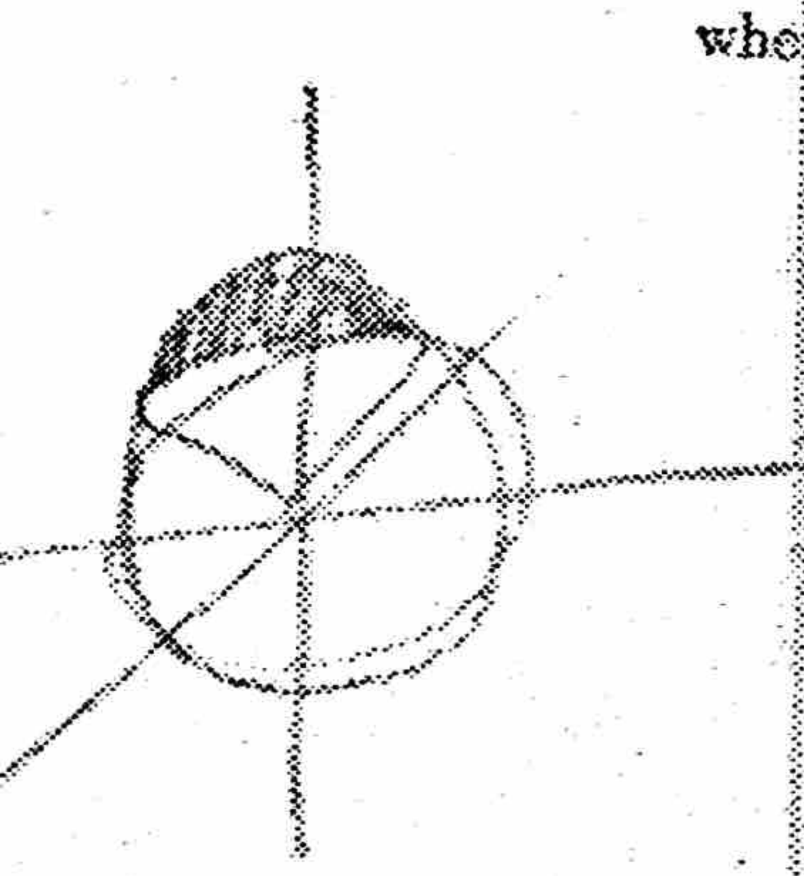
$(2\pi)(1+1)\left(\frac{1}{3}\rho^3\right) = \frac{4\pi}{3}\rho^3$

**Problem 4.** (4)

Find

$$\iiint_E z\, dxdydz,$$

where $E$ is bounded by the sphere $x^2 + y^2 + z^2 = z$ and the cone $z = \sqrt{x^2 + y^2}$. = Sphere − Cone



$x^2 + y^2 + z^2 - z = 0$

$x^2 + y^2 + z^2 - z + \frac{1}{4} = \frac{1}{4}$ $\qquad \cancel{\rho\cos\phi}$

$\cos\phi = 1$

$x^2 + y^2 + (z - \frac{1}{2})^2 = \frac{1}{4}$ $\quad ? \ -1.5 \qquad \phi = 0 \text{ or } 2\pi$

$$\int_0^{2\pi}\int_0^{\pi/4}\int_0^{\rho} \rho^3 \sin\phi \cos\phi =$$

$$\frac{1}{4}\rho^4 \cdot 2\pi \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2}\,\rho^4}{4} \ ??$$

$-1.5$

**Problem 5.** (4)

Estimate the following integral

$$\iiint_E e^{\cos x \cdot \cos y \cdot \cos z}\, dV,$$

where E is the region inside the ~~cylinder~~ $x^2 + y^2 = z^2$ with $0 \le z \le 1$.

cone

0