$$2\int\left(\frac{1}{3} - x + x^2 - \frac{x^3}{3}\right)dx = 2\left[\frac{1}{3}x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{12}\right]_0^1 = 2\left[\frac{1}{3} - \frac{1}{2} + \frac{1}{3} - \frac{1}{12}\right]$$

$$= 2\left[\frac{1}{6} - \frac{1}{12}\right] = \frac{2}{12} = \boxed{\frac{1}{6}}$$

1	3
2	1
3	0
4	C
5	O
T	4

MATH 32B Midterm I, Fall 2010

Name: [redacted] **TA's Name and Section Number:** [redacted]

Dis 1C

Problem 1. (4)

Find the triple integral $\int\int\int_E z\,dx\,dy\,dz$. Here E is the solid bounded by the four planes $x = 0$, $y = 0$, $z = 0$ and $2x + 2y + z = 2$.

$z = 2 - 2x - 2y$ $\qquad$ $2y = 2 - 2x \Rightarrow y = \cancel{2}(1-x)$

Good.

$$\cancel{\int_0^{2-2x-2y}\int_0^{1-x}\int_0^1 z\,dx\,dy\,dz} = \int_0^1\int_0^{1-x}\int_0^{2-2x-2y} z\,dz\,dy\,dx$$

$$\int_0^1\int_0^{1-x}\left.\frac{z^2}{2}\right]_0^{2-2x-2y} dy\,dx = \int_0^1\int_0^{1-x}\frac{2^2(1-x-y)^2}{2}\,dy\,dx = \int_0^1\int_0^{1-x} 2(1-\cancel{2x}-\cancel{2y}+x^2+\cancel{2xy}+y^2)\,dy\,dx$$

$$\int_0^1\left[y - 2xy - y^2 + x^2y + xy^2 + \frac{y^3}{3}\right]_0^{1-x} dx = \int_0^1 (1-x) - (2x - 2x^2) - (1 - 2x + x^2) + (x^2 - x^3) + (2x^2 - 2x^3) + (2x - 4x^2 + 2x^3)\,dx$$

$$= \int_0^1 \frac{-x}{3} + \frac{8}{3}x^2 - \frac{7}{3}x^3\,dx = \left[\frac{-x^2}{6} + \frac{8}{9}x^3 - \frac{7}{12}x^4\right]_0^1 = \frac{-1}{6} + \frac{8}{9} - \frac{7}{12} = \frac{-9}{12} + \frac{8}{9} = \frac{-27}{36} + \frac{32}{36} = \boxed{\frac{5}{36}}$$

$1 - 2x + x^2$

$1 \quad 1 - 2x + x^2$

$-x - x + 2x^2 - x^3$

$1 - 3x + 3x^2 - x^3$

$$\int_0^1\int_0^{1-x}\int_0^{2-2x-2y} z\,dz\,dy\,dx$$

3

$$\frac{1}{2}\int_0^1\int_0^{1-x} z^2\Big]_0^{2-2x-2y} dy\,dx = \frac{1}{2}\int_0^1\int_0^{1-x}(2-2x-2y)^2\,dy\,dx$$

$$= \frac{1}{2}\int_0^1\int_0^{1-x} 4 - 8x - 8y + 8xy + 4x^2 + 4y^2\,dy\,dx$$

$$= 2\int_0^1\int_0^{1-x} 1 - 2x - 2y + 2xy + x^2 + y^2\,dy\,dx$$

$2 - 2x - 2y$

$2 \quad 4 - 4x - 4y$

$-2x - 4x + 4x^2 + 4xy$

$-2y - 4y + 4xy + 4y^2$

$$= 2\int_0^1 y - 2xy - y^2 + xy^2 + x^2y + \frac{y^3}{3}\Big]_{y=0}^{1-x} dx$$

$$2\int_0^1 (1-x) - 2x(1-x) - (1-x)^2 + x(1-x)^2 + x^2(1-x) + \frac{(1-x)^3}{3}$$

$$= 2\int_0^1 (1-x) - (2x - 2x^2) - (1 - 2x + x^2) + (x - 2x^2 + x^3) + (x^2 - x^3) + \frac{(1 - 3x + 3x^2 - x^3)}{3}\,dx$$

Problem 2. (4)

Find the triple integral $\iiint_E x\,dV$, where E is the region bounded by the paraboloid $x = 4(y^2+z^2)$ and the plane $x = 4$.

$x = 4y^2 + 4z^2 \qquad y = \sqrt{\frac{x}{4}} \qquad z = \sqrt{\frac{x}{4} - y^2}$

$x = 4$

cylindrical.

$$\int_0^4 \int_{-\sqrt{\frac{x}{4}}}^{\sqrt{\frac{x}{4}}} \int_{-\sqrt{x-4y^2}}^{\sqrt{\frac{x}{4}-4y^2}} x\,dz\,dy\,dx$$

$$= 4\int_0^4 \int_0^{\sqrt{\frac{x}{4}}} \int_0^{\sqrt{x-4y^2}} x\,dz\,dy\,dx = 4\int_0^4 \int_0^{\sqrt{x-4y^2}} \left[xz\right]_0^{\sqrt{x-4y^2}} dy\,dx$$

$$= 4\int_0^4 \int_0^{\sqrt{\frac{x}{4}}} \sqrt{x^3 - 4x^2y^2}\,dy\,dx = 4\int_0^4 (x^3-4x^2y^2)^{3/2}$$

Always include

$$\int_0^{2\pi}\int_0^1 \int_{4r^2}^4 x\,dx\,r\,dr\,d\theta$$

$x = 4(y^2+z^2)$, r^2

$$\int_0^{2\pi}\int_0^1 \left[\frac{x^2}{2}\right]_{4r^2}^4 r\,dr\,d\theta$$

$$\frac{1}{2}\int_0^{2\pi}\int_0^1 16r - 16r^5\,dr\,d\theta = \frac{1}{2}\int_0^{2\pi}\left[8r^2 - \frac{16}{6}r^6\right]_0^1 d\theta$$

$$= \frac{1}{2}\int_0^{2\pi}\left[\left(8-\frac{16}{6}\right)\right]d\theta = \frac{1}{2}\int_0^{2\pi}\frac{32}{6}\,d\theta = \frac{16}{6}\int_0^{2\pi}d\theta = \frac{8}{3}(2\pi)$$

$$\boxed{\frac{16\pi}{3}}$$

Correct

$\int_0^1 \int_{y-1}^{1-y} y \, dx \, dy$ $\quad M = [y(1-y) - y(y-1)]$

$[(y-y^2) - (y^2-y)]$

$2y - 2y^2$ $\quad \int_0^1 2y - 2y^2 \, dy = \left[\frac{y^2}{2} - \frac{2y^3}{3}\right]_0^1$

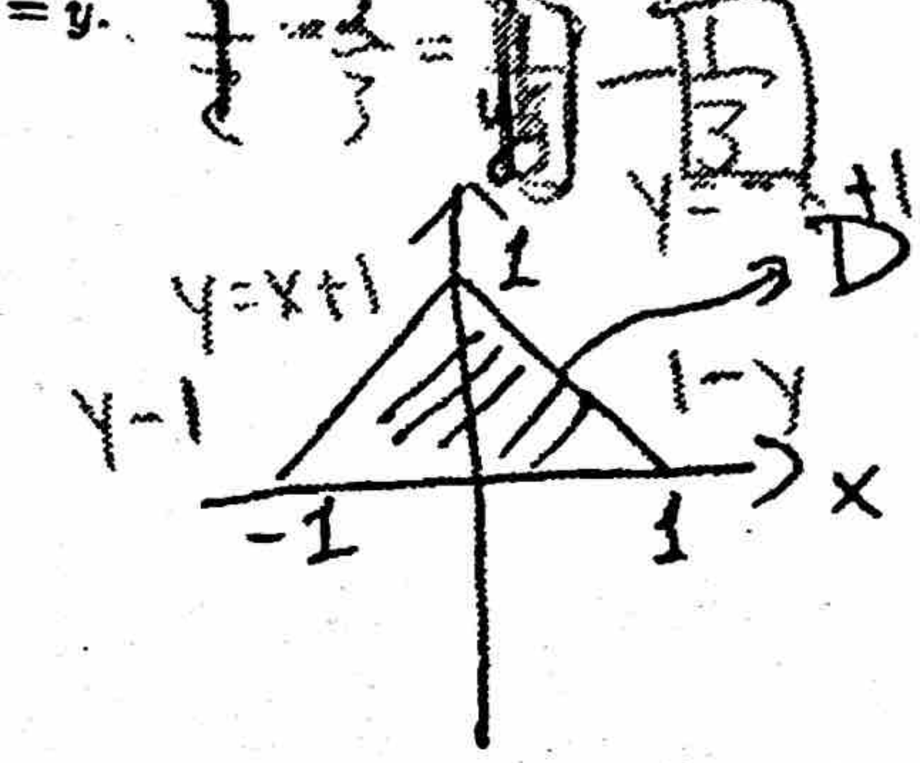
Problem 3. (4)

Compute the center of mass of D. Here the density fuction $\rho(x, y) = y$.

$1 - \frac{2}{3} = \boxed{\frac{1}{3}}$

$\bar{x} = \int_{-1}^{1} \int_{-x+1}^{x+1} y \, \rho(x,y) \, dy \, dx$

$\bar{y} = \text{"} \quad x \, \cancel{\rho(x,y)} \, dy \, dx \qquad y = -x+1$



$\bar{x} = 2\int_{-1}^{1} \left[\frac{y^2}{2}\right]_0^{x+1} dx = \int_{-1}^{1} x^2 + 2x + 1 \, dx = 2\left[\frac{x^3}{3} + x^2 + x\right]_0^1 = 2\left[\frac{1}{3} + 1 + 1\right.$

$2\left(\frac{7}{3}\right) = \cancel{\boxed{\frac{14}{3}}}$

$\bar{y} = 4\int_0^1 \int_0^{x+1} xy \, dy \, dx = 4\int_0^1 \left[\frac{xy^2}{2}\right]_0^{x+1} = 2\int_0^1 x^3 + 2x^2 + x \, dx$

$2\left[\frac{y^3}{3} - \frac{y^4}{4}\right]_0^1 = 2\left[\frac{1}{3} - \frac{1}{4}\right] = \frac{2}{3} - \frac{2}{4} = \frac{1}{6} = \quad 2\left[\frac{x^4}{4} + \frac{2x^3}{3} + \frac{x^2}{2}\right]_0^1$

$= \frac{1}{6} \cdot \frac{6}{1} \cdot \frac{1}{3} = \boxed{\frac{1}{2}}$

$(\bar{x}, \bar{y}) = \boxed{\left(0, \frac{1}{2}\right)}$

$2\left[\frac{1}{4} + \frac{2}{3} + \frac{1}{2}\right] = \frac{1}{2} + \frac{4}{3} + 1$

$= 2\frac{5}{6} = \cancel{\boxed{\frac{17}{6}}}$

$\bar{x} = \int_0^1 \int_{y-1}^{1-y} \frac{xy \, dx \, dy}{\text{mass}} = \frac{1}{2}\int_0^1 \left[x^2 y\right]_{x=y-1}^{1-y} dy$

$= \frac{1}{2}\int_0^1 [(1-y)^2 y - (y-1)^2 y] \, dy = \boxed{0}$

$\cancel{y - 2y^2 + y^3 - (y^3 - 2y^2 + y)}$

$\bar{y} = \int_0^1 \int_{y-1}^{1-y} \frac{y^2 \, dx \, dy}{\text{mass}} = \int_0^1 \left[xy^2\right]_{x=y-1}^{1-y} dy = \int_0^1 [(y^2 - y^3) - (y^3 - y^2)]$

$= \int 2y^2 - 2y^3 \, dy$

Problem 4. (4)

Find the iterated integral

$$\int_0^k \int_{-\sqrt{k^2-x^2}}^{\sqrt{k^2-x^2}} \sin(x^2+y^2)\,dy\,dx.$$

r^2

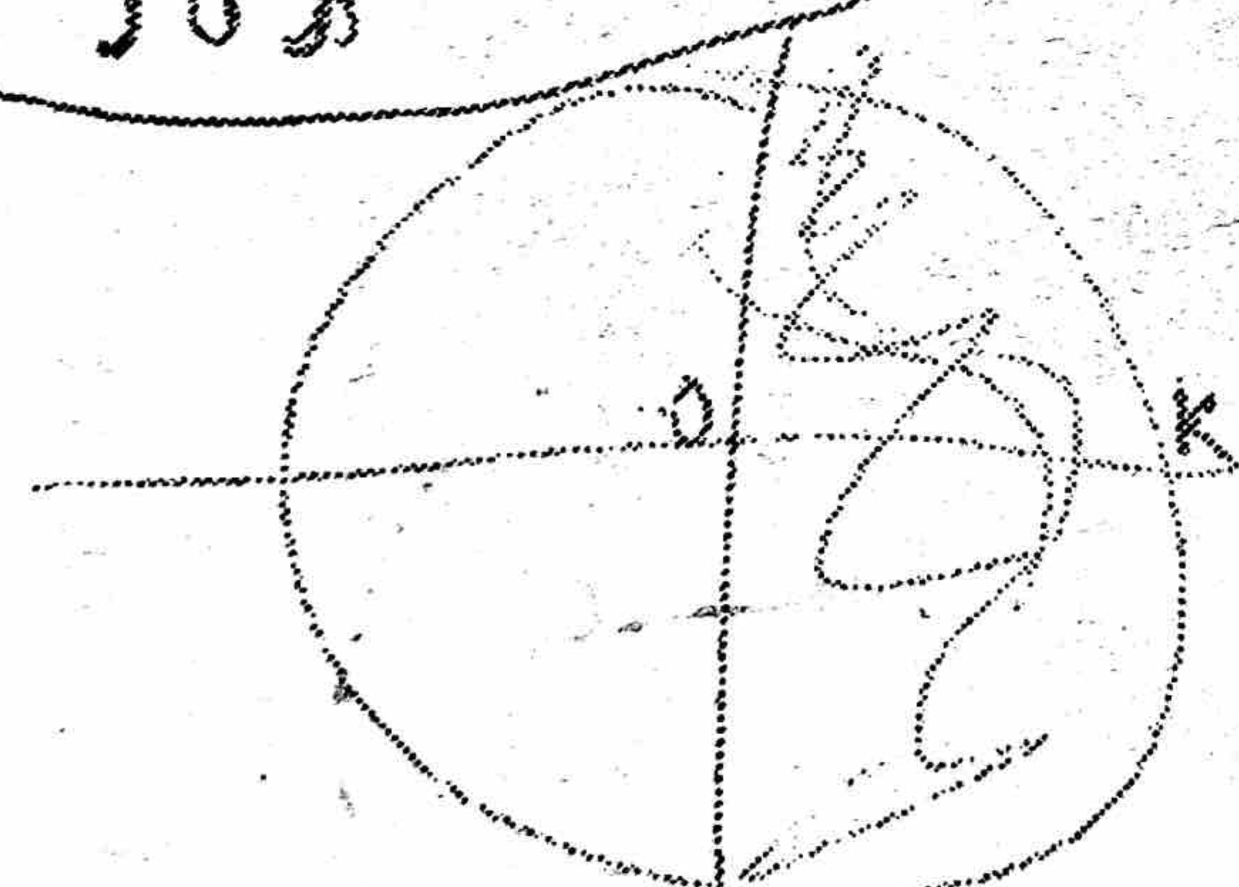
Hint: Convert the iterated integal into a proper double integral and then evaluate the double integral.

$$2\int_0^k \int_0^{\sqrt{k^2-x^2}} \sin(x^2+y^2)\,dy\,dx$$

$$2\int_0^k \int \qquad x = r\cos\theta$$

$$2\int_0^{\pi/2}\int_0^k \sin r^2\, r\,dr\,d\theta$$

0



$$\int_0^{\pi/2} d\theta \int_0^{k^2} \sin u\,du$$

$$\frac{\pi}{2}\left[-\cos u\right]_0^{k^2}$$

$$\frac{\pi}{2}(-\cos k^2 + 1) \qquad u = r^2,\ du = 2r\,dr$$

$$\boxed{\frac{(1-\cos k^2)\pi}{2}}$$

Problem 5. (4)

Find

$$\iiint_E z\,dxdydz,$$

where E is bounded by the sphere $x^2+y^2+z^2=z$ and the cone $z=\sqrt{x^2+y^2}$.

$x^2+y^2+(x^2+y^2)=z$

$2(x^2+y^2)=z$

$\int_{\sqrt{x^2+y^2}}^{2(x^2+y^2)} \int_0^{\sqrt{\frac{z}{2}-x^2}} \int_0^{\sqrt{\frac{z}{2}-y^2}} z\,dxdydz$

$\int_{\sqrt{x^2+y^2}}^{2(x^2+y^2)} \int_0^{\sqrt{\frac{z}{2}-x^2}} [zx]_0^{\sqrt{\frac{z}{2}-y^2}} dydz$

$x^2+y^2+z^2=z$

$x^2+y^2+z^2-z+\frac{1}{4}=0+\frac{1}{4}$

$x^2+y^2+(z-\frac{1}{2})^2=(\frac{1}{2})$

$\int_0^{2\pi}\int_0^{\pi/4}\int_0^{\cos\phi} \rho\cos\phi \quad \rho^2\sin\phi\,d\rho\,d\phi\,d\theta$

$\int_0^{2\pi}\int_0^{\pi/4} \left[\frac{\rho^4}{4}\cos\phi\sin\phi\right]_0^{\cos\phi} d\phi\,d\theta$

$\frac{1}{4}\int_0^{2\pi}\int_0^{\pi/4} \cos^5\phi\sin\phi\,d\phi\,d\theta$

$u=\cos\phi$

$du=-\sin\phi\,d\phi$

$\frac{1}{4}\int_0^{2\pi}\int_1^{\sqrt{2}/2} -u^5\,du\,d\theta$

$=\frac{-1}{4}\int_0^{2\pi}\left[\frac{u^6}{6}\right]_1^{\sqrt{2}/2} d\theta = \frac{-1}{4}\int_0^{2\pi}\left(\frac{\left(\frac{\sqrt{2}}{2}\right)^6}{6}-\frac{1}{6}\right) = \frac{-1}{4}\left[\frac{-7}{48}\theta\right]_0^{2\pi}$

$=\frac{7}{\cancel{2}\cdot\cancel{4}\cdot 48}\cancel{2}\pi = \frac{7\pi}{96}$

$\frac{\cancel{8}}{\cancel{8}\cdot 6}=\frac{1}{48}-\frac{8}{48}=\frac{-7}{48}$