Test 02

Math 32b @ UCLA (SPRING 2021)

Assigned: May 21, 2021.

Instructions/Admonishment

1. SHOW ALL WORK.
   A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit.
2. Duration: 24 hours.
3. The following is my own work, without the aid of any other person.
   Signature: ____

Problem 1 CHANGE OF VARIABLES.

Let $\mathscr{D} = \{(x, y) \in \mathbb{R}^2; |y| + |y - x| \leq 1\}$ and $\mathscr{I} = \displaystyle\iint_{\mathscr{D}} \frac{(2y - x)^2}{x^2 + 3} dA$.

(i) Sketch the graph of the domain $\mathscr{D}$ and then use it to give a change of variables u and v.

(ii) Evaluate the double integral $\mathscr{I}$.

Problem 2 A CONSERVATIVE VECTOR FIELD.

Consider the vector field $\mathbf{F}(x,y,z) = \left\langle 2xy + z^2, x^2, 2xz \right\rangle$.

(i) Show that $\mathbf{F}$ is a conservative vector field and find ϕ such that $\mathbf{F} = \nabla\phi$.

(ii) Use (i) evaluate $\displaystyle\int_{\mathscr{C}} \mathbf{F}\cdot d\mathbf{r}$, where $\mathscr{C}$ consists of the line segment from $(1,0,-2)$ to $(1,1,0)$ followed by the curve given by $\mathbf{r}(t) = \langle e^t, \cos(t), t\rangle$ for $0 \leq t \leq 1$.

Problem 3 DIFFERENT FORMS OF GREEN's THEOREM.

Consider the region $\mathscr{R}$ bounded above by the curve $y = 5 - x^2$ and below by the curve $y = x^2 - 3$ and the vector field $\mathbf{F} = \left\langle xy, xy + 2x \right\rangle$. Using Green's Theorem

(i) find the counterclockwise circulation of $\mathbf{F}$ around the boundary of $\mathscr{R}$,

(ii) find the outward flux of $\mathbf{F}$ across the boundary of $\mathscr{R}$.

Problem 4 THE GRAVITATIONAL POTENTIAL.

The potential function for the gravitational force field due to a mass M at the origin acting on a mass m is $\phi = \dfrac{GMm}{|\mathbf{r}|}$, where $\mathbf{r} =< x, y, z >$ is the position vector of the mass m and G is the gravitational constant.

(i) Compute the gravitational force $\mathbf{F} = -\nabla\phi$

(ii) Show that the field is irrotational, that is $\nabla \times \mathbf{F} = \mathbf{0}$.