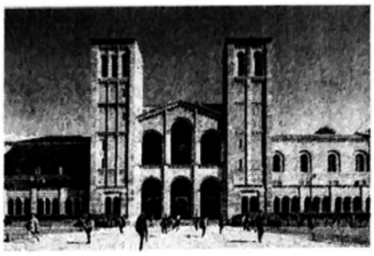
Test 01

MATH 32B @ @ $\mathscr{U}CLA$ (Winter 2022): Midterm 01

Assigned: January 25, 2022.



Instructions/Oath

1. SHOW ALL WORK.
   A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit.
2. Duration: 24 hours.
3. The following is my own work, without the aid of any other person.
   Signature:

Problem 1 *Iterated Double Integral* .

Let $\mathscr{I} = \int_0^3 \int_0^{2-\frac{2}{3}y} dxdy + \int_{-2}^0 \int_0^{y+2} dxdy.$

(i) Reverse the order of integration to combine the sum above into one double integral.

(ii) Evaluate $\mathscr{I}$.

Problem 2 *Double Integral in Polar Coordinates* .

Let $\mathscr{R}$ be the region in xy-plane inside both the circle of radius $r = 2$ centered at $(0, 0)$ and the circle $r = 4\cos\theta$.

(i) Sketch the region $\mathscr{R}$.

(ii) Use polar coordinates to calculate the area $\mathscr{S} = \int\int_{\mathscr{R}} dA$.

Problem 3 ***The Cylindrical Solid***

A solid $\mathscr{W}$ lies within the cylinder $x^2 + y^2 = 1$, below the plane $z = 4$, and above the paraboloid $z = 1 - x^2 - y^2$. The density $\rho(x, y, z)$ at any point is proportional to its distance from the axis of the cylinder and is given by $\rho(x, y, z) = K\sqrt{x^2 + y^2}$, where K is the proportionality constant.

(i) Express the domain $\mathscr{W}$ in cylindrical coordinates.

(ii) Integrate $\rho(x, y, z)$ over $\mathscr{W}$ using cylindrical coordinates.

Problem 4 ***Iterated Triple Integral***

Consider the iterated integral $\mathscr{A} = \int_0^1 \int_0^{x^2} \int_0^y f(x, y, z)\,dz\,dy\,dx$.

Rewrite $\mathscr{A}$ as an equivalent iterated integral in the order $dx\,dz\,dy$. Explain your work.