TEST 1

MATH 32B @ UCLA (WINTER 2021)

Assigned: February 03, 2020.

Instructions/Admonishment

1. SHOW ALL WORK.
   A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit.

2. Duration: 24 hours.

3. The following is my own work, without the aid of any other person.
   Signature: _____

Exercise 1 DOUBLE INTEGRAL IN CARTESIAN SYSTEM OF COORDINATES.

Consider the integral $\int_0^2 \int_0^x dydx + \int_2^3 \int_0^{6-2x} dydx$.

(i) Reverse the order of integration to combine the sum above into one double integral.

(ii) Evaluate the integral.

Exercise 2 DOUBLE INTEGRAL IN POLAR COORDINATES.

Consider the intergal $\displaystyle\int_0^{\sqrt{2}/2}\int_x^{\sqrt{1-x^2}} dydx + \int_{-\sqrt{2}/2}^{\sqrt{2}/2}\int_{|y|}^{\sqrt{1-y^2}} dxdy$

(i) Use the polar coordinates to combine the integrals into a single double integral.

(ii) Evaluate the integral. (Assume $-\pi \leq \theta \leq \pi$)

Exercise 3 TRIPLE INTEGRAL IN CYLINDRICAL COORDINATES.

Consider the region $\mathcal{W}$ that lies between the sphere $x^2+y^2+z^2=4$, above the plane $z=0$, and inside the cylinder $x^2+y^2=1$.

(i) Sketch the region $\mathcal{W}$.

(ii) Use cylindrical coordinates to integrate $f(x,y,z)=z$ over $\mathcal{W}$.

Exercise 4 TRIPLE INTEGRAL IN SPHERICAL COORDINATES.

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Consider the solid $\mathcal{W}$ bounded below by the xy-plane, on the sides by the sphere $\rho = 2$, and above by the cone $\phi = \pi/3$.

(i) Find the spherical coordinate limits for the integral that calculates the volume of the region $\mathcal{W}$.

(ii) Evaluate the integral.

Exercise 5 TRIPLE INTEGRAL IN CARTESIAN COORDINATES

Cnnsider the integral $\int_{-1}^{1}\int_{x^2}^{1}\int_{0}^{1-y} f(x,y,z)dzdydx$.

(i) Rewrite the integral as an equivalent integral in the order $\int_{\square}^{\square}\int_{\square}^{\square}\int_{\square}^{\square} f(x,y,z)dydxdz$.

(ii) Explain how you got the new limits of integration..