

Test 01

MATH 32B @ UCLA (Spring 2021)

Assigned: April 23, 2021.

Instructions/Admonishment

1. SHOW ALL WORK.

A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit.

2. Duration: 24 hours.

3. The following is my own work, without the aid of any other person.

Signature: _____

Problem 1 Iterated Double Integral.

$$\text{Let } \mathcal{I} = \int_0^3 \int_0^{2-\frac{2}{3}y} dx dy + \int_{-2}^0 \int_0^{y+2} dx dy.$$

- (i) Reverse the order of integration to combine the sum above into one double integral.
- (ii) Evaluate \mathcal{I} .

Problem 2 Double Integral in Polar Coordinates.

Let \mathcal{R} be the region in xy -plane inside both the circle of radius $r = 2$ centered at $(0, 0)$ and the circle $r = 4 \cos \theta$.

(i) Sketch the region \mathcal{R} .

(ii) Use polar coordinates to calculate the area $\mathcal{S} = \iint_{\mathcal{R}} dA$.

Problem 3 The Cylindrical Solid.

A solid \mathcal{W} lies within the cylinder $x^2 + y^2 = 1$, below the plan $z = 4$, and above the paraboloid $z = 1 - x^2 - y^2$. The density $\rho(x, y, z)$ at any point is proportional to its distance from the axis of the cylinder and is given by $\rho(x, y, z) = K\sqrt{x^2 + y^2}$, where K is the proportionality constant.

- (i) Express the domain \mathcal{W} in cylindrical coordinates.
- (ii) Integrate $\rho(x, y, z)$ over \mathcal{W} using cylindrical coordinates.

Problem 4 Iterated Triple Integral.

Consider the iterated integral $\mathcal{A} = \int_0^1 \int_0^{x^2} \int_0^y f(x, y, z) dz dy dx$.

- (i) Rewrite \mathcal{A} as an equivalent iterated integral in the order $dx dz dy$.
- (ii) Explain how you got the new limits of integration.