Name: [redacted]

UID: [redacted]

Signature: [redacted]

Instructions: You have 24 hours to complete this exam, from 12:00 AM to 11:59 PM (Pacific Daylight Time) on Friday, March 20, 2020. There are 12 problems worth a total of 120 points. This exam is open book and open notes. You must justify your answers and show all of your work to receive full credit. Simplify your answers as much as possible. You may lose points for answers that are not simplified. Write your solutions in the space indicated on the exam template. If your answer continues onto another page, write an easily visible note under the original question. You may use the last three pages of the exam template for scratch work. If you do not want something you write on the exam template to be graded, you must clearly cross it out. You must scan your completed exam template and upload your solutions to Gradescope by 11:59 PM (Pacific Daylight Time) on Friday, March 20, 2020.

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	

Question	Points	Score
8	10	
9	10	
10	10	
11	10	
12	10	
Total:	120	

[redacted]

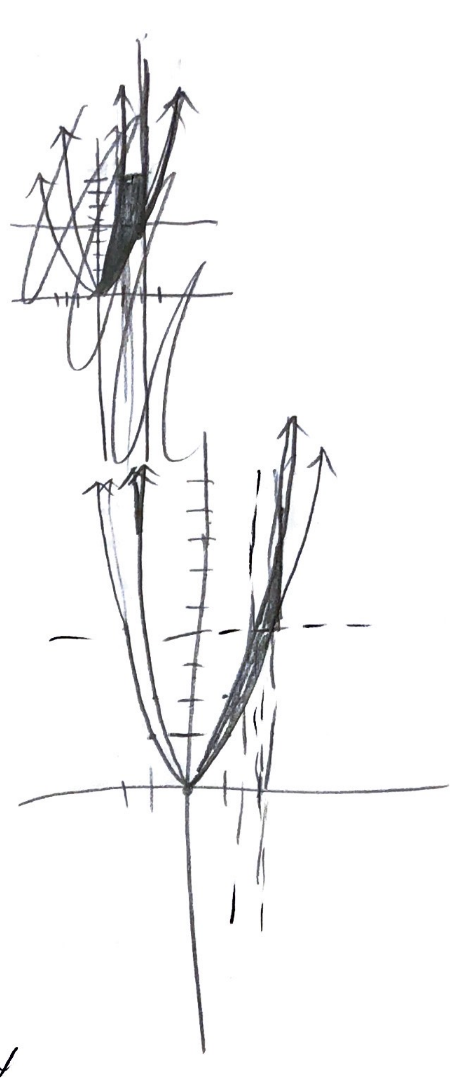
$2x^2 = 8$
$x = 2$

1. (10 points)

$2x^2 = 4$
$x^2 = 2$

$x = \sqrt{y/2}$ $\quad y/2 = x^2$ $\quad y = 2x^2$

$x = \sqrt{y}$ $\quad y = x^2$



$$\int_0^{\sqrt{2}} \int_{x^2}^{2x^2} \frac{1}{1+x^3}\, dy\, dx$$

$$+ \int_{\sqrt{2}}^{2} \int_{x^2}^{2x^2} \frac{1}{1+x^3}\, dy\, dx$$

$$= \int_0^{\sqrt{2}} \frac{y}{1+x^3} \Big|_{x^2}^{2x^2} dx$$

$$+ \int_{\sqrt{2}}^{2} \frac{y}{1+x^3} \Big|_{x^2}^{2x^2} dx$$

$$= \int_0^{\sqrt{2}} \frac{x^2}{1+x^3}\, dx + \int_{\sqrt{2}}^{2} \frac{x^2}{1+x^3}\, dy$$

$$= \frac{1}{3}\ln|1+x^3| \Big|_0^{\sqrt{2}} + \frac{1}{3}\ln|1+x^3| \Big|_{\sqrt{2}}^{2}$$

$$= \frac{1}{3}\ln|1+2\sqrt{2}| + \frac{1}{3}\ln|9| - \frac{1}{3}\ln|1+2\sqrt{2}|$$

$$= \frac{1}{3}\ln 9 = \boxed{\frac{2}{3}\ln 3}$$

2. (10 points)

$\cancel{z^2 = 1-(x^2+y^2)}$

$$I_z = \iiint_\Omega \delta(x,y,z)\cdot r_z^2 \, dV = \int_0^{2\pi}\int_0^1\int_0^{1-r_z} r_z^2 \, r_z \, dz\,dr\,d\theta$$

$$I_z = \int_0^{2\pi}\int_0^1 z r_z^3 \Big|_0^{1-r_z} dr\,d\theta$$

$$I_z = \int_0^{2\pi}\int_0^1 (1-r_z) r_z^3 \, dr\,d\theta$$

$$I_z = \int_0^{2\pi}\int_0^1 r_z^3 - r_z^4 \, dr\,d\theta$$

$$I_z = \int_0^{2\pi} \frac{1}{4} r_z^4 - \frac{1}{5} r_z^5 \Big|_0^1 d\theta$$

$$I_z = \int_0^{2\pi} \frac{1}{4} - \frac{1}{5} \, d\theta$$

$$\boxed{I_z = \frac{\pi}{10}}$$

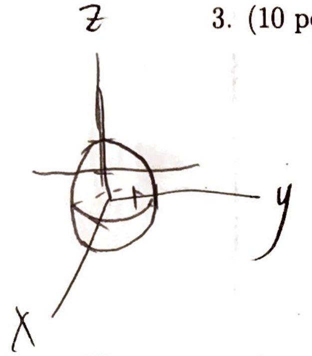
$z=0$

$1=r$

[redacted]

$z=1$
$r^2=3$

$r^2+z^2=4$
$z^2=3$



3. (10 points)

symmetric about x and y axis

$$z_{CM}=\frac{1}{mass}\cdot\int_0^{2\pi}\int_0^{\sqrt3}\int_1^{\sqrt{4-r^2}} z\cdot r\,dz\,dr\,d\theta$$

$$z_{cm}=\frac{1}{mass}\cdot\int_0^{2\pi}\int_0^{\sqrt3}\frac12 z^2 r\Big|_1^{\sqrt{4-r^2}}\,dr\,d\theta$$

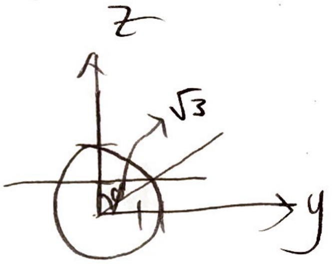
$$=\frac{1}{mass}\cdot\int_0^{2\pi}\int_0^{\sqrt3}\frac12(4-r^2)r-\frac12 r\,dr\,d\theta$$

$$=\frac{1}{mass}\int_0^{2\pi}\int_0^{\sqrt3} 2r-\frac{r^3}{2}-\frac12 r\,dr\,d\theta$$

$$=\frac{1}{mass}\int_0^{2\pi}\frac34 r^2-\frac18 r^4\Big|_0^{\sqrt3}\,d\theta$$

$$=\frac{1}{mass}\int_0^{2\pi}\frac94-\frac98\,d\theta=\frac{1}{mass}\cdot2\pi\cdot\frac98=\frac{1}{mass}\cdot\frac{9\pi}{4}$$

$$=\frac{3}{5\pi}\cdot\frac{9\pi}{4}$$



$$\boxed{z_{cm}=\frac{27}{20} \qquad CM=\left(0,0,\frac{27}{20}\right)}$$

$\rho=2$

$z=1$ $\quad \rho\cos\phi=z=1$

$r^2=3$ $\quad \rho=\sec\phi$

$\sin\phi=\frac{\sqrt3}{2}$

$\phi=\frac{\pi}{3}$

$$mass=\int_0^{2\pi}\int_0^{\pi/3}\int_{\sec\phi}^{2}\delta(x,y,z)\,\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$

$$=\int_0^{2\pi}\int_0^{\pi/3}\frac13\rho^3\sin\phi\Big|_{\sec\phi}^{2}\,d\phi\,d\theta$$

$$=\int_0^{2\pi}\int_0^{\pi/3}\frac83\sin\phi-\frac13\sec^3\phi\sin\phi\,d\phi\,d\theta$$

$$=\int_0^{2\pi}\left(-\frac83\cos\phi-\frac16\sec^2\phi\right)\Big|_0^{\pi/3}\,d\theta$$

$$=2\pi\left(-\frac43-\frac23+\frac83+\frac16\right)$$

$$=2\pi\left(\frac23+\frac16\right)$$

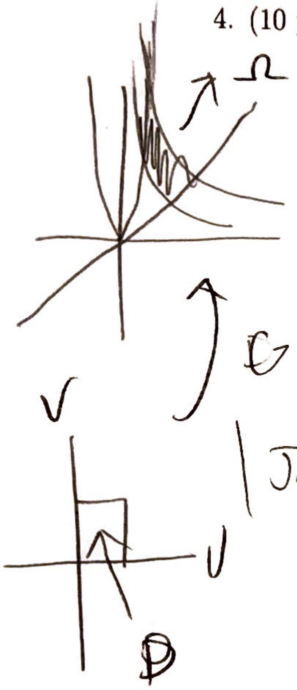
$$=\frac{5\pi}{3}$$

$\frac{12\cos\phi\sin\phi}{36\cos^4\phi}$

$\frac{1}{6\cos^2\phi}$

[redacted]

4. (10 points)



$U = xy \qquad 1 \le U \le 3 \qquad \frac{U}{V} \le V \le 2\left(\frac{U}{V}\right)^2$

$V = y \qquad x \le V \le 2x^2 \qquad V^2 \ge U \qquad V \ge \sqrt{U}$

$V^3 \le 2U^2 \qquad V \le 2^{1/3} U^{2/3}$

$$\iint_D U \, dA_{UV} = \int_1^3 \int_{\sqrt{U}}^{2^{1/3}U^{2/3}} U \left(|Jac(G)|\right) dv\,du$$

$$|Jac(G)| = \left|\frac{1}{Jac(G^{-1})}\right| = \left|\frac{1}{\frac{\partial U}{\partial x}\frac{\partial V}{\partial y} - \frac{\partial U}{\partial y}\frac{\partial V}{\partial x}}\right| = \left|\frac{1}{y \cdot 1 - x \cdot 0}\right| = \left|\frac{1}{y}\right| = \left|\frac{1}{V}\right| = \frac{1}{V}$$

always pos from $V = 1$ to $V = 2^{1/3}$

$$\iint_\Omega xy\,dA = \int_1^3 \int_{\sqrt{U}}^{2^{1/3}U^{2/3}} \frac{U}{V}\,dv\,du$$

$$= \int_1^3 U \ln|V| \Big|_{\sqrt{U}}^{2^{1/3}U^{2/3}} dU$$

$$= \int_1^3 U\left(\ln\left(2^{1/3}U^{2/3}\right) - \frac{1}{2}\ln U\right) dU$$

$$= \int_1^3 U\left(\frac{\ln 2}{3} + \frac{2}{3}\ln U - \frac{1}{2}\ln U\right) dU = \int_1^3 U\left(\frac{\ln 2}{3} + \frac{1}{6}\ln U\right) dU$$

$$= \left(\frac{1}{2}\left(\frac{\ln 2}{3}\right)(U^2) + \frac{1}{12}U^2 \ln U - \frac{1}{24}U^2\right)\Bigg|_1^3$$

$$= \left(\frac{3}{2}\ln 2 + \frac{3}{4}\ln 3 - \frac{9}{24} + \frac{1}{24} - \frac{\ln 2}{6}\right)$$

$$= \frac{4}{3}\ln 2 + \frac{3\ln 3}{4} - \frac{1}{3}$$

$$\boxed{\iint_\Omega xy\,dA = \frac{4}{3}\ln 2 + \frac{3\ln 3}{4} - \frac{1}{3}}$$

$a = \ln U$

$db = U\,dU$

$b = \frac{1}{2}U^2$

$da = \frac{1}{U}dU$

$\frac{1}{2}U^2 \ln 2 - \frac{1}{2}\int U\,dU$

[redacted]

5. (10 points)

$\vec{F} = (y^2, z, -3xy)$ already not 0

$$\operatorname{curl}(\vec{F}) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^2 & z & -3xy \end{vmatrix} = (-3x-1,$$

$\operatorname{curl}(\vec{F}) \neq 0$, not cons.

$r(t) = (t+1, 3t, 1-2t) \quad 0 \le t \le 1$

$$\int_C y^2 dx + z dy - 3xy dz = \int_0^1 9t^2 + 3(1-2t) - 3(t+1)(3t)(-2)\, dt$$

$$= \int_0^1 9t^2 + 3 - 6t + 18t(t+1)\, dt$$

$$= \int_0^1 27t^2 + 12t + 3\, dt$$

$$= 9t^3 + 6t^2 + 3t \Big|_0^1$$

$$= 18$$

$$\boxed{\int_C y^2 dx + z dy - 3xy dz = 18}$$

[redacted]

6. (a) (5 points)

$$\frac{\partial F_2}{\partial x} = \frac{\partial F_1}{\partial y}, \quad \frac{\partial F_3}{\partial y} = \frac{\partial F_2}{\partial z}, \quad \frac{\partial F_1}{\partial z} = \frac{\partial F_3}{\partial x}$$

$-e^x\sin y = -e^x\sin y, \quad 4yz = 4yz, \quad 1 = 1$

Thus, $\operatorname{curl}(\vec{F}) = 0$, and because $\vec{F}$ is defined for all points in the domain, $\vec{F}$ is also simply connected. Because $\vec{F}$ is simply connected and $\operatorname{curl}(\vec{F}) = 0$, $\vec{F}$ is a conservative vector field.

(b) (5 points) cons, thus path-independent

$\int_C \vec{F}\cdot d\vec{r} = f(r(\pi/2)) - f(r(0)) \qquad \nabla f = \vec{F}$

$$\frac{\partial f}{\partial x} = e^x\cos y + z$$

$$f = e^x\cos y + xz + h(y,z)$$

$$\frac{\partial f}{\partial y} = -e^x\sin y + \frac{\partial h(y,z)}{\partial y} = 2yz^2 - e^x\sin y$$

$$\frac{\partial h(y,z)}{\partial y} = 2yz^2$$

$$h(y,z) = y^2z^2 + g(z)$$

$$f = e^x\cos y + xz + y^2z^2 + g(z)$$

$$\frac{\partial f}{\partial z} = x + 2y^2z + g'(z) = x + 2y^2z + e^z$$

$$g'(z) = e^z$$

$$g(z) = e^z$$

$$f(x,y,z) = e^x\cos y + xz + y^2z^2 + e^z$$

$$-f(1,0,0) + f(0,1,\pi^2/4)$$

$$= -(e^1\cdot 1 + 0 + 0 + 1)$$

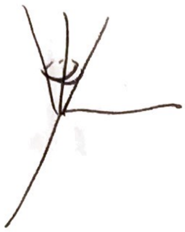
$$+ \left(\cos 1 + 0 + \frac{\pi^4}{16} + e^{\pi^2/4}\right)$$

$$= -e - 1 + \cos 1 + \frac{\pi^4}{16} + e^{\pi^2/4}$$

$$\boxed{\int_C \vec{F}\cdot d\vec{r} = -e + e^{\pi^2/4} + \frac{\pi^4}{16} - 1 + \cos 1}$$

[redacted]

7. (10 points)



$$\iint_S x\,dS$$

$$G(r,\theta) = (r\cos\theta, r\sin\theta, r)$$

$$\vec{T}_r = (\cos\theta, \sin\theta, 1)$$

$$\vec{T}_\theta = (-r\sin\theta, r\cos\theta, 0)$$

$$\vec{N} = \vec{T}_r \times \vec{T}_\theta = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \cos\theta & \sin\theta & 1 \\ -r\sin\theta & r\cos\theta & 0 \end{vmatrix}$$

$z = r$

$2 = r$

$$\vec{N} = (-r\cos\theta, -r\sin\theta, r\cos^2\theta + r\sin^2\theta)$$

$$\vec{N} = (-r\cos\theta, -r\sin\theta, r)$$

$$\|\vec{N}\| = \sqrt{r^2\cos^2\theta + r^2\sin^2\theta + r^2} = r\sqrt{2}$$

$$\iint_S x\,dS = \int_0^{\pi/2}\int_0^2 r\cos\theta \cdot r\sqrt{2}\,dr\,d\theta$$

$\swarrow$

$$\iint_S f(G(r,\theta)) \cdot \|\vec{N}(r,\theta)\| \cdot dA_{r\theta}$$

$$= \sqrt{2}\int_0^{\pi/2}\int_0^2 r^2\cos\theta\,dr\,d\theta$$

$$= \sqrt{2}\int_0^{\pi/2} \frac{1}{3}r^3\cos\theta\Big|_0^2\,d\theta$$

$$= \sqrt{2}\int_0^{\pi/2} \frac{8}{3}\cos\theta\,d\theta$$

$$= \frac{8\sqrt{2}}{3} \cdot \sin\theta\Big|_0^{\pi/2}$$

$$\boxed{= \frac{8\sqrt{2}}{3}}$$

[redacted]

8. (10 points)

$$\iint_S \vec{F}\cdot d\vec{S}$$

$\operatorname{div}(\vec{F}) = y+0+y \neq 0$ so no vector potential

$$G(x,y) = (x, y, 1-x-y)$$

$$\vec{T_x} = (1,0,-1)$$

$$\vec{T_y} = (0,1,-1)$$

$$\vec{N} = \vec{T_x}\times\vec{T_y} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{vmatrix} = (1,1,1) \rightarrow \text{pointing upwards } \checkmark$$

$$\iint_S \vec{F}\cdot d\vec{S} = \iint_S (xy,\ x(1-x-y),\ y(1-x-y))\cdot(1,1,1)\, dA_{xy}$$

$$= \iint_S xy + x - x^2 - xy + y - xy - y^2\, dA_{xy}$$

$$= \iint_S x + y - xy - x^2 - y^2\, dA_{xy}$$

change variables

$$= \int_0^{2\pi}\int_0^2 r\cos\theta + r\sin\theta - r^2\cos\theta\sin\theta - r^2\ r\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^2 r^2\cos\theta + r^2\sin\theta - r^3\cos\theta\sin\theta - r^3\,dr\,d\theta$$

$$= \int_0^{2\pi} \frac{1}{3}r^3\cos\theta + \frac{1}{3}r^3\sin\theta - \frac{1}{4}r^4\cos\theta\sin\theta - \frac{1}{4}r^4\Big|_0^2\, d\theta$$

$$= \int_0^{2\pi} \frac{8}{3}\cos\theta + \frac{8}{3}\sin\theta - 4\cos\theta\sin\theta - 4\, d\theta$$

$$= \left(\frac{8}{3}\sin\theta - \frac{8}{3}\cos\theta + 2\cos^2\theta - 4\theta\right)\Big|_0^{2\pi}$$

$$= -\frac{8}{3} + 2 - 8\pi + \frac{8}{3} - 2 - 0 = -8\pi$$

$$\boxed{\iint_S \vec{F}\cdot d\vec{S} = -8\pi}$$

[redacted]

9. (10 points)

$$\frac{\partial F_2}{\partial x} = \frac{\partial F_1}{\partial y}$$

$\cancel{\text{zero}}$ $2-2x \neq -2x$ not conservative

$$\oint_C \vec{F}\cdot d\vec{r} = \iint_\Omega \operatorname{curl}(\vec{F})\cdot\hat{k}\, dA_{xy}$$ GREEN'S THEOREM

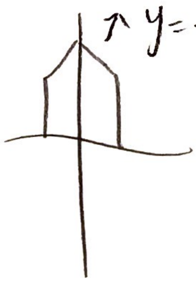
$$= \iint_\Omega 2-2x+2x\, dA_{xy}$$

$$= 2\iint_\Omega dA_{xy} = 2\cdot\left(2\cdot\left(\frac{2+3}{2}\right)\right) = 10$$

[ALT]

$$2\iint_\Omega dA_{xy} = 2\cdot 2\cdot \int_0^1\int_0^{-x+3} dy\,dx$$

$y=-x+3$



$$= 4\cdot \int_0^1 y\Big|_0^{-x+3} dx$$

$$= 4\cdot\int_0^1 -x+3\, dx$$

$$= 4\left(-\frac{1}{2}x^2+3x\Big|_0^1\right)$$

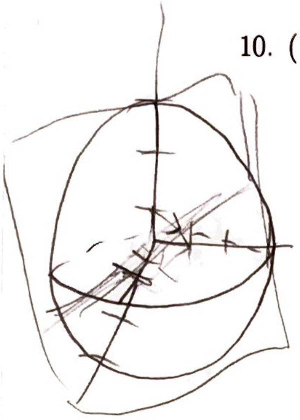
$$= 4\left(-\frac{1}{2}+3\right)$$

$$= 10$$

$$\boxed{\oint \vec{F}\cdot d\vec{r} = 10}$$

$\iint_\Omega \operatorname{curl}(\vec{F})\cdot\vec{n}\,dS$ [redacted]

10. (10 points)



$\vec{F}=(2z,2x,-1)$

$\frac{\partial F_2}{\partial x}=\frac{\partial F_1}{\partial y} \quad \frac{\partial F_3}{\partial y}=\frac{\partial F_2}{\partial z} \quad \frac{\partial F_1}{\partial z}=\frac{\partial F_3}{\partial x}$

$2 \neq 0$

not conservative

$\oint_C \vec{F}\cdot d\vec{r} = \iint_\Omega \operatorname{curl}(\vec{F})\cdot\vec{n}\,dS$

$\operatorname{curl}(\vec{F})=\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2z & 2x & -1 \end{vmatrix} = (0,2,2)$

STOKES THEOREM

$G(x,y)=(x,y,-x-y)$

$\vec{T}_x=(1,0,-1)$

$\vec{T}_y=(0,1,-1)$

$\vec{N}=\vec{T}_x\times\vec{T}_y=\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{vmatrix}=(1,1,1)$

oriented CC, so $\vec{N}$ dir is good

$\vec{n}=\frac{\vec{N}}{\|\vec{N}\|}=\frac{(1,1,1)}{\sqrt{3}}$

$\iint_\Omega (0,2,2)\cdot\frac{(1,1,1)}{\sqrt{3}}\,dS$

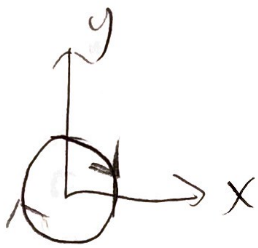
$=\iint_\Omega \frac{4}{\sqrt{3}}\,dS$

$=\frac{4}{\sqrt{3}}\iint_\Omega dS = \frac{4\pi(\sqrt{3})^2}{\sqrt{3}} \quad \frac{12\pi}{\sqrt{3}}$

$$\boxed{\oint 2z\,dx+2x\,dy-dz=\frac{12}{\sqrt{3}}\pi}$$

[redacted]

11. (10 points)



normal into page

Ω ~~this~~ is a closed surface, so DIVERGENCE THEOREM

$$\iint_S \vec{F}\cdot d\vec{S} = \iiint_\Omega \operatorname{div}(\vec{F})\, dV$$

$$\operatorname{div}(\vec{F}) = \nabla\cdot\vec{F} = z+z+2z = 4z$$

~~cyl~~ Spherical:

$$4z = 4\rho\cos\phi$$

$$= \int_0^{2\pi}\int_0^{\pi/2}\int_0^1 4\rho\cos\phi\ \rho^2\sin\phi\, d\rho\, d\phi\, d\theta$$

$$= 4\int_0^{2\pi} d\theta \int_0^{\pi/2}\cos\phi\sin\phi\, d\phi \int_0^1 \rho^3\, d\rho$$

$$= 8\pi\cdot\left(\frac{1}{2}\sin^2\theta\Big|_0^{\pi/2}\right)\cdot \frac{1}{4}\rho^4\Big|_0^1$$

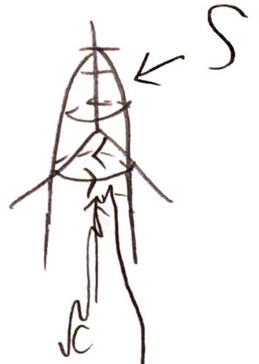
$$= 8\pi\cdot\frac{1}{2}\cdot\frac{1}{4}$$

$$\boxed{\iint_S \vec{F}\cdot d\vec{S} = \pi}$$

[redacted]

12. (10 points)

not closed surface, $\text{div}(\vec{F}) \neq 0$, so no vector potential



$$\iint_S \vec{F}\cdot d\vec{S} = \oint_C \cdot d\vec{r}$$

$$r_c(t) = (\sqrt{3}\cos t, \sqrt{3}\sin t, 0) \quad 0 \le t \le 2\pi$$

$$r_c'(t) = (-\sqrt{3}\sin t, \sqrt{3}\cos t, 0)$$

$$\int_0^{2\pi} \vec{F}(\vec{r}(t)) \cdot r'(t)\,dt$$

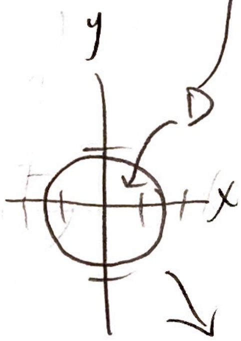
$$\int_0^{2\pi} (0,0,1)\cdot(-3,$$

$z=0$

$r^2=3$

$r=\sqrt{3}$

DIV. THEOREM



$$\iint_S \vec{F}\cdot d\vec{S} + \iint_D \vec{F}\cdot d\vec{S} = \iiint_\Omega \text{div}(\vec{F})\cdot dV$$

$$\iint_D \vec{F}\cdot d\vec{S} = \int_0^{2\pi}\int_0^{\sqrt{3}} (0,0,1)\cdot(0,0,-r)\cdot dA_{r\theta}$$

$$= \int_0^{2\pi}\int_0^{\sqrt{3}} -\tfrac{1}{2}r^2\,dr\,d\theta$$

$$= 2\pi \cdot \left(-\tfrac{1}{2}r^2\Big|_0^{\sqrt{3}}\right)$$

$$= -3\pi$$

$$G(r,\theta) = (r\cos\theta, r\sin\theta, 0)$$

$$\vec{T}_\theta = (-r\sin\theta, r\cos\theta, 0)$$

$$\vec{T}_r = (\cos\theta, \sin\theta, 0)$$

$$\vec{N} = \vec{T}_\theta \times \vec{T}_r = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -r\sin\theta & r\cos\theta & 0 \\ \cos\theta & \sin\theta & 0 \end{vmatrix} = (0,0,-r\sin^2\theta - r\cos^2\theta)$$

$$= (0,0,-r)$$

outward pointing

(necessary for Div Theorem)

[redacted]

$r^2=3$

You may use this page for scratch work.

$$\iiint_\Omega \operatorname{div}(\vec{F})\cdot dV$$

$$= \int_0^{2\pi}\int_0^{\sqrt{3}}\int_0^{3-r^2} 2z \cdot r\,dz\,dr\,d\theta$$

$$= 2\pi \cdot \int_0^{\sqrt{3}} z^2 r \Big|_0^{3-r^2} dr$$

$$= 2\pi \cdot \int_0^{\sqrt{3}} (3-r^2)^2 r\,dr$$

$$= 2\pi \int_0^{\sqrt{3}} (9-6r^2+r^4) r\,dr$$

$$= 2\pi \int_0^{\sqrt{3}} 9r-6r^3+r^5\,dr$$

$$= 2\pi \left(\frac{9}{2}r^2-\frac{3}{2}r^4+\frac{1}{6}r^6 \Big|_0^{\sqrt{3}}\right)$$

$$= 2\pi\left(\frac{27}{2}-\frac{27}{2}+\frac{27}{6}\right)$$

$$= \frac{27\pi}{3} = 9\pi$$

$$\iint_S \vec{F}\cdot d\vec{S} = \iiint_\Omega \operatorname{div}(\vec{F})\cdot dV - \iint_D \vec{F}\cdot d\vec{S}$$

$$\boxed{\iint_S \vec{F}\cdot d\vec{S} = 9\pi + 3\pi}$$

$$\boxed{\iint_S \vec{F}\cdot d\vec{S} = 12\pi}$$

[redacted]

You may use this page for scratch work.

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You may use this page for scratch work.