MATH 32B Midterm 1B

Rishab Sukumar

TOTAL POINTS

50 / 50

QUESTION 1

1 Riemann Sums 8 / 8

✓ + **2 pts** (correct) Partition

✓ + **1 pts** (correct) Area of regular partition

✓ + **2 pts** (correct) Sample points

✓ + **1 pts** Correct written sum and product

✓ + **1 pts** Correct answer

✓ + **1 pts** Clear and fully explained

+ **0 pts** No points

QUESTION 2

2 Fubini's Theorem 13 / 13

✓ + **1 pts** Readability

✓ + **8 pts** Bounds of Integration

✓ + **4 pts** Correct Calculation

QUESTION 3

3 Polar Coordinates 13 / 13

✓ + **2 pts** f(x,y) = 1/r^2

✓ + **2 pts** dx dy = r dr dtheta

✓ + **3 pts** correct limits for r

✓ + **2 pts** correct limits for theta

✓ + **2 pts** correctly integrate 1/r

✓ + **1 pts** correct integral (only if limits and integrand correct)

✓ + **1 pts** solution fully explained, using English sentences and units where appropriate

+ **2 pts** reasonably correct picture of integration domain (bonus, only if at least one limit incorrect)

+ **0 pts** no points

+ **2 pts** 4*R^2 <= r^2 <= 16*R^2 (only if no picture and r limits incorrect)

+ **2 pts** some progress towards a solution in Cartesian coordinates

QUESTION 4

4 Volume of a solid 16 / 16

✓ + **2 pts** Correct limits for $$z$$ using $$dzdydx$$ or $$dzdxdy$$

✓ + **2 pts** Correct order for $$z$$ limits using $$dzdydx$$ or $$dzdxdy$$

✓ + **4 pts** Correct limits for $$y$$ if using $$dzdydx$$ or $$dydx$$ (ignoring order of limits)

✓ + **2 pts** Correct order for $$y$$ limits if using $$dzdydx$$ or $$dydx$$

✓ + **1 pts** Correct limits for $$x$$ if using $$dzdydx$$ or $$dydx$$ (ignoring order of limits)

✓ + **1 pts** Correct order for $$x$$ limits if using $$dzdydx$$ or $$dydx$$

+ **2 pts** Splitting the integral if using $$dzdxdy$$ or $$dxdy$$

+ **1 pts** Correct limits for $$y$$ if using $$dzdxdy$$ or $$dxdy$$ (ignoring order of limits)

+ **1 pts** Correct order for $$y$$ limits if using $$dzdxdy$$ or $$dxdy$$

+ **2 pts** Correct limits for $$x$$ if using $$dzdxdy$$ or $$dxdy$$ (ignoring order of limits)

+ **2 pts** Correct order for $$x$$ limits if using $$dzdxdy$$ or $$dxdy$$

✓ + **1 pts** Correctly integrating in $$x$$ (requires $$x$$ limits and all previous integrations to be correct to get points)

✓ + **1 pts** Correctly integrating in $$y$$ (requires $$y$$ limits and all previous integrations to be correct to get points)

✓ + **1 pts** Correctly integrating in $$z$$ (requires correct $$z$$ limits to get points)

+ **5 pts** Correct integrand if using double integral.

+ **2 pts** Integrand correct modulo a sign error if using double integral.

+ **1 pts** Picture(s) of the domain (bonus, only if not achieved 15/15 on previous points)

✓ **+ 1 pts** **Solution fully explained, using English sentences and units where appropriate.**

+ 0 pts No explanation, no picture and incorrect integral.

Math 32B - Lecture 1
Fall 2018
Midterm 1
10/15/2018

Name: Rishab Sukumar
TA Section: 1B
ID: 304902259

Time Limit: 50 Minutes

Version (B)

This exam contains 10 pages (including this cover page) and 4 problems. There are a total of 50 points available.

Check to see if any pages are missing. Enter your name and TA Section at the top of this page.

You may **not** use your books, notes or a calculator on this exam.

Please **switch off your cell phone** and place it in your bag or pocket for the duration of the test.

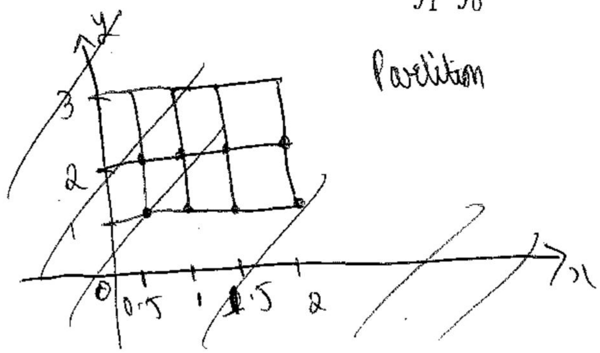
- Attempt all questions.
- Write your solutions clearly, in full English sentences, using units where appropriate.
- You may use scratch paper if required.

The majority of points on each problem will be for setting up the integrals correctly. At least one point on each problem will be for clearly explaining your solution, as on the homeworks.

1. (8 points) You are given the following values of a continuous function $f(x, y)$:

2	3	3.5	4	4.5	5
1	2	2.5	3	3.5	4
0	1	1.5	2	2.5	3
y \ x	1	1.5	2	2.5	3

Using a regular partition and the **lower-right** vertices of the subrectangles as sample points, approximate the integral $\int_1^3 \int_0^2 f(x, y)\, dydx$ by computing the Riemann sum $S_{4,2}$



Partition

Calculate width $\Delta x = \frac{3-1}{4} = 0.5$

No. of divisions

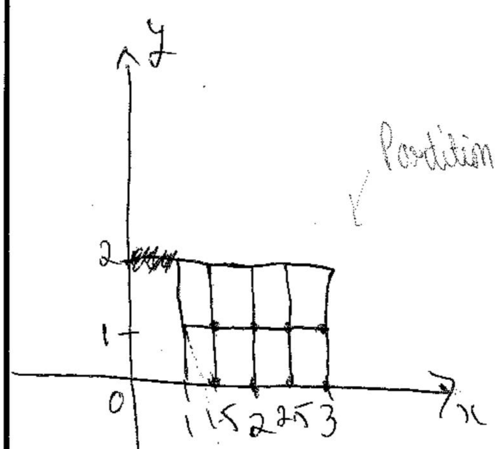
$\Delta y = \frac{2-0}{2} = 1$

$\therefore \Delta A = \Delta x \, \Delta y = 1 \times 0.5$
$= 0.5 = \frac{1}{2}$

Riemann sum, $S_{4,2} = \sum_{i=1}^{4} \sum_{j=1}^{2} f(P_{ij})\, \Delta A$

Sample points

where P_{ij} are lower right vertices



Partition

$= \frac{1}{2}\left[f(1.5,0) + f(1.5,1) + f(2,0) + f(2,1) + f(2.5,0) + f(2.5,1) + f(3,0) + f(3,1)\right]$

$= \frac{1}{2}\left[1.5 + 2.5 + 2 + 3 + 2.5 + 3.5 + 3 + 4\right]$

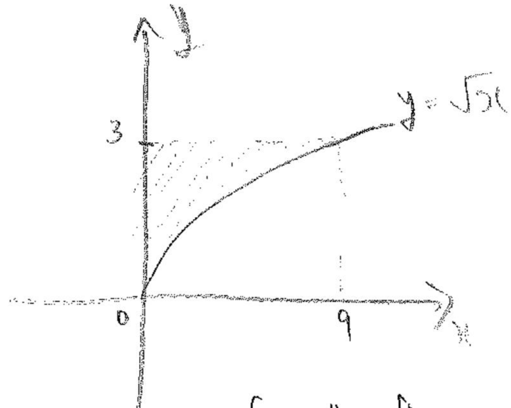
$= \frac{1}{2}\left[22\right]$

$= 11$

2. (20 points) Find $\int_0^9 \int_{\sqrt{x}}^3 \cos(y^3)\, dy dx$. *(Hint: You might wish to switch the order of integration.)*

By limits given, $\sqrt{x} \leq y \leq 3$

$0 \leq x \leq 9$



since $\sqrt{x} \leq y$

$x \leq y^2$

at $x=0$, $y=0$

$\therefore$ $0 \leq y \leq 3$

and $0 \leq x \leq y^2$

It is clear from the figure that $D: \{ 0 \leq x \leq y^2, 0 \leq y \leq 3 \}$

By Fubini's theorem,
changing order of integrals

$$\int_0^3 \int_0^{y^2} \cos y^3 \, dx\, dy$$

$$= \int_0^3 x \cos y^3 \Big|_0^{y^2} dy$$

$$= \int_0^3 y^2 \cos y^3 - 0 \quad dy$$

$$= \int_0^3 y^2 \cos y^3 \, dy$$

Using u-substitution

let $y^3 = u$

$3y^2\, dy = du$

$\therefore \quad y^2\,dy = \frac{1}{3}\,du$

Changing limits,

when $y=0$, $u=0$

when $y=3$, $u=27$

$\therefore \frac{1}{3}\int_0^{27} \cos u\,du$

$= \frac{1}{3} (\sin u) \Big|_0^{27}$

$= \frac{1}{3}(\sin 27 - \sin 0)$

$= \frac{1}{3}\sin 27$

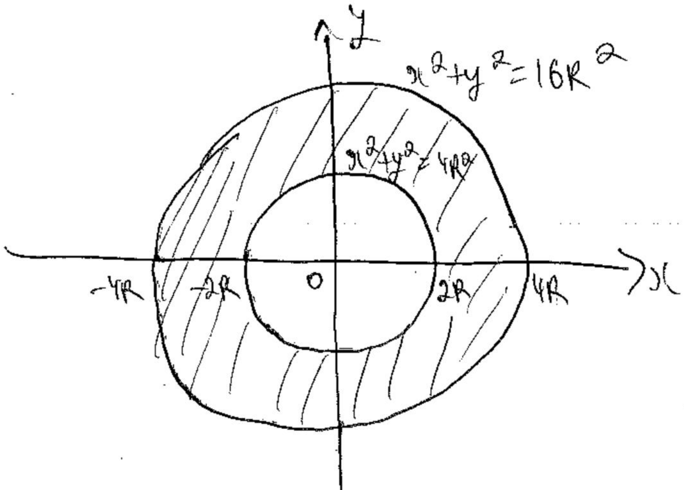
3. (13 points) For $R > 0$ let $\mathcal{D} = \{(x, y) : 4R^2 \leq x^2 + y^2 \leq 16R^2\}$ and $f(x, y) = \dfrac{1}{x^2 + y^2}$. Show that the value of $\iint\limits_{\mathcal{D}} f(x, y)\, dA$ does not depend on the value of R.

$4R^2 \leq x^2+y^2 \leq 16R^2$

Thus, we are dealing with 2 circles

$x^2+y^2 = 4R^2$

and $x^2+y^2 = 16R^2$



Changing variables into polar coordinates,

we know that $x^2+y^2 = r^2$

$\therefore 4R^2 \leq r^2 \leq 16R^2$

$\therefore 2R \leq r \leq 4R$

From the figure it is clear that $0 \leq \theta \leq 2\pi$

$\therefore$ Domain in polar coordinates, $D: \{0 \leq \theta \leq 2\pi,\ 2R \leq r \leq 4R\}$

Write function in polar coordinates,

$$\frac{1}{x^2+y^2} = \frac{1}{r^2}$$

$$\therefore \iint_D f(x,y)\, dA$$

$$= \int_{\theta=0}^{2\pi} \int_{r=2R}^{4R} \frac{1}{r^{\cancel{2}}}\, \cancel{r}\, dr\, d\theta$$

$$= \int_{\theta=0}^{2\pi} \int_{r=2R}^{4R} \frac{1}{r}\, dr \quad d\theta$$

$$= \int_{\theta=0}^{2\pi} \ln r \Big|_{2R}^{4R} \quad d\theta$$

$$= \int_{\theta=0}^{2\pi} \ln|4R| - \ln|2R| \ d\theta$$

$$= \int_{\theta=0}^{2\pi} \ln \left| \frac{\overset{2}{\cancel{4R}}}{\cancel{2R}} \right| \quad d\theta$$

$$= \int_{\theta=0}^{2\pi} \ln 2 \ d\theta$$

$$= \theta \ln 2 \Big|_0^{2\pi}$$

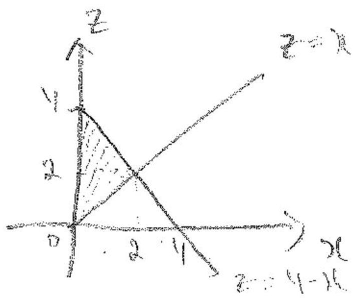
$$= 2\pi \ln 2 - 0$$

$$= 2\pi \ln 2$$

Thus, we see that $2\pi \ln 2$ does not contain R.
$\iint_D f(x,y)\ dA$ does not depend on R.

4. (16 points) Find the volume of the region bounded by the surfaces $x = 0$, $x = 1+y$, $x = 4-2y$, $z = x$ and $z = 4 - x$.

$x = 0, \quad x = 1+y, \quad x = 4-2y, \quad z = x, \quad z = 4-x$



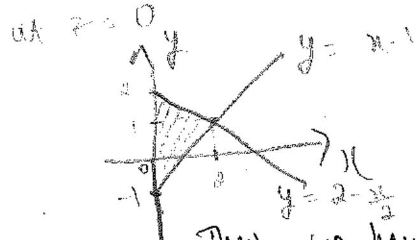
when $x = 0$, $y = -1$

$z = 0$ ← We now consider the xy plane

$x = 4-2y$

$0 = 4-2y$

at z = 0



$2y = 4$

$y = 2$

Thus, we have domain D: $\{ 0 \le x \le 2,\ x-1 \le y \le 2-\frac{x}{2},\ x \le z \le 4-x \}$

From the figures above

The volume of the region can be calculated as

$$\iiint_D 1 \cdot dV$$

$$= \int_{x=0}^{2} \int_{y=x-1}^{y=2-\frac{x}{2}} \int_{z=x}^{4-x} 1 \; dz\,dy\,dx$$

$$= \int_{x=0}^{2} \int_{y=x-1}^{2-\frac{x}{2}} z \Big|_{0}^{4-x} dx\,dy$$

$$= \int_{x=0}^{2} \int_{y=x-1}^{2-\frac{x}{2}} 4-x-x \; dy\,dx$$

$$= \int_0^2 \int_{x-1}^{2-\frac{x}{2}} 4-2x \; dy\,dx$$

~~$= \int_{y=-1}^{2} 4x - \frac{x^2}{2} \Big|_0^{1+y} dy$~~

$$= \int_{y=-1}^{2} 4(1+y) - \frac{(1+y)^2}{2} - 0 + 0 \, dy$$

$$= \int_{y=-1}^{2} 4 + 4y - \left[\frac{1+y^2+2y}{2}\right] dy$$

$$= \int_{y=-1}^{2} \frac{8+8y-1-y^2-2y}{2} \, dy$$

$$= \int_{y=-1}^{2} \frac{7+6y-y^2}{2} \, dy$$

$$= \frac{1}{2}\int_{y=-1}^{2} 7+6y-y^2 \, dy$$

$$= \frac{1}{2}\left[7y + \frac{\cancel{6}^{3}y^2}{\cancel{2}} - \frac{y^3}{3}\Big|_{-1}^{2}\right]$$

$$= \frac{1}{2}\left[7y + 3y^2 - \frac{y^3}{3}\Big|_{-1}^{2}\right]$$

$$= \frac{1}{2}\left[7(2) + 3(2^2) - \frac{2^3}{3} - \left[7(-1) + 3(-1)^2 - \frac{(-1)^3}{3}\right]\right]$$

$$= \frac{1}{2}\left[14 + 12 - \frac{8}{3} - \left[-7 + 3 + \frac{1}{3}\right]\right]$$

$$= \frac{1}{2}\left[26 - \frac{8}{3} + 7 - 3 - \frac{1}{3}\right]$$

$$= \frac{1}{2}\left[30 - \frac{9}{3}\right] = \frac{1}{2}\left[30 - 3\right] = \frac{27}{2}$$

$$\int_0^2 4y - 2xy \Big|_{x-1}^{2-\frac{x}{2}} dx$$

$$= \int_0^2 4\left(2-\frac{x}{2}\right) - 2x\left(2-\frac{x}{2}\right) - \left[4(x-1) - 2x(x-1)\right] dx$$

$$= \int_0^2 8 - 2x - 4x + x^2 - \left[4x - 4 - 2x^2 + 2x\right] dx$$

$$= \int_0^2 8 - 2x - 4x + x^2 - 4x + 4 + 2x^2 - 2x \, dx$$

$$= \int_0^2 12 - 12x + 3x^2 \, dx$$

$$= 12x - \frac{\cancel{12}^{6} x^2}{\cancel{2}} + \frac{\cancel{3} x^3}{\cancel{3}} \Big|_0^2$$

$$= 12x - 6x^2 + x^3 \Big|_0^2$$

$$= 12(2) - 6(2^2) + 2^3 - 0 + 0 - 0$$

$$= \overset{24}{\cancel{8}} - 6(4) + 8$$

$$= \overset{24}{\cancel{8}} - 24 + 8$$

$$= +8$$