Math 32B Final Clockwise

SEI CHANG

TOTAL POINTS

75 / 90

QUESTION 1

1 Fubini's Theorem 6 / 6

✓ + **6 pts** **Correct answer (2/3)(e^27 - 1).**

+ **2 pts** (Partial credit) New x limits are 0 to 9.

+ **2 pts** (Partial credit) New y limits are 0 to sqrt(x).

+ **1 pts** (Partial credit) New y integral is sqrt(x)*exp(x^(3/2)).

+ **1 pts** (Partial credit, only applies if new limits are incorrect) Reasonably correct picture.

+ **0 pts** No points.

+ **3 pts** (Partial credit) Incorrect limits: 0 <= x <= 9, sqrt(x) <= y <= 3

QUESTION 2

2 Stokes' Theorem 4 / 8

+ **8 pts** Correct answer 248.

✓ + **4 pts** **(Partial credit) Answer for _inward_ pointing normal 208.**

+ **0 pts** No points.

+ **7 pts** (Partial credit) Correct method and orientations, but arithmetic error

+ **3 pts** (Partial credit) Line integral over C1 is equal to sum of line integrals and surface integral, with some (incorrect) choice of signs.

+ **2 pts** (Partial credit, only if no other points apply) Mention or state Stokes theorem.

QUESTION 3

3 Line integral 6 / 12

✓ + **4 pts** **Correct parametrization**

+ **2 pts** Partial crerdits for parametrization

+ **4 pts** Correct integral formula

✓ + **2 pts** **Partial credits for integral**

+ **4 pts** Correct calculation and final answer

+ **2 pts** Partial credits for calculation

+ **1 pts** Almost makes no sense

+ **0 pts** Nothing correct

- **1 pts** Tiny calculation error

QUESTION 4

4 Moment of inertia 14 / 14

✓ + **1 pts** **a) Correct limits $$0\leq\rho\leq\frac1{10}$$**

✓ + **1 pts** **a) Correct limits $$0\leq\theta<2\pi$$**

✓ + **1 pts** **a) Correct upper bound $$\phi\leq \pi$$**

✓ + **2 pts** **a) Correct lower bound $$\phi\geq \frac{2\pi}3$$**

✓ + **1 pts** **b) Correctly using part (a) to obtain limits (credit given even if limits wrong, provided they are consistent)**

✓ + **1 pts** **b) Correct integrand $$3(x^2+y^2)$$ (must substitute $$\delta=3$$ into formula from formula sheet to gain credit)**

✓ + **2 pts** **b) Correctly converting $$x^2 + y^2$$ to $$\rho^2\cos^2\theta\sin^2\phi + \rho^2\sin^2\theta\sin^2\phi$$ in spherical coordinates**

✓ + **1 pts** **b) Correctly simplifying $$3(x^2+y^2)$$ to $$3\rho^2\sin^2\phi$$**

✓ + **2 pts** **b) Correct Jacobian $$\rho^2\sin\phi$$ in spherical coordinates**

✓ + **1 pts** **b) Correct answer of $$\frac{\pi}{400000}\,\mathrm{kg}\,\mathrm{m}^2$$ (units required for points, only awarded if rest of computation correct)**

✓ + **1 pts** **Solution thoroughly explained, using full sentences**

✓ + **1 pts** **Correct picture(s) of region (bonus point, only awarded if points lost elsewhere)**

+ **0 pts** No credit due

QUESTION 5

5 Probability 12 / 14

✓ + 2 pts Correct limits (max 4 pts)

+ 1 pts Correct limits

+ 1 pts Correct limits

✓ + 2 pts Correct integrand (max 5 pts)

✓ + 2 pts correct integrand

✓ + 1 pts Correct integrand

✓ + 2 pts Computations (max 5 pts)

✓ + 2 pts Computations

✓ + 1 pts Computations

+ 0 pts No credit due

QUESTION 6

6 Divergence Theorem 14 / 14

✓ + 4 pts Correct divergence

✓ + 7 pts Correct parametrization of $$
\mathcal{W}$$

✓ + 3 pts Correct evaluation of triple integral

+ 2 pts Bonus: Drew accurate picture (must include both cylinders and both planes, and accurate portrayal of their intersections [the larger cylinder and two planes meet in a single point])

+ 0 pts No credit

QUESTION 7

7 Vector line integral 9 / 12

✓ + 4 pts Write F as a sum of vortex field and a conservative field

+ 2 pts Vortex field has integral 2pi over this C

✓ + 2 pts Compute curl_z F_2 or show F_2 is conservative

✓ + 3 pts Conclude (e.g. by Green's theorem or using that F_2 is conservative) that the integral over C of F_2 is 0

+ 1 pts Arrive at correct answer, 2pi, by valid method

+ 0 pts Incorrect

+ 2 pts Mostly correct argument that integral of F_2 is 0

+ 1 pts curl_z F_2 minor error

QUESTION 8

8 Surface integral 10 / 10

✓ + 3 pts Decompose flux integral

+ 1 pts Partial credit for decomposition

✓ + 2 pts Do component integrals

+ 1 pts Partial credit for component integrals

✓ + 1 pts Combine integrals

✓ + 2 pts Used divergence theorem (part (b))

✓ + 1 pts Correct (and justified) div(F) (part (b))

✓ + 1 pts Clear and well-explained solution

+ 0 pts No credit due

b_j, not n_j.

Math 32B - Lectures 3 & 4
Winter 2019
Final Exam
3/17/2019

Name: Sei Chang
SID: 305-912-856
TA Section: 3F

Time Limit: 180 Minutes

Version 0

This exam contains 20 pages (including this cover page) and 8 problems. There are a total of 90 points available.

Check to see if any pages are missing. Enter your name, SID and TA Section at the top of this page.

You may **not** use your books, notes or a calculator on this exam.

Please **switch off your cell phone** and place it in your bag or pocket for the duration of the test.

- Attempt all questions.
- Write your solutions clearly, in full English sentences, using units where appropriate.
- You may write on both sides of each page.
- You may use scratch paper if required.

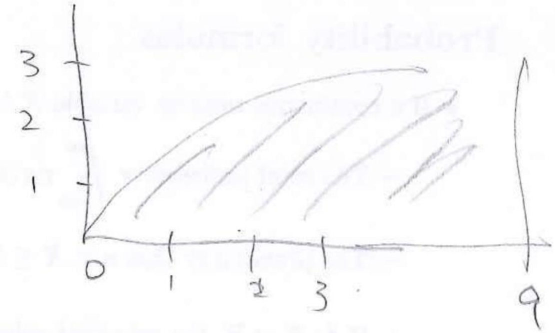
Mechanics formulas

- If $\mathcal{D}$ is a lamina with mass density $\delta(x,y)$ then
  - The mass is $M = \iint_{\mathcal{D}} \delta(x,y)\,dA$.
  - The y-moment is $M_y = \iint_{\mathcal{D}} x\,\delta(x,y)\,dA$.
  - The x-moment is $M_x = \iint_{\mathcal{D}} y\,\delta(x,y)\,dA$.
  - The center of mass is $(x_{\text{CM}}, y_{\text{CM}}) = \left(\frac{M_y}{M}, \frac{M_x}{M}\right)$.
  - The moment of inertia about the x-axis is $I_x = \iint_{\mathcal{D}} y^2\,\delta(x,y)\,dA$.
  - The moment of inertia about the y-axis is $I_y = \iint_{\mathcal{D}} x^2\,\delta(x,y)\,dA$.
  - The polar moment of inertia is $I_0 = \iint_{\mathcal{D}} (x^2+y^2)\,\delta(x,y)\,dA$.
- If $\mathcal{W}$ is a solid with mass density $\delta(x,y,z)$ then
  - The mass is $M = \iiint_{\mathcal{W}} \delta(x,y,z)\,dV$.
  - The yz-moment is $M_{yz} = \iiint_{\mathcal{W}} x\,\delta(x,y,z)\,dV$.
  - The xz-moment is $M_{zx} = \iiint_{\mathcal{W}} y\,\delta(x,y,z)\,dV$.
  - The xy-moment is $M_{xy} = \iiint_{\mathcal{W}} z\,\delta(x,y,z)\,dV$.
  - The center of mass is $(x_{\text{CM}}, y_{\text{CM}}, z_{\text{CM}}) = \left(\frac{M_{yz}}{M}, \frac{M_{zx}}{M}, \frac{M_{xy}}{M}\right)$.
  - The moment of inertia about the x-axis is $I_x = \iiint_{\mathcal{W}} (y^2+z^2)\,\delta(x,y,z)\,dV$.
  - The moment of inertia about the y-axis is $I_y = \iiint_{\mathcal{W}} (x^2+z^2)\,\delta(x,y,z)\,dV$.
  - The moment of inertia about the z-axis is $I_z = \iiint_{\mathcal{W}} (x^2+y^2)\,\delta(x,y,z)\,dV$.

Probability formulas

- If a continuous random variable X has probability density function $p_X(x)$ then
  - The total probability $\int_{-\infty}^{\infty} p_X(x)\,dx = 1$.
  - The probability that $a < X \leq b$ is $\mathbb{P}[a < X \leq b] = \int_a^b p_X(x)\,dx$.
  - If $f\colon \mathbb{R} \to \mathbb{R}$, the expected value of $f(X)$ is $\mathbb{E}[f(X)] = \int_{-\infty}^{\infty} f(x)\,p_X(x)\,dx$.
- If continuous random variables X, Y have joint probability density function $p_{X,Y}(x,y)$ then
  - The total probability $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p_{X,Y}(x,y)\,dxdy = 1$
  - The probability that $(X,Y) \in \mathcal{D}$ is $\mathbb{P}[(X,Y) \in \mathcal{D}] = \iint_{\mathcal{D}} p_{X,Y}(x,y)\,dA$.
  - If $f\colon \mathbb{R}^2 \to \mathbb{R}$, the expected value of $f(X,Y)$ is $\mathbb{E}[f(X,Y)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,p_{X,Y}(x,y)\,dxdy$.

1. (6 points) Find $\int_0^3 \int_{y^2}^{9} e^{x^{\frac{3}{2}}}\, dx dy$.

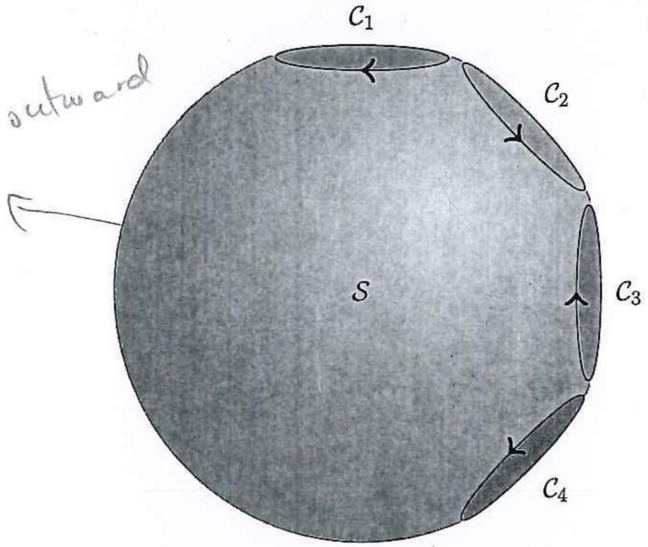


To find new region, we set $x = y^2$ to $y = \sqrt{x}$.
Since given, we know x goes from 0 to 9, we can then transform the ~~equation~~ limits to

$$\int_0^9 \int_0^{\sqrt{x}} e^{x^{\frac{3}{2}}}\, dy dx = \int_0^9 \left[x^{\frac{1}{2}} e^{x^{\frac{3}{2}}} \right] dx$$

$$= \left[\frac{2}{3} e^{x^{\frac{3}{2}}} \right]_0^9 = \frac{2}{3}\left[e^{27} - 1 \right] = \frac{2}{3} e^{27} - \frac{2}{3}$$

2. (8 points) Let S be a part of the unit sphere $x^2 + y^2 + z^2 = 1$ oriented with outward pointing normal, with four holes bounded by the curves $\mathcal{C}_1, \mathcal{C}_2, \mathcal{C}_3, \mathcal{C}_4$ oriented as in the following picture:



Suppose that for a vector field $\mathbf{F}$ we have

$$\iint_S \operatorname{curl}\mathbf{F}\cdot d\mathbf{S} = 20, \qquad \oint_{\mathcal{C}_2} \mathbf{F}\cdot d\mathbf{r} = 305, \qquad \oint_{\mathcal{C}_3} \mathbf{F}\cdot d\mathbf{r} = 104, \qquad \oint_{\mathcal{C}_4} \mathbf{F}\cdot d\mathbf{r} = 27.$$

Find $\oint_{\mathcal{C}_1} \mathbf{F}\cdot d\mathbf{r}$.

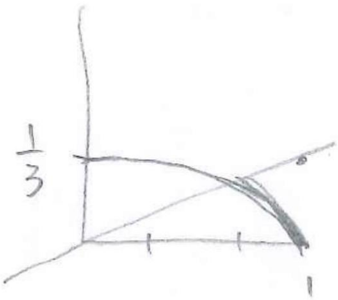
Using Stoke's Theorem, we can determine that $\iint_S \text{curl} F \cdot dS = \oint_C F\cdot dr$. $\oint_C F \cdot dr$ in this case is the sum of all the integrals from the different boundaies. Hence we get

$$\iint_S \text{curl} F \cdot dS = -\oint_{C_1} F\cdot dr + \int_{C_2} F\cdot dr - \int_{C_3} F\cdot dr + \int_{C_4} F\cdot dr$$

$$\oint_{C_1} F\,dr = 305 - 20 - 104 + 27$$

$$= 7 + 201 = \boxed{208}$$

3. (12 points) Let C be the part of the ellipse $x^2+9y^2=1$ bewteen $y=0$ and $y=\frac{1}{3}x$ in the first quadrant. Find $\int_C x\sqrt{\frac{1}{9}x^2+9y^2}\,ds$.



We can parameterize ellipse as $x=t$. Since $9y^2=1-x^2$ we can use $y=\frac{1}{3}\sqrt{1-x^2}$. Hence we get the param. of

$r(t)=\langle t, \frac{1}{3}\sqrt{1-t^2}\rangle$

We then find the limits, for which we get:

① $x^2=1,\ y=0$ ② $x^2+x^2=1,\ y=\frac{1}{3}x$

$\downarrow$ $x=\sqrt{\frac{1}{2}}$

$x=1$ based on drawing.

So we get $\int_{\sqrt{\frac{1}{2}}}^{1} t\sqrt{\frac{1}{9}t^2+9\left(\frac{1}{9}(1-t^2)\right)}\,dt$

$$=\int_{\sqrt{\frac{1}{2}}}^{1} t\sqrt{\frac{1}{9}t^2+1-t^2}\,dt=\int_{\sqrt{\frac{1}{2}}}^{1} t\sqrt{-\frac{8}{9}t^2+1}\,dt=\left[-\frac{3}{8}\left(-\frac{8}{9}t^2+1\right)^{\frac{3}{2}}\right]_{\sqrt{\frac{1}{2}}}^{1}$$

$$=-\frac{3}{8}\left[\left(\frac{1}{9}\right)^{\frac{3}{2}}-\left(-\frac{8}{9}\left(\frac{1}{2}\right)+1\right)^{\frac{3}{2}}\right]=-\frac{3}{8}\left[\frac{1}{27}-\left(\frac{5}{9}\right)^{\frac{3}{2}}\right]$$

$$= \boxed{\frac{3}{8}\left(\frac{5}{7}\right)^{\frac{3}{2}} - \frac{1}{72}}$$

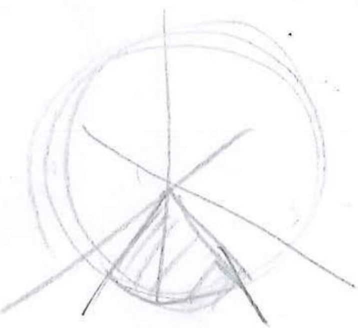
4. (14 points) The solid $\mathcal{W}$ lies in the region where $x^2 + y^2 + z^2 \leq \frac{1}{100}$ and $\sqrt{3}z \leq -\sqrt{x^2+y^2}$, where distance is measured in meters, and has constant density $\delta(x,y,z) = 3\,\mathrm{kg\,m^{-3}}$.

(a) Write $\mathcal{W}$ using spherical coordinates.

(b) Find the moment of inertia of $\mathcal{W}$ about the z-axis. *(Do not forget to use the correct units.)*

a. Using spherical we get $\rho^2 \leq \frac{1}{100}$ so $\rho \leq \frac{1}{10}$

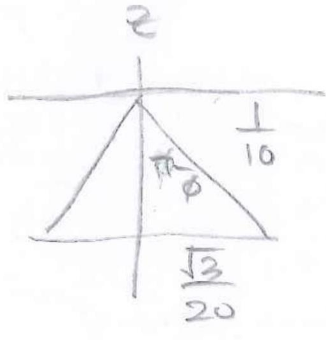
Then we try to find the intersection of the two regions:



$$x^2+y^2+\frac{x^2+y^2}{3} = \frac{1}{100} \quad \text{(substitution)}$$

$$\frac{4}{3}(x^2+y^2) = \frac{1}{100}$$

$$x^2+y^2 = \frac{3}{400} \rightarrow \text{circle of radius } \frac{\sqrt{3}}{20}.$$



Using projection, we can determine angle to be $\arcsin\left(\frac{\frac{\sqrt{3}}{20}}{\frac{1}{10}}\right) = \frac{\sqrt{3}}{2}$

$= \pi - \phi = \frac{\pi}{3}$. So $\phi = \frac{2\pi}{3}$

Hence our region is $\left\{0 \leq \theta \leq 2\pi,\ \frac{2\pi}{3} \leq \phi \leq \pi,\ 0 \leq \rho \leq \frac{1}{10}\right\}$

Now to calculate inertia,

we get $\iiint_W x^2+y^2\,\delta(x,y,z)\,dV = 3\iiint_W x^2+y^2\,dV.$

$x = \rho\cos\theta\sin\phi$ So $x^2+y^2 = \rho^2\sin^2\phi$

$y = \rho\sin\theta\sin\phi$

$$= 3\int_0^{2\pi}\int_{\frac{2}{3}\pi}^{\pi}\int_0^{\frac{1}{10}} \underbrace{\rho^2\sin\phi}_{\text{Jacobian}}\,(\underbrace{\rho^2\sin^2\phi}_{x^2+y^2})\,d\rho\, d\phi\, d\theta$$

Since there is no θ in integrand, $= 6\pi\int_{\frac{2}{3}\pi}^{\pi}\int_0^{\frac{1}{10}} \rho^4\sin^3\phi\, d\rho\, d\phi$

Since limits are constant we get

$$= 6\pi\int_0^{\frac{1}{10}} \rho^4\, d\rho \int_{\frac{2\pi}{3}}^{\pi} \sin^3\phi\, d\phi = 6\pi\left[\frac{\rho^5}{5}\right]_0^{\frac{1}{10}}\int_{\frac{2\pi}{3}}^{\pi}\sin^3\phi\, d\phi$$

Now we calculate $\int_{\frac{2\pi}{3}}^{\pi}\sin^3\phi\, d\phi$:

$$= \int_{\frac{2\pi}{3}}^{\pi} \sin\phi\,(1-\cos^2\phi)\, d\phi$$

$$= \int_{\frac{2\pi}{3}}^{\pi} \sin\phi - \sin\phi\cos^2\phi\, d\phi$$

$$= \left[-\cos\phi + \frac{1}{3}\cos^3\phi\right]_{\frac{2\pi}{3}}^{\pi}$$

$$= \left[\overbrace{1-\frac{1}{3}}^{\frac{2}{3}} - \left(\frac{1}{2} - \overbrace{\frac{1}{3}\left(\frac{1}{8}\right)}^{\frac{1}{24}}\right)\right]$$

$$= \frac{2}{3} - \frac{1}{2} + \frac{1}{24} = \frac{1}{6} + \frac{1}{24} = \boxed{\frac{5}{24}}$$

So then the moment of inertia is

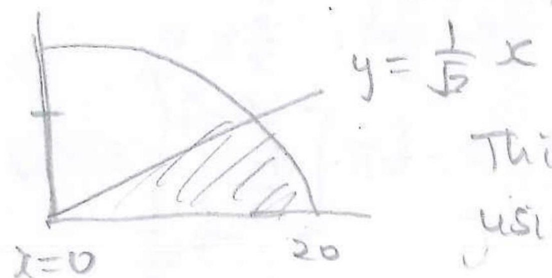
$$\cancel{6}\pi\left(\frac{1}{\cancel{5}(10^5)}\right)\left(\frac{\cancel{5}}{\cancel{24}_{4}}\right)$$

$$= \frac{\pi}{4(10^5)}$$

5. (14 points) A shot put throwing sector $\mathcal{D} \subset \mathbb{R}^2$ is bounded by the curves $x = 0$, $y = \frac{1}{\sqrt{3}}x$ and $x^2 + y^2 = 400$ in the first quadrant. On any given throw, the position at which my shot lands may be modelled by a pair of random variables (X, Y) with joint probability density

$$p_{X,Y}(x,y) = \begin{cases} \dfrac{3}{175}\dfrac{xy^2}{(x^2+y^2)^{\frac{3}{2}}} & \text{if } (x,y) \in \mathcal{D} \\ 0 & \text{otherwise,} \end{cases}$$

so that the distance I throw is $\boxed{\sqrt{X^2+Y^2}}$. $\boxed{\text{Find } \mathbb{E}[\sqrt{X^2+Y^2}]}$.



This region is best when using r, θ. First we get integral

$$\frac{3}{175}\iint (x^2+y^2)^{\frac{1}{2}}\left(\frac{xy^2}{(x^2+y^2)^{\frac{3}{2}}}\right) dydx = \frac{3}{175}\iint \frac{xy^2}{(x^2+y^2)}\, dydx$$

$$= \frac{3}{175}\iint \frac{r\cos\theta\,(r^2\sin^2\theta)}{r^2}(r)\, drd\theta$$

$$= \frac{3}{175}\iint r^2\cos\theta\sin^2\theta\, drd\theta.$$ Then we find region:

$y = \frac{1}{\sqrt{3}}x, \rightarrow r\sin\theta = \frac{1}{\sqrt{3}} r\cos\theta \quad \tan\theta = \frac{1}{\sqrt{3}}$

So $\theta = \frac{\pi}{6}$. Then we got the limits

$$\frac{3}{175}\int_0^{\frac{\pi}{6}}\int_0^{20} r^2\cos\theta\sin^2\theta\, drd\theta.$$ To evaluate, we separate the integrals since the limits are constant

$$= \frac{3}{175}\int_0^{20} r^2\,dr \int_0^{\pi/6} \sin^2\theta\cos\theta\,d\theta$$

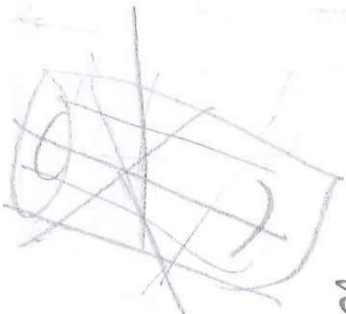
$$= \frac{3}{175}\left[\frac{r^3}{3}\right]_0^{20}\left[\frac{1}{3}\sin^3\theta\right]_0^{\frac{\pi}{6}} = \frac{1}{3(175)}(20)^3\left(\frac{1}{8}\right)$$

$$= \frac{\cancel{1000}^{200}}{3(\cancel{175}_{35})} = \frac{200}{105} = \boxed{\frac{40}{21}}$$

6. (14 points) Let $\boxed{\mathcal{S}}$ be the boundary of the region $\mathcal{W}$ bounded by the cylinders $y^2 + z^2 = 1$, $y^2 + z^2 = 9$ and the planes $x = 3$, $y = x$ oriented with outward pointing normal. Find the flux of the vector field $\mathbf{F} = \left\langle \frac{y}{x^2+y^2}, -\frac{x}{x^2+y^2}, 2z \right\rangle$ across $\mathcal{S}$.

The flux across F is $\iint F \cdot dS$.

First we test with divergence Theorem, $\iiint_W \operatorname{div} F \, dV = \iint F \cdot dS$.



So we find that div F =

$\frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$.

$= -\frac{2xy}{(x^2+y^2)^2} + \frac{2xy}{(x^2+y^2)^2} + 2 = 2$. Since div F = 2,

we can simply find the region W since

$\iint F \cdot dS = \iiint_W 2 \, dV$ $\qquad y^2+z^2=1 \quad \theta \to$ angle bt y & z

We find the region as $\{1 \le r \le 3,\ r\sin\theta \le x \le 3,\ 0 \le \theta \le 2\pi\}$

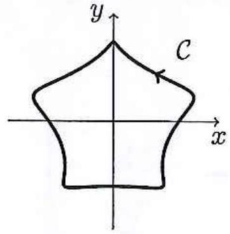
$$= 2\int_0^{2\pi}\int_1^3\int_{r\sin\theta}^3 r \, dx\, dr\, d\theta = 2\int_0^{2\pi}\int_1^3 3r - r^2\sin\theta \, dr\, d\theta$$

$$= 2\int_0^{2\pi}\left[\frac{3r^2}{2} - \frac{r^3}{3}\sin\theta\right]_1^3 d\theta = 2\int_0^{2\pi} \frac{27}{2} - \frac{\cancel{27}^{\,9}}{\cancel{3}}\sin\theta - \left(\frac{3}{2} - \frac{\sin\theta}{3}\right) d\theta$$

$$= 2\int_0^{2\pi} \frac{24}{2} - \frac{26}{3}\sin\theta \, d\theta = 2\left[12\theta + \frac{26}{3}\cos\theta\right]_0^{2\pi}$$

$$= 2\left[24\pi + \frac{26}{3} - \frac{26}{3}\right] = \boxed{48\pi}$$

7. (12 points) Let $\mathcal{C}$ be the curve



i j k
∂x ∂y ∂z
F_1 F_2 F_3

Find $\oint_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$ where

$$\mathbf{F}(x, y) = \left\langle -\frac{y}{x^2+y^2} + \sin(x^5) + 2ye^{2xy},\ \frac{x}{x^2+y^2} + e^{\cos(y)} + 2xe^{2xy} \right\rangle.$$

(*Hint: Try writing* $\mathbf{F}$ *as a sum of two vector fields that we know how to integrate around* $\mathcal{C}$.)

We can write $F_1(x,y) = \langle 2ye^{2xy}, 2xe^{2xy} \rangle$
Since we know that F_1 is conservative with a potential function of $f = e^{2xy}$, $\oint_C F_1 \cdot dr = 0$
Then we can write $F_2 = \langle \sin(x^5), e^{\cos(y)} \rangle$
Since we know $\int_C F_2 \cdot dr = \iint \mathrm{curl}_z F_2 \, dA$,
$\mathrm{curl}_z F_2 = \frac{\partial F_{22}}{\partial x} - \frac{\partial F_{21}}{\partial y} = 0$. So $\int_C F_2 \cdot dr = 0$.
We now remain with $F_3 = \langle -\frac{y}{x^2+y^2}, \frac{x}{x^2+y^2} \rangle$
$\mathrm{curl}_z F_3 = \frac{\partial F_{32}}{\partial x} - \frac{\partial F_{31}}{\partial y}$

$$= \frac{1}{x^2+y^2} - \frac{2x^2}{(x^2+y^2)^2} - \left(-\frac{1}{x^2+y^2} + \frac{2y^2}{(x^2+y^2)^2} \right) = \frac{2}{x^2+y^2} - \frac{2x^2+2y^2}{(x^2+y^2)^2}$$

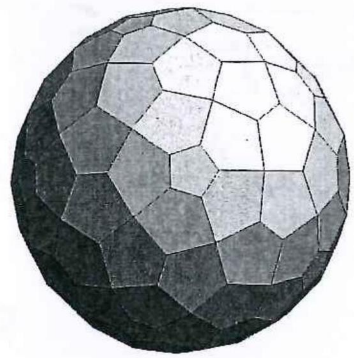
$$= \frac{2(x^2+y^2)^{2} - 2x^2 - y^2}{(x^2+y^2)^2} = 0$$

Since $\text{curl}_z F = 0$, $\iint \text{curl}_z F \, dA = \oint_c F_3 \cdot dr = 0$.

So $\oint_c F \cdot dr = \oint_c F_1 \cdot dr + \oint_c F_2 \cdot dr + \oint_c F_3 \cdot dr$

$$= 0 + 0 + 0 = \boxed{0}$$

8. (10 points) Recall that a polyhedron is a solid bounded by several planar surfaces, for example



$\langle x, y\rangle + \langle x, y\rangle$

$\langle x, x\rangle + \langle y, y\rangle$

Let $\mathcal{W} \subset \mathbb{R}^3$ be a polyhedron with boundary $\mathcal{S}$ composed of k planar surfaces $\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_k$ so that

$$\mathcal{S} = \mathcal{S}_1 \cup \mathcal{S}_2 \cup \cdots \cup \mathcal{S}_k.$$

We orient $\mathcal{S}$ with the outward unit normal.

For each $j = 1, \ldots, k$ define the constant unit vector $\mathbf{b}_j$ so that $\mathbf{b}_j$ is equal to the outward unit normal to $\mathcal{S}$ on the surface $\mathcal{S}_j$. Define the constant vector $\mathbf{N}_j = \text{Area}(\mathcal{S}_j)\,\mathbf{b}_j$.

(a) Let $\mathbf{F} = \mathbf{N}_1 + \mathbf{N}_2 + \cdots + \mathbf{N}_k$. Show that

$$\|\mathbf{F}\|^2 = \iint_{\mathcal{S}} \mathbf{F} \cdot d\mathbf{S}. \quad = \text{div} F \, dV$$

(b) Using your answer to part (a), show that $\mathbf{F} = 0$.

a. $\iint_S F \cdot dS = \iint_S F \cdot n \, dS \rightarrow$ this can be broken to smaller surfaces. We know normal to plane is $\langle a_k, b_k, c_k\rangle$, So we can get

$$\iint_{S_1} \langle A_1 a_1 + \ldots + A_k a_k, A_1 b_1 \ldots A_k b_k, A_1 c_1 \ldots A_k c_k\rangle$$

To sum everything we get

$\iiint_V$

So $\iint_S F \cdot d\vec{S} = \iint_{S_1} F \cdot n_1 \, dS_1 + \iint_{S_2} F \cdot n_2 \, dS_2 \ldots \iint_{S_k} F \cdot n_k \, dS_k$

$= F \cdot n_1 \underbrace{\iint 1 \, dS_1}_{\downarrow} + F \cdot n_2 \iint 1 \, dS_2 \ldots F \cdot n_k \iint 1 \, dS_k$

$= F \cdot \frac{N_1}{\cancel{A_1}} (\cancel{A_1}) + F \cdot \frac{N_2}{\cancel{A_2}} (\cancel{A_2}) \ldots F \cdot \frac{N_k}{\cancel{A_k}} (\cancel{A_k})$

$= F \cdot N_1 + F \cdot N_2 \ldots F \cdot N_k$

$= F \cdot (N_1 + N_2 \ldots N_k) = F \cdot F = \|F\|^2$

b. $\iint_S F \cdot dS = \iiint_W \operatorname{div}(F) \, dV$

Since F is constant vector field, $\operatorname{div} F = 0$. So $\iiint_W \operatorname{div} F \, dV = 0$, then

$\iint_S F \cdot dS = \|F\|^2 = 0.$

This means F is 0.