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MATH 32B Midterm I, Spring 2015

Name:

Circle Your TA's Name and Section Number: **Joshua Keneda 2A 2B, Eden Prywes 2C 2D, Yilon Yang 2E 2F**

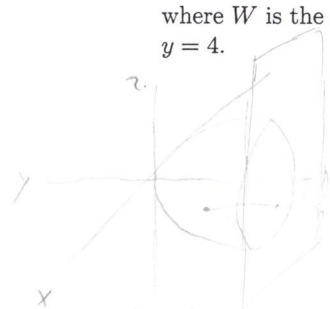
Justify All Your Answers. No Points Will Be Given Without Sufficient Reasoning/Calculations.

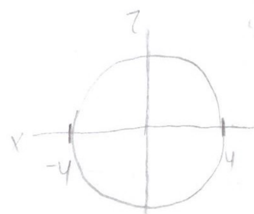
Problem 1. (4)

Find the triple integral

$$\iiint_W \sqrt{x^2+z^2}\, dV,$$

where W is the region bounded by the paraboloid $4y = (x^2+z^2)$ and the plane $y = 4$.





$4(4) = x^2 + z^2$

$16 = x^2 + z^2$

$x = r\cos\theta$

$z = r\sin\theta$

D: $0 \le \theta \le 2\pi$, $0 \le r \le 4$

$4y = r^2$

$y = \frac{r^2}{4}$

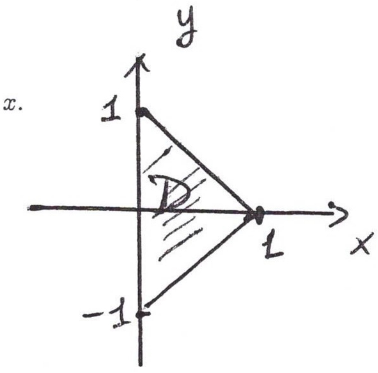
$$\int_0^{2\pi}\int_{r=0}^{4}\int_{y=\frac{r^2}{4}}^{4} r \cdot r \, dy\, dr\, d\theta = 2\pi \int_0^4 r^2 y \Big|_{\frac{r^2}{4}}^{4} dr\, d\theta$$

$$= 2\pi \int_0^4 4r^2 - \frac{r^4}{4}\, dr\, d\theta = 2\pi\left(\frac{4r^3}{3} - \frac{r^5}{20}\Big|_0^4\right)$$

$$= 2\pi\left(\frac{4(4)^3}{3} - \frac{4^5}{20}\right)$$

Problem 2. (4)

Compute the center of mass of D. Here the density fuction $\rho(x, y) = x$.



D: $0 \le x \le 1$

$x-1 \le y \le 1-x$

$M(D) = \int_0^1 \int_{x-1}^{1-x} x \, dy \, dx$

$= \int_0^1 xy \Big|_{x-1}^{1-x} dx = \int_0^1 x - x^2 - x^2 + x \, dx$

$= \int_0^1 2x - 2x^2 \, dx$

$= x^2 - \frac{2x^3}{3} \Big|_0^1 = 1 - \frac{2}{3} - (0) = \frac{1}{3}$

$M_x = \int_0^1 \int_{x-1}^{1-x} xy \, dy \, dx$

$= \int_0^1 x \frac{y^2}{2} \Big|_{x-1}^{1-x} dx = \frac{1}{2} \int_0^1 x\left(1 - 2x + x^2 - (x^2 - 2x + 1)\right) dx = \frac{1}{2} \int_0^1 x(0) \, dx$

$= 0$

$M_y = \int_0^1 \int_{x-1}^{1-x} x^2 \, dy \, dx = \int_0^1 x^2 y \Big|_{x-1}^{1-x} dx = \int_0^1 x^2 - x^3 - x^3 + x^2 \, dx$

$= \int_0^1 2x^2 - 2x^3 \, dx = 2\left(\frac{x^3}{3} - \frac{x^4}{4}\right)\Big|_0^1 = 2\left(\frac{1}{3} - \frac{1}{4}\right) = 2\left(\frac{4}{12} - \frac{3}{12}\right) = \frac{1}{6}$

$\left(\frac{1}{6} \cdot \frac{3}{1}, 0\right) = \boxed{\left(\frac{1}{2}, 0\right)}$

Problem 3. (4)

Find the iterated integral

$$\int_0^k \int_{y=-\sqrt{k^2-x^2}}^{\sqrt{k^2-x^2}} (x^2+y^2)\cos(x^4+y^4+2x^2y^2)\,dy\,dx.$$

Here k is a positive constant.

$(x^2+y^2)^2$

$x^4+2x^2y^2+y^4$

r^2

$r\,dr\,d\theta$

$x = r\cos\theta$

$y = r\sin\theta$

$(x^2+y^2)^2 = (r^2)^2 = r^4$

$k^2 - x^2 = k^2 - r^2\cos^2\theta$

$y^2 = k^2 - x^2$

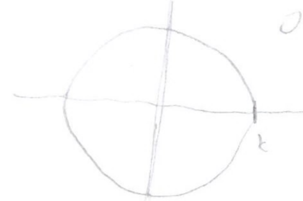
$y^2 + x^2 = k^2$

$r^2 = k^2$

$k = r$

$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$0 \le r \le k$



$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^k r^3 \cos(r^4)\,dr\,d\theta$$

$$\int_{-\pi/2}^{\pi/2} \frac{1}{4} \sin r^4 \Big|_0^k \,d\theta$$

$$\int_{-\pi/2}^{\pi/2} d\theta \cdot \left(\frac{1}{4}\sin k^4 - \sin 0\right)$$

$$\left(\frac{\pi}{2}+\frac{\pi}{2}\right)\cdot\frac{\sin k^4}{4} = \boxed{\frac{\pi \sin k^4}{4}}$$

Problem 4. (4)

$\sqrt{\rho^2\sin^2\phi} = \rho\sin\phi$

Find

$$\iiint_W \sqrt{x^2+y^2}\,dV.$$

Here W is the solid with $z \geq 0$, above the cone $x^2+y^2=z^2$ and below the sphere $x^2+y^2+z^2=R^2$. Here R is a positive constant.

$0 \le \phi \le \frac{\pi}{4}$

$0 \le \theta \le 2\pi$

$0 \le \rho \le R$

$x^2+y^2+z^2=R^2$

$\rho^2 = R^2$

$\rho = R$

$x = \rho\sin\phi\cos\theta$

$y = \rho\sin\phi\sin\theta$

$\rho^2\sin^2\phi\cos^2\theta + \rho^2\sin^2\phi\sin^2\theta$

$= \rho^2\sin^2\phi$

$$\int_0^{\pi/4}\int_0^{2\pi}\int_0^R \overbrace{\rho^2\sin^2\phi\,(\rho^2\sin\phi)}^{\rho^4}\,d\rho\,d\theta\,d\phi$$

$$= 2\pi\int_0^{\pi/4}\sin^3\phi\,d\phi \cdot \frac{\rho^5}{5}\Big|_0^R$$

$\sin^3 = \sin(1-\cos^2\phi)$

$= \sin - \sin\cos^2$

$$= 2\pi\frac{R^5}{5}\left(\int_0^{\pi/4}\sin\phi - \sin\phi\cos^2\phi\,d\phi\right)$$

$$= \frac{2\pi R^5}{5}\left(-\cos\phi + \frac{\cos^3\phi}{3}\Big|_0^{\pi/4}\right)$$

$$= \frac{2\pi R^5}{5}\left(-\frac{\sqrt{2}}{2} + \frac{\left(\frac{\sqrt{2}}{2}\right)^3}{3} - \left(-1 + \frac{1}{3}\right)\right)$$

$$= \frac{2\pi R^5}{5}\left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{12} + \frac{2}{3}\right) = \frac{2\pi R^5}{5}\left(\frac{\sqrt{2} - 6\sqrt{2} + 4}{12}\right)$$

Problem 5. (4)

Estimate the following integral

$$\iiint_W e^{\sin x \cos y \sin z}\, dV,$$

where W is the solid bounded by the cylinder $x^2 + y^2 = R^2$, the plane $z = -h$ and the plane $z = h$. Here R and h are two positive constants.

$0 \le \theta \le 2\pi$

$0 \le r \le R$

$-h \le z \le h$

$$\int_0^{2\pi}\int_0^R\int_{-h}^{h} e^{\sin x \cos y \sin z}\, dz\, r\, dr\, d\theta$$

$$-1 \le \sin x \cos y \sin z \le 1$$

$$e^{-1} \le e^{\sin x \cos y \sin z} \le e^{1}$$

$$\int_0^{2\pi}\int_0^R\int_{-h}^{h} r e^{-1}\, dz\, dr\, d\theta \le \iiint_W e^{\sin x \cos y \sin z}\, dV \le \iiint_W e^{1}\, dV$$

$$= 2\pi \int_0^R r\, dr \cdot \frac{1}{e} z \Big|_{-h}^{h} = 2\pi\left(\frac{r^2}{2}\Big|_0^R\right)\cdot\left(\frac{h}{e} - \frac{-h}{e}\right) = \frac{2\pi R^2 h}{e}$$

$$\int_0^{2\pi}\int_0^R\int_{-h}^{h} r e\, dz\, dr\, d\theta = 2\pi\left(\frac{r^2}{2}\Big|_0^R\right)\cdot e z\Big|_{-h}^{h} = \pi R^2 (he - -he)$$

$$= 2\pi R^2 h \cdot e$$

$$\boxed{\frac{2\pi R^2 h}{e} \le \iiint_W e^{\sin x \cos y \sin z}\, dV \le 2\pi R^2 h \cdot e}$$