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904 446 502

17.5

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2	2.5
3	4
4	3
5	4

MATH 32B Midterm I, Fall 2015

Name:

Circle Your TA's Name and Section Number: **Yuming Zhang 2A 2B, Matthew Stoffregen 2C 2D, Qianchang Wang 2E 2F**

Justify All Your Answers. No Points Will Be Given Without Sufficient Reasoning/Calculations.

Problem 1. (4)

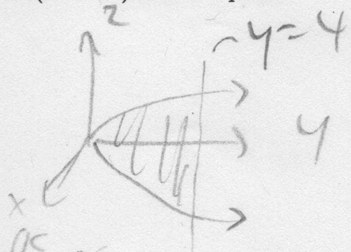
Find the triple integral

$$\iiint_E x^2\, dV,$$

where E is the region bounded by the paraboloid $y = 4(x^2 + z^2)$ and the plane $y = 4$.

$y = 4(x^2+z^2) = 4 \Rightarrow x^2+z^2 = 1$

$\Rightarrow r^2 = 1$



$x = r\cos\theta,\ z = r\sin\theta,\ y = y \qquad 0 \le \theta \le 2\pi,\ 0 \le r \le 1$

$4r^2 \le y \le 4$

$2\cos^2\theta - 1 = \cos 2\theta$

$\cos^2\theta = \frac{\cos 2\theta + 1}{2}$

$I = \int_0^{2\pi}\int_0^1\int_{4r^2}^4 r^2\cos^2\theta \cdot r\, dy\, dr\, d\theta$

$= \int_0^{2\pi} \cos^2\theta\, d\theta \int_0^1 r^3(4-4r^2)\, dr$

1

$= \int_0^{2\pi} \frac{\cos 2\theta + 1}{2}\, d\theta \int_0^1 4r^3 - 4r^5\, dr$

$= \frac{1}{2}\left(\frac{1}{2}\sin 2\theta + \theta\right)\Big|_0^{2\pi} \left(r^4 - \frac{2}{3}r^6\right)\Big|_0^1$

$= \frac{1}{2}(2\pi)\left(1 - \frac{2}{3}\right) = \frac{\pi}{3}$

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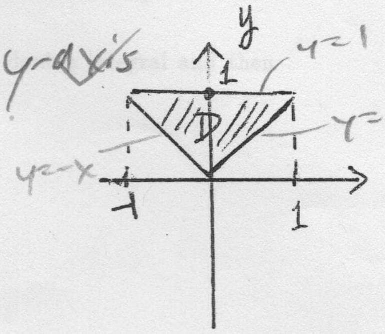
Problem 2. (4)

Compute the center of of mass of D given by the following picture. Here the density fuction $\rho(x,y) = y^2$.



$x_{cm} = 0$ because of symmetry over y-axis

$M(D) = 2\int_0^1 \int_x^1 y^2\,dy\,dx$

$= 2\int_0^1 \frac{1}{3}y^3\Big|_x^1 dx$

$= \frac{2}{3}\int_0^1 1 - \frac{1}{3}x^3\,dx$

$= \frac{2}{3}\left(x - \frac{1}{12}x^4\right)\Big|_0^1$

$= \frac{2}{3}\left(1 - \frac{1}{12}\right) = \frac{2}{3}\cdot\frac{11}{\cancel{12}\,6} = \frac{11}{18}$

$M_x = \int_{-1}^0 \int_{-x}^1 y^3\,dy\,dx + \int_0^1 \int_x^1 y^3\,dy\,dx$

$= \int_{-1}^0 \frac{1}{4}y^4\Big|_{-x}^1 dy\,dx + \int_0^1 \frac{1}{4}y^4\Big|_x^1$

$= \frac{1}{4}\int_{-1}^0 1 - x^4\,dx + \frac{1}{4}\int_0^1 1 - x^4\,dx$

$= \frac{1}{4}\left[\left(x - \frac{1}{5}x^5\right)\Big|_{-1}^0 + \left(x - \frac{1}{5}x^5\right)\Big|_0^1\right]$

$= \frac{1}{4}\left(-\left(-1 + \frac{1}{5}\right) + \left(1 - \frac{1}{5}\right)\right)$

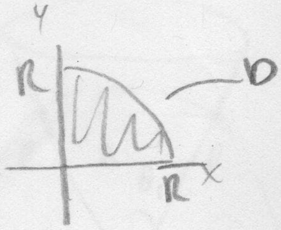
$= \frac{1}{4}\left(2 - \frac{2}{5}\right) = \frac{1}{4}\left(\frac{8}{5}\right) = \frac{2}{5}$ ✓

$\therefore y_{cm} = \frac{M_x}{M(D)} = \frac{\frac{2}{5}}{\frac{11}{8}}$

Problem 3. (4)

Find the iterated integral $\int_0^R \int_0^{\sqrt{R^2-x^2}} e^{(x^2+y^2)}dydx$. Here R is a positive constant.

Hint: Convert the iterated integral into a proper double integral and then evaluate the double integral.



$f(x,y) = e^{x^2+y^2} = e^{r^2}$

$0 \leq r \leq R, \; 0 \leq \theta \leq \frac{\pi}{2}$

$I = \int_0^{\frac{\pi}{2}} \int_0^R e^{r^2} \cdot r \, dr \, d\theta$

$= \frac{\pi}{2} \left(\frac{1}{2} e^{r^2}\right) \Big/_0^R$

$= \frac{\pi}{4}\left(e^{R^2} - 1\right)$

Problem 4. (4)

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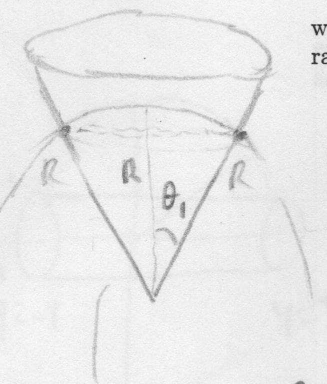
Find the triple integral

$$\iiint_W x^2\,dV,$$

$\rho = R$

where W is the region in $\mathbf{R}^3$ bounded by the sphere $x^2+y^2+z^2=R^2$ of the radius R and the cone $x^2+y^2=z^2$ with $z \geq 0$.

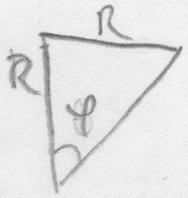


$x^2+y^2=z^2 \Rightarrow r^2=z^2 \Rightarrow z=r$

$0 \leq \varphi \leq \frac{\pi}{4} \quad 0 \leq \theta \leq 2\pi,\ 0 \leq \rho \leq R$

$r^2+z^2=R^2 \quad 0 \leq \theta \leq 2\pi,\ 0 \leq r \leq R$

$z=\sqrt{R^2-r^2},\ 0 \leq z \leq \sqrt{R^2-r^2}$



$\varphi = \tan^{-1}\frac{R}{R} = \frac{\pi}{4}$

$I = \int_0^{2\pi}\int_0^R\int_0^{\sqrt{R^2-r^2}} r^2\cos^2\theta \cdot r\,dr\,d\theta$

$I = \int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_0^R \rho^2 \sin^2\varphi\cos^2\theta \cdot \rho^2 \sin\cancel{\theta}\,d\rho\,d\varphi\,d\theta$

$\cos 2\varphi = 1 - 2\sin^2\varphi$

$\frac{1-\cos 2\varphi}{2} = \sin^2\varphi$

$= \int_0^{2\pi} \cos^2\theta\sin\theta\,d\theta \int_0^{\frac{\pi}{4}} \sin^2\varphi\,d\varphi \int_0^R \rho^4\,d\rho$

$= -\frac{1}{3}\cos^3\theta\Big/_0^{2\pi} \int_0^{\frac{\pi}{4}} \frac{1}{2}(1-\cos 2\varphi)\,d\varphi \cdot \frac{1}{5}\rho^5$

$= -\frac{1}{3}((-1)^3 - 1) \cdot \frac{1}{2}(\varphi - \frac{1}{2}\sin 2\varphi)\Big/_0^{\frac{\pi}{4}} \cdot \frac{1}{5}\rho^5$

$= \frac{2}{3} \cdot \frac{1}{2}(\frac{\pi}{4} - \frac{1}{2}\cdot 1) \cdot \frac{1}{5}\rho^5$

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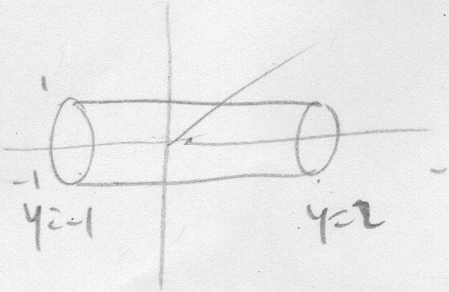
Problem 5. (4)

Estimate the following integral

$$\iiint_E e^{sinxcosysinz}\,dV,$$

where E is the region inside the cylinder $x^2 + z^2 = 1$ between the planes $y = -1$ and $y = 2$.



$Vol(E) = \pi(1)^2(3) = 3\pi$

$-1 \le \sin x \cos y \sin z \le 1,$

$\therefore e^{-1} \le e^{\sin x \cos y \sin z} \le e$

$\therefore \frac{3\pi}{e} \le \iiint_E e^{\sin x \cos y \sin z} \le 3\pi e$