Math 32B-1 Exam 2 - Blue

KADE BIERMANN ADAMS

TOTAL POINTS

64 / 100

QUESTION 1

Problem 1 - Page 1 15 pts

1.1 Part (a) 2 / 4

+ **1 pts** Correct definition of curl.
✓ + **1 pts** Correct computation of curl.
+ **1 pts** F is defined on R^3 (a simply connected domain).
✓ + **1 pts** Conclusion that F is conservative.
+ **0 pts** Blank or completely incorrect.

1.2 Part (b) 3 / 3

✓ + **2 pts** Correct function f.
✓ + **1 pts** Some justification (either partial integrals or computation of the gradient).
+ **1 pts** Partial credit: one summand of f correct.
+ **0 pts** Completely incorrect or blank.

1.3 Part (c) 2 / 4

+ **2 pts** Reference to Fundamental Theorem of Line Integrals.
✓ + **1 pts** Correct formula involving endpoints.
✓ + **1 pts** Correct final answer.
+ **0 pts** Blank or completely incorrect.

1.4 Part (d) 4 / 4

✓ + **1 pts** div(curl G) = 0
✓ + **1 pts** Correctly calculate div(F).
✓ + **1 pts** Observe 2 is not zero.
✓ + **1 pts** Conclude no such G exists based on computation.
+ **0 pts** Blank or completely incorrect.

QUESTION 2

2 Problem 2 - Page 2 11 / 15

✓ + **3 pts** Correctly computed $$\frac{\partial F_1}{\partial y}$$
✓ + **3 pts** Correctly computed $$\frac{\partial F_2}{\partial x}$$
✓ + **1 pts** Concluding the partial derivatives are equal (must have correctly computed derivatives; circling/marking the two calculations doesn't suffice).

✓ + **4 pts** Correctly determined the connected components of the domain.
+ **4 pts** Correctly explaining why the vector field is conservative on its domain (must have correctly explained what the domain is).
+ **0 pts** No credit

The components are simply connected. Review the definition.

QUESTION 3

3 Problem 3 - Page 2 13 / 15

+ **2 pts** Correct expression for work (must explicitly state work is equal to some line integral, writing something to the effect of 'work =' or 'W =')
✓ + **7 pts** Valid parametrization of curve
✓ + **2 pts** Calculated tangent vector given a parametrization
✓ + **2 pts** Plug into expression for computing vector line integral
✓ + **2 pts** Correct final answer
+ **0 pts** No credit

QUESTION 4

4 Problem 4 - Page 3 12 / 15

+ **15 pts** Correct
✓ + **3 pts** Full Credit Criterion 1: Student uses Green's Theorem to convert the line integral into an area

integral and provides correct formula.

✓ + 5 pts **Full Credit Criterion 2: After using Green's Theorem, the student has the correct area integral written down.**

✓ + 4 pts **Full Credit Criterion 3: Correct Bounds are set up to evaluate the double integral**

+ **2 pts** Full Credit Criterion 4: Integrals are correctly evaluated, i.e. no calculus mistakes were made and every time you take an integral you get the correct expression.

+ **1 pts** Full Credit Criterion 5: Correct Answer is given

+ **5 pts** Partial Credit 1: The student failed to use Green's Theorem, but still correctly evaluated the integral by calculating three separate line integrals managing to get the correct answer

+ **1 pts** Partial Credit 2: The student failed to use Green's Theorem and instead attempted to evaluate three separate line integrals, but in the end managing to get the incorrect answer.

+ **7 pts** Partial Credit 3: The student oriented the boundary of C in the negative direction, but still managed to get the correct answer for this case (which is the negative of the actual correct answer).

+ **0 pts** Incomplete or Completely incorrect

You had the correct formula from Green's Theorem, but changed the integrand in the last line.

QUESTION 5

5 Problem 5 - Page 3 **0 / 15**

+ **15 pts** Correct

+ **3 pts** Full Credit Criterion 1: The student clearly draws the region they want to integrate over. This should be a half ellipse with the two arcs oriented in the correct direction.

+ **4 pts** Full Credit Criterion 2: Using their picture, the student converts the line integral over the full boundary of the half ellipse into an area integral using Green's Theorem. They should 0 for this area integral.

+ **3 pts** Full Credit Criterion 3: The student indicates that the sum of the line integrals over the arcs of the region equals the line integral over the whole boundary of the region. This should be an equation.

+ **4 pts** Full Credit Criterion 4: The student calculates the line integral over the x-axis from the region drawn in criterion 1.

+ **1 pts** Full Credit Criterion 5: The final answer is correct.

+ **3 pts** Partial Credit 1: The student misread the question and just evaluated the area integral over a half ellipse. If they got zero, they get can get this partial credit option. Note this is different from criterion 2 above.

✓ + 0 pts **Incomplete or no meaningful progress**

You did not answer the actual question begin asked.

QUESTION 6

6 Problem 6 - Page 4 **17 / 25**

✓ + 4 pts **Part a: Parametrization formula correct**

✓ + 3 pts **Part a: Parametrization domain correct**

+ **2 pts** Part a: Attempted to calculate the normal vector induced by the parametrization

+ **2 pts** Part a: Induced tangent vectors calculated correctly

+ **1 pts** Part a: Induced tangent vectors calculated with a minor error

+ **2 pts** Part a: Normal correctly calculated from induced tangent vectors, with correct orientation

+ **1 pts** Part a: Normal calculated from induced tangent vectors with some errors.

✓ + 5 pts **Part a: Set up the integral correctly**

+ **2.5 pts** Part a: Set up the integral with some errors

+ **2 pts** Part a: Calculated the integral correctly

✓ + 1 pts **Part a: Integral calculated with some errors**

✓ + 3 pts **Part b: Attempted to calculate the integral of 1 over the surface (correctly using the parametrization)**

✓ + 2 pts **Part b: Correctly calculated the integral.**

+ **1 pts** Part b: Integral calculated with some errors

+ **0 pts** Completely incorrect.

- 1 Point adjustment

No work is shown for computing the normal, so no rubric points are awarded for that. One point is given for obtaining the correct answer, somehow (+1pt adjustment).

However, since you tried to include the top and bottom disks when we explicitly said not to, two points are deducted (-2pt adjustment, covers same error in both parts).

Math 32B - Spring 2019

Exam 2

Full Name: Kade Adams

UID: 505123178

Circle the name of your TA and the day of your discussion:

Patrick Hiatt   Eli Sadovnik   Frederick Vu

Tuesday   Thursday

Instructions:

- Read each problem carefully.
- Show all work clearly and circle or box your final answer where appropriate.
- Justify your answers. A correct final answer without valid reasoning will not receive credit.
- Simplify your answers as much as possible.
- Include units with your answer where applicable.
- Calculators are not allowed but you may have a 3×5 inch notecard.

Page	Points	Score
1	15	
2	30	
3	30	
4	25	
Total:	100	

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You may use this page for scratch work. Work found on this page will not be graded unless clearly indicated in the exam.

1. (15 points) Let $\mathbf{F}(x, y, z) = \langle yz, xz, xy+2z \rangle$ and let C be the line segment from $(1, 0, -2)$ to $(4, 5, 3)$.

(a) Show that the vector field $\mathbf{F}$ is conservative using curl.

$\text{Curl}(\vec{F}) = \langle x-x, y-y, z-z \rangle = \vec{0}$

therefore $\vec{F}$ is conservative

(b) Find a function f such that $\mathbf{F} = \nabla f$.

$\int yz \, dx$ $\qquad \int xz \, dy$ $\qquad \int xy + 2z \, dz$

$xyz + g(y,z)$ $\qquad xyz + h(x,z)$ $\qquad xyz + z^2 + f(x,y)$

$\boxed{f = xyz + z^2}$

(c) Use part (b) to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$.

$\int_C F \cdot d\vec{r} = \int_C \nabla f \cdot d\vec{r} = f(\vec{r}(b)) - f(\vec{r}(a)) = (1+3(1))(5(1))(-2+5(1)) + (-2+5(1))^2 - (-2)^2$

$= 4 \cdot 5 \cdot 3 + 3^2 - (-2)^2$

$60 + 9 - 4 = \boxed{65}$

$\vec{r}(t) = \langle 1+3t, 5t, -2+5t \rangle \quad 0 \leq t \leq 1$

$f(\vec{r}(t)) = (1+3t)(5t)(-2+5t) + (-2+5t)^2$

(d) Is there a vector field $\mathbf{G}$ defined on $\mathbb{R}^3$ such that $\text{curl}\, \mathbf{G} = \mathbf{F}$?

If $\text{curl}\, G = F$

Then $\text{div}(\text{curl}\, G) = \text{div}(F)$

$0 = \text{div}(F)$

$\text{div}(F) = 0 + 0 + (0 + 2) = 2$

$\text{div}(F) \neq 0$

$\boxed{\text{No}}$

/15

2. (15 points) Consider the vector field

$$\mathbf{F}(x,y) = \langle F_1, F_2 \rangle = \left\langle \frac{y}{x^2-y^2}, \frac{-x}{x^2-y^2} \right\rangle.$$

(a) Show that $\dfrac{\partial F_1}{\partial y} = \dfrac{\partial F_2}{\partial x}$.

$$\frac{\partial F_1}{\partial y} = \frac{\partial F_2}{\partial x} \checkmark$$

$$\frac{\partial F_1}{\partial y} = \frac{(x^2-y^2) - (y(-2y))}{(x^2-y^2)^2} = \frac{x^2-y^2+2y^2}{(x^2-y^2)^2} = \boxed{\frac{x^2+y^2}{(x^2-y^2)^2}}$$

$$\frac{\partial F_2}{\partial x} = \frac{-(x^2-y^2) - (-x\,2x)}{(x^2-y^2)^2}$$

$$\frac{-x^2+y^2+2x^2}{(x^2-y^2)^2} = \boxed{\frac{x^2+y^2}{(x^2-y^2)^2}}$$

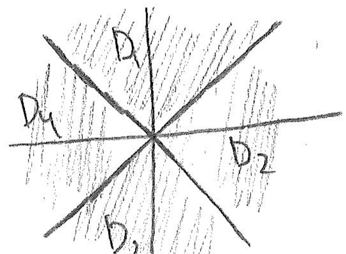
(b) Show that $\mathbf{F}$ is defined on four distinct connected domains in the plane. On each of these domains, is $\mathbf{F}$ conservative? *Hint:* Are these domains simply connected?

for $F(x,y)$ $x^2 \neq y^2$ so $D =$

$x \neq y$

$x \neq -y$

$-x = y$



• The domains are not simply connected as they are seperated by the lines $y=x$ & $y=-x$ creating 4 trinagular Domains

3. (15 points) Find the work done by the force field $\mathbf{F}(x,y) = \langle x^2, ye^x \rangle$ in moving a particle along the parabola $x = y^2+1$ from $(1,0)$ to $(2,1)$.

$y = t$

$x = t^2+1$

$\vec{r}(t) = \langle t^2+1, t \rangle \quad 0 \le t \le 1$

$\vec{F}(\vec{r}(t)) = \langle (t^2+1)^2, te^{t^2+1} \rangle$

$\vec{r}\,'(t) = \langle 2t, 1 \rangle$

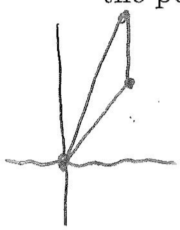
$$\int_C \vec{F}\cdot d\vec{r} = \int_0^1 \vec{F}(\vec{r}(t))\cdot\vec{r}\,'(t)\,dt = \int_0^1 \langle t^4+2t^2+1, te^{t^2+1} \rangle \cdot \langle 2t, 1 \rangle\, dt$$

$$= \int_0^1 2t^5+4t^3+2t+te^{t^2+1}\,dt = \frac{1}{3}t^6 + t^4 + t^2 + \frac{1}{2}e^{t^2+1} \Big|_0^1$$

$$= \left[\frac{1}{3}+1+1+\frac{e^2}{2}\right] - \left[\frac{e}{2}\right] = \frac{1}{3}+\frac{3}{3}+\frac{3}{3}+\frac{e^2}{2}-\frac{e}{2} = \boxed{\frac{7}{3}+\frac{e^2-e}{2}}$$

/30

4. (15 points) Use Green's Theorem to evaluate the line integral $\int_C xy^2\,dx + 2x^2y\,dy$ along the positively oriented boundary C of the triangle with vertices $(0,0)$, $(2,2)$, and $(2,4)$.



$\vec{F} = \langle xy^2, 2x^2y$

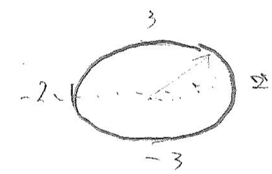
$\oint_C xy^2 dx + 2x^2y\,dy = \iint_D 4xy - 2xy\, dA = \iint_D 2xy$

$\int_0^2 \int_{y=x}^{y=2x} 4xy\,dy\,dx = \int_0^2 2xy^2 \Big|_x^{2x} dx = \int_0^2 4x^2 - 2x^2\,dx = \int_0^2 2x^2\,dx = \frac{2}{3}x^3\Big|_0^2$

$\boxed{\frac{16}{3}}$

5. (15 points) Let C be the curve parameterized by $\mathbf{r}(t) = \langle 2\cos t, 3\sin t\rangle$ for $0 \le t \le \pi$. Find $\int_C \left(e^x + y^2\cos x\right)\,dx + \left(e^{2y} + 2y\sin x\right)\,dy$. *Hint:* Complete C to form a closed curve and use Green's Theorem.

$\vec{r}(t) =$ ellipse



$\vec{F} = \langle e^x + y^2\cos x, e^{2y} + 2y\sin x\rangle$

$\oint_C \vec{F}\cdot d\vec{r} = \iint_D 0 + 2y\cos x - (0 + 2y\cos x)$

$\iint_D 1\,dA = \text{Area}(D)$

$\int_0^{2\pi}\int_0^{\sqrt{4\cos^2\theta + 9\sin^2\theta}} r\,dr\,d\theta$

$\int_0^{2\pi} \frac{1}{2}r^2 \Big|^{\sqrt{4\cos^2\theta + 9\sin^2\theta}} d\theta = \int_0^{2\pi} 2\cos^2\theta + \frac{9}{2}\sin^2\theta\,d\theta$

$\int_C \vec{F}\cdot d\vec{r} = \frac{1}{2}\oint_C \vec{F}\cdot d\vec{r} = \frac{1}{2}\text{Area}(D)$

$\left[\theta + \frac{1}{2}\sin(2\theta) + \frac{9}{2}\left(\frac{\theta}{2} - \frac{1}{4}\sin 2\theta\right)\right]_0^{2\pi}$

$\frac{1}{2}\text{Area}(D) = \frac{1}{2}\frac{13\pi}{2} = \boxed{\frac{13\pi}{4}}$

$2\pi + \frac{9}{2}\cdot\pi$ Area =

/30

6. (25 points) Consider the vector field $\mathbf{F}(x, y, z) = \langle x, y, e^z \rangle$ and the oriented surface $\mathcal{S}$ given by the cylinder $x^2 + y^2 = 4$ for $1 \le z \le 6$ with outward pointing normal. (Note: the surface $\mathcal{S}$ does not include the top or bottom of the cylinder).

(a) Find the flux $\iint_{\mathcal{S}} \mathbf{F} \cdot d\mathbf{S}$.

$\vec{r}(z,\theta) = \langle 2\cos\theta, 2\sin\theta, z \rangle$

$N = \langle 2\cos\theta, 2\sin\theta, 0 \rangle$

$\vec{F}(\vec{r}(z,\theta)) = \langle 2\cos\theta, 2\sin\theta, e^z \rangle$

$\iint_S \vec{F} \cdot d\vec{S}$

$$\int_1^6 \int_0^{2\pi} \langle 2\cos\theta, 2\sin\theta, e^z \rangle \cdot \langle 2\cos\theta, 2\sin\theta, 0 \rangle \, d\theta \, dz$$

$$\int_1^6 \int_0^{2\pi} 4\cos^2\theta + 4\sin^2\theta + \cancel{e} \, d\theta \, dz = \int_1^6 \int_0^{2\pi} 4 \, d\theta \, dz = \int_1^6 4\theta \Big|_0^{2\pi} dz = \int_1^6 8\pi dr = 8\pi r \Big|_1^6 = 48\pi$$

Top circle

$g(r,\theta) = \langle r\cos\theta, r\sin\theta, 6 \rangle$

$$\begin{array}{c|ccc|} & \vec{i} & \vec{j} & \vec{k} \\ g_r & \cos\theta & \sin\theta & 0 \\ g_\theta & -r\sin\theta & r\cos\theta & 0 \end{array}$$

$\vec{n} = \langle 0, 0, r \rangle$

$$\int_0^2 \int_0^{2\pi} \langle r\cos\theta, r\sin\theta, e^6 \rangle \cdot \langle 0,0,r \rangle \overset{d\theta\,dr}{=} \int_0^2 \int_0^{2\pi} re^6 \, d\theta \, dr = e^6 \int_0^2 r\theta \Big|_0^{2\pi} = e^6 \int_0^2 2\pi r = e^6 \left[\pi r^2 \Big|_0^2 \right] = e^6 \pi 4$$

bottom circle

$h(r,\theta) = \langle r\cos\theta, r\sin\theta, 1 \rangle$

$\vec{n} = \langle 0, 0, -r \rangle$

$$\int_0^2 \int_0^{2\pi} \langle r\cos\theta, r\sin\theta, e \rangle \cdot \langle 0,0,-r \rangle \overset{d\theta\,dr}{=} \int_0^2 \int_0^{2\pi} -re \, d\theta \, dr = \int_0^2 -re\theta \Big|_0^{2\pi} = \int_0^2 -2\pi er = -\pi e r^2 \Big|_0^2$$

$= -4\pi e$

$\boxed{\text{Flux} = 48\pi - 4\pi e + 4\pi e^6}$

(b) Use a surface integral to compute the surface area of $\mathcal{S}$. (You should be able to check your answer by computing the surface area another way).

Cylinder

$$SA = \iint_D \|\vec{n}\| \, dA = \int_1^6 \int_0^{2\pi} 2 \, d\theta \, dr = 2 \int_1^6 2\pi = 2(2\pi r \Big|_1^6) = 2((12\pi - 2\pi) = 20\pi$$

Top

$$SA = \int_0^2 \int_0^{2\pi} r \, d\theta \, dr = \int_0^2 r \int_0^{2\pi} 1 = 2 \cdot 2\pi = 4\pi$$

Bottom

$$SA = \int_0^2 \int_0^{2\pi} r \, d\theta \, dr = 4\pi$$

$SA = 4\pi + 4\pi + 20\pi = \boxed{28\pi}$

Check:

$\pi r^2 + \pi r^2 + 2\pi r L$

$4\pi + 4\pi + 4(5)\pi$

$4\pi + 4\pi + 20\pi$

28π

/25