Math 32B - Spring 2019

Exam 1

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Circle the name of your TA and the day of your discussion:

Patrick Hiatt　　Eli Sadovnik　　Frederick Vu

Tuesday　　Thursday

Instructions:

- Read each problem carefully.
- Show all work clearly and circle or box your final answer where appropriate.
- Justify your answers. A correct final answer without valid reasoning will not receive credit.
- Simplify your answers as much as possible.
- Include units with your answer where applicable.
- Calculators are not allowed but you may have a 3×5 inch notecard.

Page	Points	Score
1	25	
2	25	
3	25	
4	25	
Total:	100	

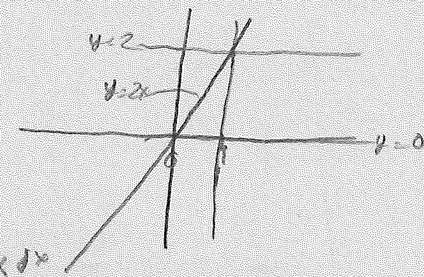
1. (10 points) Evaluate the iterated integral

$$\int_0^2 \int_{y/2}^1 \cos\left(\frac{\pi}{4}x^2\right) dx\, dy.$$

$0 \le y \le 2$

$\frac{y}{2} \le x \le 1$

$$\int_0^1 \int_0^{2x} \cos\left(\tfrac{\pi}{4}x^2\right) dy\, dx =$$



$$\int_0^1 \cos\left(\tfrac{\pi}{4}x^2\right) y \Big|_0^{2x} dx = \int_0^1 2x \cos\left(\tfrac{\pi}{4}x^2\right) dx$$

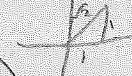
$u = \frac{\pi}{4}x^2$

$du = \frac{\pi}{2} x\, dx$

$\frac{2\,du}{x\pi} = dx$

$0 \le x \le 1$

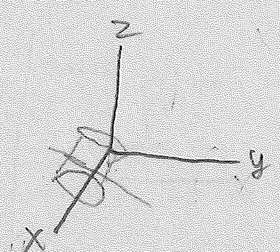
$0 \le y \le 2x$



$$= \frac{4}{\pi}\int_0^{\pi/4} \cos(u)\, du = \frac{4}{\pi}\left(\sin u \Big|_0^{\pi/4}\right) = \frac{4}{\pi}\left(\frac{1}{\sqrt{2}}\right) = \frac{4}{\sqrt{2}\,\pi}$$

2. (15 points) Use a triple integral to find the volume of the solid enclosed by $x = y^2 + z^2$ and $x = 16$.

$x \ge 0$



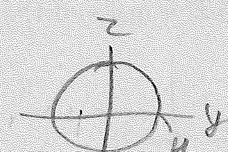
x-simple

$y^2 + z^2 \le x \le 16$

$r^2 \le x \le 16$

whole circle so

$0 \le \theta \le 2\pi$



$y = r\cos\theta$

$z = r\sin\theta$

$x = x$

$0 \le r \le 4$

$$\int_0^{2\pi}\int_0^4\int_{r^2}^{16} r\, dx\, dr\, d\theta = \int_0^{2\pi}\int_0^4 (r^3 - 16r)\, dr\, d\theta = 2\pi\left(\int_0^4 r^3 - 16r\, dr\right)$$

$$= 2\pi\left(\frac{r^4}{4} - 8r^2 \Big|_0^4\right)$$

$$= 2\pi\left(\frac{256}{4} - 8(16)\right)$$

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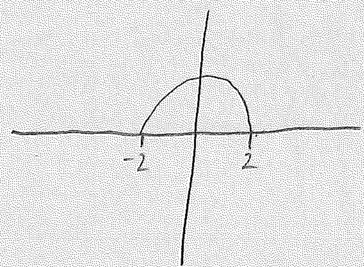
3. (10 points) Evaluate the iterated integral.

$$\int_{-2}^{2}\int_{0}^{\sqrt{4-x^2}} \frac{1}{\sqrt{1+x^2+y^2}}\,dy\,dx$$



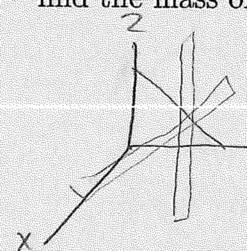
$0 \le \theta \le \pi$

$r = 2$

$$\int_0^{\pi}\int_0^{2} (1+r^2)^{-1/2}\, r\, dr\, d\theta \qquad u = r^2+1, \quad du = 2r\,dr$$

$$\frac{1}{2}\int_0^{\pi}\int_0^{5} u^{-1/2}\, du\, d\theta = \left(\frac{1}{2}\right)(\pi)\left(2\sqrt{u}\,\Big|_0^5\right) = \frac{\pi}{2}\cdot 2\sqrt{5} = \pi\sqrt{5}$$

4. (15 points) Consider the solid in the first octant bounded by the coordinate planes and the planes $z = 4-y$, $x = 1$, and $y = 2$. If the solid has density function $\delta(x,y,z) = 3xy$, find the mass of the solid. $x \ge 0 \quad y \ge 0 \quad z \ge 0$

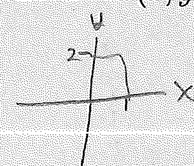


z-simple

$0 \le z \le 4-y$

$0 \le x \le 1$

$0 \le y \le 2$



$$M = \int_0^1\int_0^2\int_0^{4-y} 3xy\, dz\, dy\, dx$$

$$= \int_0^1\int_0^2 3xy(4-y)\, dy\, dx = \int_0^1\int_0^2 12xy - 3xy^2\, dy\, dx = \int_0^1\int_0^2 3x(4y-y^2)\, dy\, dx$$

$$= \int_0^1 3x\, dx \int_0^2 4y - y^2\, dy$$

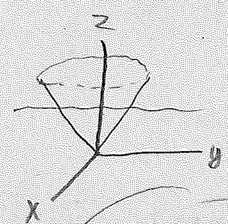
$$= \frac{3}{2}x^2\Big|_0^1 \cdot 2y^2 - \frac{y^3}{3}\Big|_0^2$$

$$= \frac{3}{2}\cdot\left(4 - \frac{8}{3}\right) = \frac{3}{2}\left(\frac{4}{3}\right) = 2$$

/25

5. (25 points) Let $\mathcal{W}$ be the solid bounded by the cone $z = \sqrt{x^2+y^2}$ and the plane $z = 5$. Set up but **DO NOT EVALUATE** a triple integral to find $\iiint_{\mathcal{W}} x^2+y^2+z^2\,dV$ in each of the following coordinate systems.

1. Rectangular coordinates

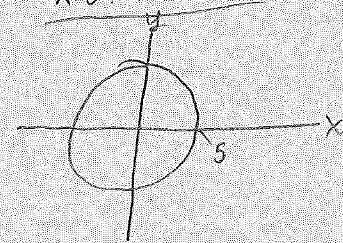


$\sqrt{x^2+y^2} \le z \le 5$

$-5 \le x \le 5$

$-\sqrt{25-x^2} \le y \le \sqrt{25-x^2}$

xy-projection



$$\int_{-5}^{5}\int_{-\sqrt{25-x^2}}^{\sqrt{25-x^2}}\int_{\sqrt{x^2+y^2}}^{5} (x^2+y^2+z^2)\,dz\,dy\,dx$$

2. Cylindrical coordinates

$r \le z \le 5$

$0 \le \theta \le 2\pi$

$0 \le r \le 5$

$$\int_{0}^{2\pi}\int_{0}^{5}\int_{r}^{5} (r^2+z^2)\,r\,dz\,dr\,d\theta$$

3. Spherical coordinates

$0 \le \rho \le 5$

$0 \le \theta \le 2\pi$

$z \ge 0 \quad 0 \le \phi \le \pi$

$\rho\cos\phi = \rho^2\sin^2\phi\cos^2\theta + \rho^2\sin^2\phi\sin^2\theta$

$\rho\cos\phi = \rho^2(\sin^2\phi\cos^2\theta + \sin^2\phi\sin^2\theta)$

$\rho\cos\phi = \rho^2\sin^2\phi\,(1)$

$\frac{\cos\phi}{\sin^2\phi} = \rho$

$\frac{\cot\phi}{\sin\phi} = \rho$

$$\int_{0}^{\pi}\int_{0}^{2\pi}\int_{0}^{\frac{\cot\phi}{\sin\phi}} \rho^2(\rho^2\sin\phi)\,d\rho\,d\theta\,d\phi$$

/25

6. (10 points) Use a double integral to find the area inside one loop of the polar rose $r = 3\sin(4\theta)$.

↑ 8 loops total, so integral from $0 \le \theta \le \pi$ gives 4 loops, so take that integral and divide by 4

$$\text{Area} = \frac{1}{4}\int_0^{\pi}\int_0^{3\sin(4\theta)} r\,dr\,d\theta = \frac{1}{8}\int_0^{\pi} r^2\Big|_0^{3\sin(4\theta)} d\theta = \frac{9}{8}\int_0^{\pi} \sin^2(4\theta)\,d\theta =$$

$$\frac{9}{16}\int_0^{\pi}(1-\cos 8\theta)\,d\theta = \frac{9}{16}\left(\theta - \frac{1}{8}\sin 8\theta\Big|_0^{\pi}\right) = \frac{9}{16}(\pi) = \frac{9\pi}{16}$$

7. (15 points) Use a change of variables to evaluate $\iint_{\mathcal{R}} \sin\left(9x^2+4y^2\right)\,dA$ where $\mathcal{R}$ is the region in the first quadrant of the plane bounded by the ellipse $9x^2+4y^2=1$.

$$J(G^{-1}) = \begin{vmatrix} 3 & 0 \\ 0 & 4 \end{vmatrix} = 12$$

$u = 3x \quad v = 4y$

$u^2 + v^2 \le 1$

$J = 1/J(G^{-1}) = 1/12$

$u = r\cos\theta \quad v = r\sin\theta$

$r^2 = 1$

$r = 1$

$0 \le r \le 1$

whole space so

$0 \le \theta \le 2\pi$

$$\frac{1}{12}\int_0^{2\pi}\int_0^1 \sin(r^2)\,r\,dr\,d\theta = \left(\frac{1}{12}\right)(2\pi)\left(\int_0^1 \sin(r^2)\,r\,dr\,d\theta\right)$$

$u = r^2$
$du = 2r\,dr$

$$= \left(\frac{\pi}{12}\right)\left(\int_0^1 \sin(u)\,du\right) = \left(\frac{\pi}{12}\right)\left(-\cos u\Big|_0^1\right)$$

$$= \left(\frac{\pi}{12}\right)\left(-\cos(1)+1\right)$$

$$= \frac{\pi}{12}\left(1-\cos(1)\right)$$

/25

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You may use this page for scratch work. Work found on this page will not be graded unless clearly indicated in the exam.