Math 32B-1 Exam 1 - Yellow

KADE BIERMANN ADAMS

TOTAL POINTS

88.5 / 100

QUESTION 1

1 Question 1 pg 1 10 / 10

✓ + **10 pts** Completely Correct

+ **0 pts** Completely incorrect, or incomplete.

+ **2 pts** Full Credit Criterion 1: Student draws a correct picture of the domain of integration, or otherwise demonstrates they understand what the region looks like.

+ **5 pts** Full Credit Criterion 2: Student correctly changes the order of integration. In particular, the new bounds are correct.

+ **1 pts** Full Credit Criterion 3: Student correctly integrates double integral.

+ **2 pts** Full Credit Criterion 4: Correct final answer is given.

+ **2 pts** Potential Partial Credit 1: When changing the order of integration, at least one of the two bounds was correct.

QUESTION 2

2 Question 2 pg1 14 / 15

+ **15 pts** Completely Correct

+ **0 pts** Completely Incorrect or Incomplete

✓ + **2 pts** Full Credit Criterion 1: Student draws a correct and legible picture of the region or otherwise demonstrates they understand the region is a paraboloid along the y-axis

✓ + **5 pts** Full Credit Criterion 2: Student sets up a triple integral to calculate volume with the correct bounds of integration.

✓ + **5 pts** Full Credit Criterion 3: Student successfully changes the triple integral to cylindrical coordinates. In particular, bounds are correct and the appropriate Jacobian is included.

✓ + **2 pts** Full Credit Criterion 4: Student successfully goes through calculation of the triple integral in cylindrical coordinates.

+ **1 pts** Full Credit Criterion 5: Final numerical answer of 128pi is given.

+ **2 pts** Potential Partial Credit 1: The student knew what the region looked like but incorrect bounds were given in rectangular coordinates. If the integral past this point was calculated correctly and a numerical answer was obtained you receive 2 partial credit points.

+ **1 pts** Potential Partial Credit 2: None of the above full credit criterion were met, but the student at least demonstrated they know the volume is given by integrating a triple integral with integrand 1. However, if work is egregiously wrong, student will just receive a 0.

QUESTION 3

3 Question 3 pg 2 9 / 10

✓ + **2 pts** Justified bounds for polar change of coordinates (accurate picture suffices), cannot justify incorrect assertions

✓ + **2 pts** Correct bounds for integral in polar (no credit if bounds are correct for incorrect region)

✓ + **3 pts** Correct integrand in polar

+ **3 pts** Correct computation of integral (not awarded if erroneous computation led to correct final answer)

✓ + **2 pts** Computation of integral with arithmetic error (e.g. forgetting a factor of 2, but not awarded for getting incorrect antiderivative or improperly u-substituting)

+ **0 pts** No credit due

QUESTION 4

4 Question 4 pg 2 15 / 15

✓ + **2 pts** Justified bounds of integrals (with picture, say), cannot justify incorrect assertions

✓ + **4 pts** Correct innermost integral bounds

✓ + **3 pts** Other two integral bounds

✓ + **5 pts** Full evaluation of iterated integral

✓ + **1 pts** Correct final answer (no point for answer to invalid integral or for arithmetic mistake)

+ **0 pts** No credit due

+ **2 pts** One mistake in innermost bounds

QUESTION 5

Question 5 pg 3 25 pts

5.1 Rectangular 3 / 7

✓ + **1 pts** Correct lower bound for x integral

✓ + **1 pts** Correct upper bound for x integral

+ **1 pts** Correct lower bound for y integral

+ **1 pts** Correct upper bound for y integral

+ **1 pts** Correct lower bound for z integral

+ **1 pts** Correct upper bound for z integral

✓ + **1 pts** Correct integrand and order of integration

+ **0 pts** Blank or completely incorrect

5.2 Cylindrical 9 / 9

✓ + **1 pts** Correct lower bound for theta

✓ + **1 pts** Correct upper bound for theta

✓ + **1 pts** Correct lower bound for r

✓ + **1 pts** Correct upper bound for r

✓ + **1 pts** Correct lower bound for z

✓ + **1 pts** Correct upper bound for z

✓ + **1 pts** Correct integrand substitution

✓ + **1 pts** Jacobian

✓ + **1 pts** Correct order of integration

+ **0 pts** Blank or incorrect

- **1 pts** Missing parentheses in integrand.

5.3 Spherical 8 / 9

✓ + **1 pts** Correct lower bound for phi

✓ + **1 pts** Correct upper bound for phi

✓ + **1 pts** Correct lower bound for theta

✓ + **1 pts** Correct upper bound for theta

✓ + **1 pts** Correct lower bound for rho

+ **1 pts** Correct upper bound for rho

✓ + **1 pts** Correct integrand substitution

✓ + **1 pts** Jacobian

✓ + **1 pts** Correct order of integration

+ **0 pts** Blank or incorrect

QUESTION 6

6 Question 6 pg 4 10 / 10

✓ + **2.5 pts** Correct theta bounds

✓ + **1 pts** Correct r bounds

✓ + **1.5 pts** Included Jacobian

+ **0.5 pts** Correct integral evaluation until double angle formula needed; no further evaluation / incorrect evaluation afterward

✓ + **2 pts** Used double angle formula correctly

+ **1 pts** Some errors in using double angle formula

✓ + **2 pts** Correct integral evaluation (besides double angle formula)

+ **1 pts** Some error(s) in integral evaluation (besides double angle formula)

✓ + **1 pts** Correct final answer (with correct work)

+ **0 pts** Blank answer / completely incorrect

You have to be careful -- since some of the loops are generated by having a *negative* radius, if you were integrating some other function, you might need to break up the integral into an integral over each loop and flip some of their signs. Since the integrand is just 1 here, it's actually okay, but it's worth being aware of.

QUESTION 7

7 Question 7 pg 4 10.5 / 15

✓ + **3.5 pts** Correctly identify the transformation under which the domain becomes a quarter-disk OR which makes the argument of sin equal to r^2

+ **3 pts** Identify the new domain correctly

+ **1.5 pts** Identify the new domain as the entire unit disk (OR region inside an ellipse if transforming to make the integrand simple) rather than the quarter-disk in the 1st quadrant

✓ + **5.5 pts** **Correctly compute the Jacobian**

+ **3 pts** Correctly compute the inverse of the Jacobian, but mistake it for the Jacobian itself

+ **3 pts** Correctly wrote out integral using the change-of-variables rule

✓ + **1.5 pts** **Some errors in the application of the change-of-variables formula**

+ **1 pts** Bonus point for significant amount of effort spent trying to evaluate an (at least mostly correct) incomputable integral. Note: if an error made the integral computable and no harder than what was meant to be, then you cannot get this point.

+ **0 pts** Blank answer / completely incorrect

Math 32B - Spring 2019

Exam 1

Full Name: Kade Adams

UID: 505123178

Circle the name of your TA and the day of your discussion:

Patrick Hiatt   Eli Sadovnik   Frederick Vu

Tuesday   Thursday

Instructions:

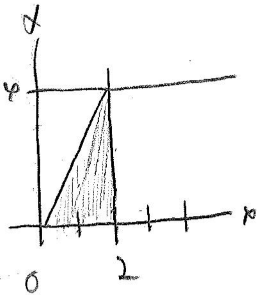
- Read each problem carefully.
- Show all work clearly and circle or box your final answer where appropriate.
- Justify your answers. A correct final answer without valid reasoning will not receive credit.
- Simplify your answers as much as possible.
- Include units with your answer where applicable.
- Calculators are not allowed but you may have a 3 × 5 inch notecard.

Page	Points	Score
1	25	
2	25	
3	25	
4	25	
Total:	100	

$y=0 \quad y=4$ $\quad x=y/2 \rightarrow 2x=y$

$x=2$

1. (10 points) Evaluate the iterated integral



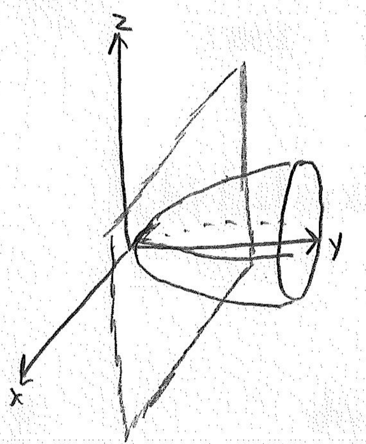
$$\int_0^4 \int_{y/2}^2 \cos\left(\frac{\pi}{16}x^2\right)\, dx\, dy.$$

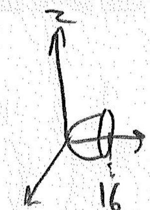
$$\int_0^2 \int_0^{2x} \cos\left(\tfrac{\pi}{16}x^2\right) dy\, dx = \int_0^2 y\cos\left(\tfrac{\pi}{16}x^2\right)\Big|_0^{2x} dx$$

$$= \int_0^2 2x\cos\left(\tfrac{\pi}{16}x^2\right) dx = \tfrac{16}{\pi}\sin\left(\tfrac{\pi}{16}x^2\right)\Big|_0^2 = \tfrac{16}{\pi}\sin\left(\tfrac{\pi}{4}\right) - \tfrac{16}{\pi}\sin(0)$$

$$= \frac{16}{\pi}\,\frac{\sqrt{2}}{2} = \boxed{\frac{8\sqrt{2}}{\pi}}$$

2. (15 points) Use a triple integral to find the volume of the solid enclosed by $y = x^2 + z^2$ and $y = 16$.





$x^2+z^2=r^2$

$y=y$

$$\int_0^{2\pi}\int_0^{16}\int_0^{\sqrt{y}} r\, dr\, dy\, d\theta = \int_0^{2\pi}\int_0^{16} \tfrac{1}{2}r^2\Big|_0^{\sqrt{y}} dy\, d\theta$$

$$\int_0^{2\pi}\int_0^{16} \tfrac{1}{2}y\, dy\, d\phi = \int_0^{2\pi} \tfrac{1}{4}y^2\Big|_0^{16} d\theta = \int_0^{2\pi} 32\, d\theta = 32\theta\Big|_0^{2\pi} = \boxed{64\pi}$$

$\frac{16}{2}\cdot\frac{16}{2} = \frac{8}{2}\cdot\frac{8}{2} = \frac{64}{2} = 32$

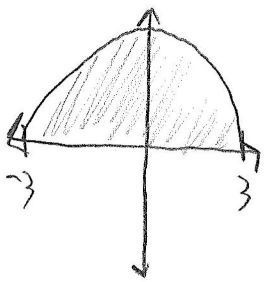
/25

$x=3 \quad x=-3 \quad y=\sqrt{9-x^2} \rightarrow y^2=9-x^2$

$y=0 \qquad x^2+y^2=9$

3. (10 points) Evaluate the iterated integral.

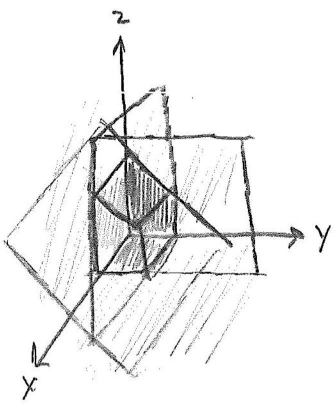
$$\int_{-3}^{3}\int_{0}^{\sqrt{9-x^2}} \frac{1}{\sqrt{1+x^2+y^2}}\,dy\,dx$$

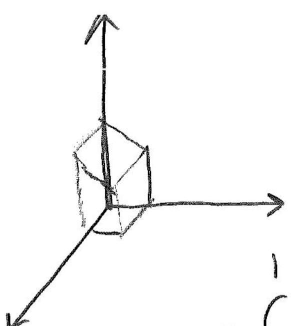


$$\int_0^{\pi}\int_0^{3} \frac{1}{\sqrt{1+r^2}}\, r\,dr\,d\theta \rightarrow \int_0^{\pi}\int_0^{3} r(1+r^2)^{-1/2}\,dr\,d\theta$$

$$= \int_0^{\pi} \frac{(1+r^2)^{1/2}}{2}\Big|_0^3\,d\theta = \int_0^{\pi} \frac{\sqrt{10}}{2}-\frac{1}{2}\,d\theta = \frac{\sqrt{10}}{2}\theta-\frac{1}{2}\theta\Big|_0^{\pi} = \frac{\pi\sqrt{10}}{2}-\frac{\pi}{2} = \boxed{\frac{\pi(\sqrt{10}-1)}{2}}$$

4. (15 points) Consider the solid in the first octant bounded by the coordinate planes and the planes $z = 4-y$, $x = 1$, and $y = 2$. If the solid has density function $\delta(x, y, z) = 9xy$, find the mass of the solid. $4+2=6$





$$\int_0^1\int_0^2\int_0^{4-y} 9xy\,dz\,dy\,dx$$

$$= \int_0^1\int_0^2 9xyz\Big|_0^{4-y}\,dy\,dx$$

$$= \int_0^1\int_0^2 36xy-9xy^2\,dy\,dx = \int_0^1 18xy^2-3xy^3\Big|_0^2\,dx = \int_0^1 72x-24x\,dx$$

$$= \int_0^1 48x\,dx = 24x^2\Big|_0^1 = \boxed{24}$$

/25

$z^2 = x^2 + y^2 \quad y^2 = z^2 - x^2$

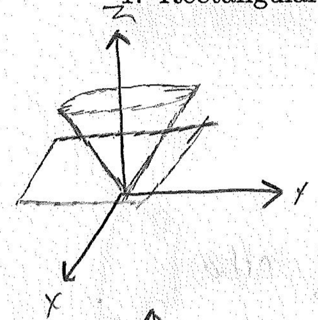
$y = \pm\sqrt{z^2 - x^2}$

5. (25 points) Let $\mathcal{W}$ be the solid bounded by the cone $z = \sqrt{x^2 + y^2}$ and the plane $z = 3$. Set up but **DO NOT EVALUATE** a triple integral to find $\iiint_{\mathcal{W}} x^2 + y^2 + z^2 \, dV$ in each of the following coordinate systems.

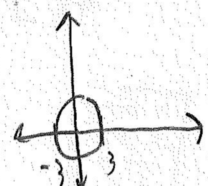
1. Rectangular coordinates

$0 \le z \le 3$

$-3 \le x \le 3$



$$\int_{-3}^{3} \int_{0}^{3} \int_{-\sqrt{z^2 - x^2}}^{\sqrt{z^2 - x^2}} x^2 + y^2 + z^2 \, dy \, dz \, dx$$



2. Cylindrical coordinates

$z = r$

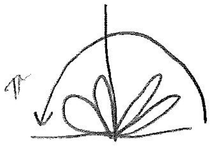
$$\int_{0}^{2\pi} \int_{0}^{3} \int_{0}^{z} r(r^2 + z^2) \, dr \, dz \, d\theta$$

3. Spherical coordinates

$z^2 = x^2 + y^2 \quad z = 3 \quad \rho\cos\phi = 3 \quad \rho = \frac{\cos\phi}{3}$

$$\int_{0}^{2\pi} \int_{0}^{\pi/4} \int_{0}^{\frac{\cos\phi}{3}} \rho^2 (\rho^2 \sin\phi) \, d\rho \, d\phi \, d\theta$$

/25



$$\sin^2(4\theta) = \frac{1-\cos(8\theta)}{2}$$

6. (10 points) Use a double integral to find the area inside one loop of the polar rose $r = 5\sin(4\theta)$.

$$\text{Area} = \frac{1}{4}\int_0^{\pi}\int_0^{5\sin(4\theta)} r\,dr\,d\theta = \frac{1}{4}\int_0^{\pi} \frac{1}{2}r^2\Big|_0^{5\sin 4\theta} d\theta = \frac{1}{4}\int_0^{\pi} \frac{25}{2}\sin^2(4\theta)\,d\theta$$

$$= \frac{25}{8}\int_0^{\pi}\sin^2(4\theta)\,d\theta = \frac{25}{16}\int_0^{\pi} 1-\cos(8\theta)\,d\theta = \frac{25}{16}\left[\theta - \frac{1}{8}\sin(8\theta)\Big|_0^{\pi}\right]$$

$$= \frac{25}{16}(\pi) = \boxed{\frac{25\pi}{16}}$$

7. (15 points) Use a change of variables to evaluate $\iint_{\mathcal{R}} \sin\left(25x^2 + 4y^2\right)\, dA$ where $\mathcal{R}$ is the region in the first quadrant of the plane bounded by the ellipse $9x^2 + 4y^2 = 1$.

$$u = 5x \quad v = 2y \quad \rightarrow \iint_R \sin(u^2+v^2)\,dA \rightarrow \frac{1}{10}\iint_{u^2+v^2=1} \sin(u^2+v^2)\,du\,dv$$

$$\text{Jac}(G^{-1}) = \begin{vmatrix} 5 & 0 \\ 0 & 2 \end{vmatrix} = 10$$

$$u^2+v^2 = r^2$$

$$r^2 = 1$$

$$\frac{1}{10}$$

$$\frac{1}{10}\int_0^{2\pi}\int_0^{1} \sin(r^2)\,r\,dr\,d\theta = \frac{1}{10}\int_0^{2\pi} -\frac{1}{2}\cos(r^2)\Big|_0^1 d\theta$$

$$= \frac{1}{10}\int_0^{2\pi} -\frac{1}{2}\cos(1) + \frac{1}{2}\,d\theta = \frac{1}{10}\left(-\frac{\theta}{2}\cos(1) + \frac{\theta}{2}\Big|_0^{2\pi}\right) = \frac{1}{10}\left(-\pi\cos(1) + \pi\right) = \boxed{\frac{\pi(1-\cos(1))}{10}}$$

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You may use this page for scratch work. Work found on this page will not be graded unless clearly indicated in the exam.