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Section	3	C

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- Fill out your name, section letter, and UID above.
- Do not open this exam packet until you are told that you may begin.
- Turn off all electronic devices and and put away all items except for a pen/pencil and an eraser.
- No phones, calculators, smart-watches or electronic devices of any kind allowed for any reason, including checking the time.
- If you have a question, raise your hand and one of the proctors will come to you. We will not answer any mathematical questions except possibly to clarify the wording of a problem.
- Quit working and close this packet when you are told to stop.

Spherical coordinates:

$$x = \rho \sin\phi \cos\theta$$
$$y = \rho \sin\phi \sin\theta$$
$$z = \rho \cos\phi$$

$$dxdydz = \rho^2 \sin\phi \, d\rho d\phi d\theta$$

Page:	1	2	3	4	5	Total
Points:	8	10	10	10	12	50
Score:						

You may use this page for scratch work.

max $0 \to 1$

1. (8 points) (a) True or False? (circle one) $\displaystyle\int_1^4 \int_0^1 \sqrt{y}\,\sin(x^2y^2)\,dx\,dy \le 6$

$0 \le x \le \sqrt{4-y^2}$

$0 \le y \le \sqrt{4-y^2}$

1×3

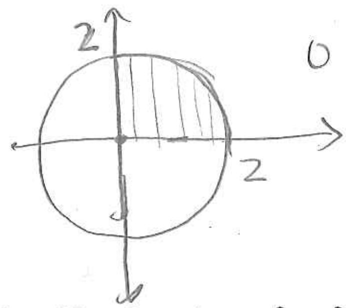
True False

area of D

$= 3\times 1$

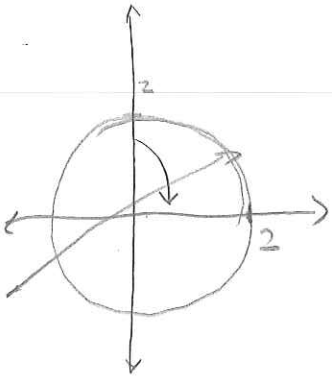
Top : $\sqrt{4} = 2$

2×3



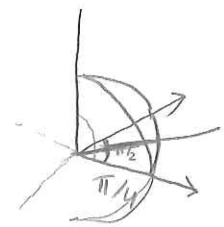
(b) Let D be the region in the positive octant $(x, y, z \ge 0)$ enclosed by the sphere $x^2+y^2+z^2 = 4$ and the planes $z = 0$, $x = 0$, and $x = y$. For **each** integral below, circle "yes" or "no" depending on whether or not it equals $\displaystyle\iiint_D x\,dV$.

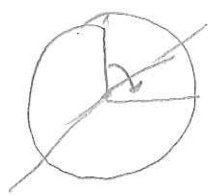
$x = \rho\cos\theta\sin\phi$ $\quad \rho^2\sin\phi$ Jacobian



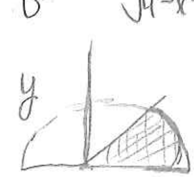
yes	no	$\displaystyle\int_0^{\pi/2}\int_0^{\pi/4}\int_0^{2} \rho^3\cos\theta\sin^2\phi\,d\rho\,d\theta\,d\phi$
yes	no	$\displaystyle\int_0^{\sqrt{2}}\int_0^{\sqrt{4-x^2}}\int_0^{\sqrt{4-x^2-y^2}} x\,dz\,dy\,dx$
yes	no	$\displaystyle\int_0^{\pi/2}\int_{\pi/4}^{\pi/2}\int_0^{2} \rho^2\cos\theta\sin\phi\,d\rho\,d\theta\,d\phi$
yes	no	$\displaystyle\int_0^{\pi/2}\int_{\pi/4}^{\pi/2}\int_0^{2} \rho^2\cos\theta\sin\phi\,d\rho\,d\phi\,d\theta$
yes	no	$\displaystyle\int_0^{2}\int_{\pi/4}^{\pi/2}\int_0^{\sqrt{4-r^2}} r^2\cos\theta\,dz\,d\theta\,dr$
yes	no	$\displaystyle\int_0^{\pi/2}\int_0^{2}\int_{\pi/4}^{\pi/2} \rho^3\cos\theta\sin^2\phi\,d\theta\,d\rho\,d\phi$

y z





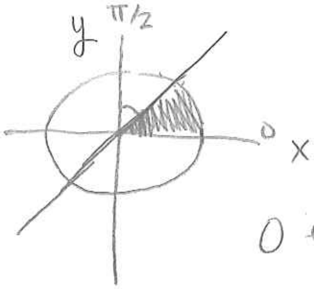
$\sqrt{4-x^2-y^2}$



$r\cos\theta = x$

$y \le x \le \sqrt{4-y^2}$

$0 \le y \le \sqrt{2}$



0 to $\frac{\pi}{2}$???

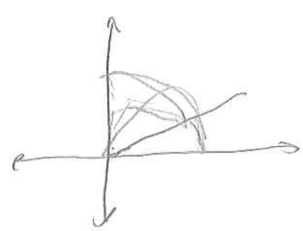
i'm so confused

$0 \le y \le x$

$0 \le x \le 2$

$0 \le \phi \le \frac{\pi}{2}$

$0 \le \theta \le \frac{\pi}{4}$



2. (10 points) Let R be the region in $\mathbb{R}^2$ which lies above the x-axis and between the circles of radius 1 and 2 centered at $(0,0)$.

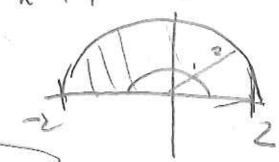
(a) Write the following integral as a sum of integrals in **rectangular coordinates**:

$$\iint_R 3y\,dA.$$

$x^2+y^2=4$

$x^2+y^2=1$

Do not evaluate these integrals. $\boxed{\text{Box your answers.}}$



$-2 \le x \le +2$

$\sqrt{1-x^2} \le y \le \sqrt{4-x^2}$

$$\boxed{\int_{-2}^{2}\int_{\sqrt{1-x^2}}^{\sqrt{4-x^2}} 3y\,dy\,dx}$$

(b) Evaluate the integral in part (a) using **polar coordinates**. $\boxed{\text{Box your answer.}}$

polar coords

integrate $\sin\theta \rightarrow -\cos\theta$

$y = r\sin\theta$

$1 \le r \le 2$

$0 \le \theta \le \pi$

$$\int_0^{\pi}\int_1^{2} 3r\sin\theta\, r\,dr\,d\theta$$

$$\left(\left(-\cos\theta\right)\Big|_0^{\pi}\right)\left(\frac{r^3}{3}\Big|_1^{2}\right)$$

$$\left[-(-1)-(-1)\right]\left(\frac{2^3}{3}-\frac{1}{3}\right)$$

$$\int_0^{\pi}\sin\theta\,d\theta\int_1^{2} r^2\,dr$$

$2\left(\frac{7}{3}\right)$

$\boxed{\cancel{44/3}}$

$\boxed{14/}$

$X = U + V + UV \qquad y = -U + 2V + 2UV$

3. (10 points) Let $G : \mathbb{R}^2 \to \mathbb{R}^2$ be the non-linear transformation

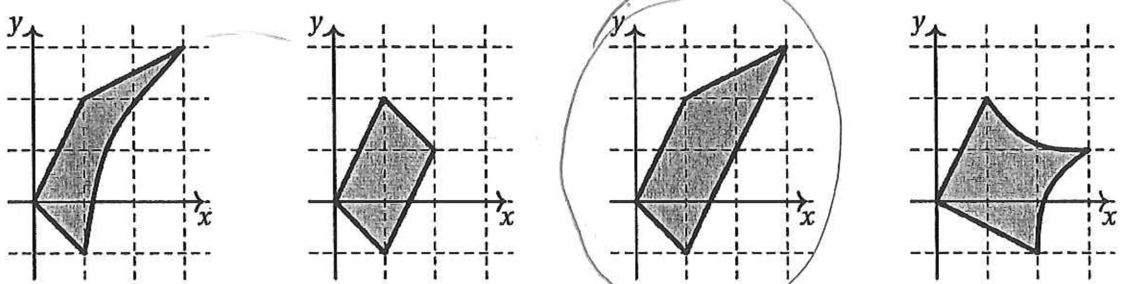
$1+1+1=3 \quad -1+2+2=3$

$$G(u,v) = (u + v + uv, -u + 2v + 2uv)\,.$$

$(0,1)$

Let R be the unit square $[0,1] \times [0,1]$ in the uv-plane and let $D = G(R)$ in the xy-plane.

(a) Circle the picture of D below. The dashed grid consists of unit squares.



$(0,1)\ (1,1)$ (b) Find the limits and integrand of the integral below so that it equals

$(0,0)\ (1,0)$

$$\iint_D \sqrt{x}\, dA$$

$G(0,0) = (0,0)$ as an integral over the square R. Do not evaluate the integral. Show your work.

$G(0,1) =$

$(1,2)$

$(1,0) =$

$(1,-1)$

$(1,1) =$

$(3,3)$

$\int_0^1 \int_0^1$

$0 \le u \le 1$

$0 \le v \le 1$

$\sqrt{x} = \sqrt{u+v+uv}$

Jacobian: $\begin{pmatrix} \nabla x \\ \nabla y \end{pmatrix} \to \begin{array}{c|cc|} & u & v \\ x & 1+v & 1+u \\ y & -1+2v & 2+2u \end{array}$

$(1+v)(2+2u) - (-1+2v)(1+u)$

$2 + 2vu + 2v + 2u - [-1 + 2vu + 2v - u]$

$$\boxed{\iint_D \sqrt{x}\, dA = \int_0^1 \int_0^1 \left(\sqrt{u+v+uv}\right)(3+3u)\, du\, dv}$$

simplified (hope I simplified correctly)

$2 + 1 + \cancel{2vu} - \cancel{2vu} + \cancel{2v} - \cancel{2v} + 2u + u$

$3 + 3u$

Or

$(1+v)(2+2u) - (-1+2v)(1+u)$

4. (10 points) (a) In **spherical coordinates**, describe the region outside the cone $x^2 + y^2 = z^2$ and inside the sphere $x^2 + y^2 + z^2 = 2$ (shown below – the sphere is translucent so you can see the cone inside).

$\rho^2 = 2$

$\rho = \sqrt{2}$

$z = \cos\phi\,\rho$

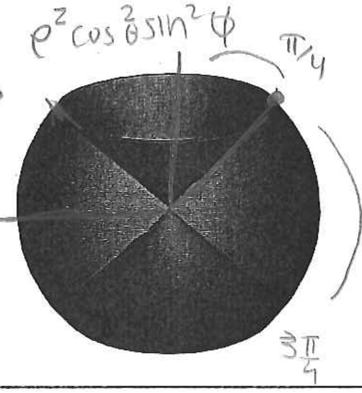
$\cos(0)$

$\rho^2\cos^2\phi = \rho^2\sin^2\theta\sin^2\phi + \rho^2\cos^2\theta\sin^2\phi$

$\rho^2\cos^2\phi = \rho^2\sin^2\phi$

$\phi = \frac{\pi}{4}$

?



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what is this plane?

$x+y = 1$ already??

$0 \le \theta \le 2\pi$

$\frac{\pi}{4} \le \phi \le \frac{3\pi}{4}$

$0 \le \rho \le \sqrt{2}$

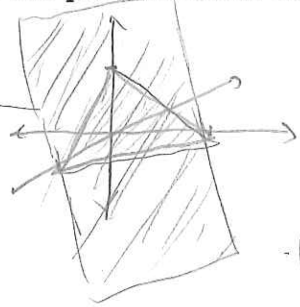
$z = 1 - x + y$

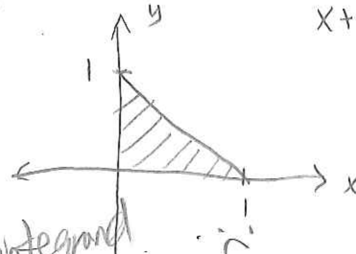
(b) Fill in the limits and integrand of the double and triple integrals below so that they both equal the volume of the region in the first octant ($x, y, z \ge 0$) below the plane $x + y + z = 1$. Be sure to follow the provided order of integration.

A & b are diff questions

$x+y=1$

$x = 1 - y$



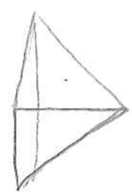


$0 \leftrightarrow 1$

~~x+~~ ?

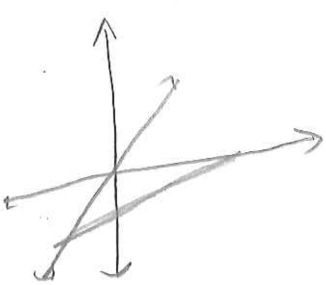
0

how to fill integrand what



$$\text{Vol} = \int_0^1 \int_0^{1-y} 1 - (x+y) \; dx\, dy$$

x+y can't be more than one?



$0 \le y \le 1 - z - x$

$0 \le x \le 1 - z$

$0 \le z \le 1$

$$\text{Vol} = \int_0^1 \int_0^{1-z} \int_0^{1-x-z} 1 \; dy\, dx\, dz$$

$0 + x + y = 1$

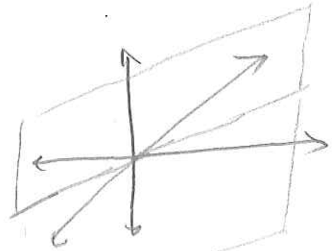
y x

5. (12 points) Multiple choice. Circle the correct answer.

(a) In spherical coordinates the plane $y = x$ can be written as

$$\rho = \frac{1}{\cos\phi} \qquad \phi = \frac{\pi}{3} \qquad \rho = 1 \qquad \theta = \frac{\pi}{4} \qquad \rho = \frac{1}{\sin\phi}.$$

$y = x$



$\rho \sin\phi \cos\theta = \rho \sin\phi \sin\theta$

(b) The Jacobian of the map $G(u, v) = (u^2 - v^2, uv)$ is

$$2u^2 + 2v^2 \qquad 2u^2 - 2v^2 \qquad 4uv \qquad 2u + 2v \qquad -4uv$$

$$\begin{array}{c|cc|} & u & v \\ \hline x & 2u & -2v \\ y & v & u \end{array}$$

$2uv - (-2uv)$

$= 4uv$

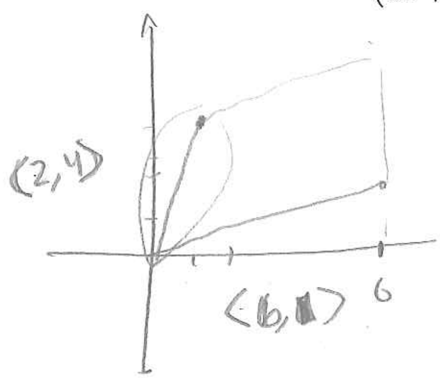
(c) In cylindrical coordinates the plane $x = 1$ can be written as

$$r = \frac{1}{\cos\theta} \qquad \theta = \frac{\pi}{3} \qquad r = 1 \qquad \theta = \frac{\pi}{4} \qquad r = \frac{1}{\sin\theta}$$

$x = 1$? $r\cos\theta = 1$

(d) The linear map which sends the unit square $[0, 1] \times [0, 1]$ to the parallelogram with vertices $(0, 0)$, $(6, 1)$, $(8, 5)$, and $(2, 4)$ is $G(u, v) =$

$$(6u + v, 2u + 4v) \qquad (6u + 2v, u + 4v) \qquad (6u + v, 4u + 2v)$$
$$(6u + 2v, 4u + v) \qquad (6u + 4v, u + 2v)$$



$(0,1) \rightarrow \langle 2u + 6v, 4u + v \rangle \quad \langle 6, 1 \rangle$

$(1,0)$ OR $\langle 6u + 2v, v + 4u \rangle$