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Section	3	B

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- Fill out your name, section letter, and UID above.
- Do not open this exam packet until you are told that you may begin.
- Turn off all electronic devices and and put away all items except for a pen/pencil and an eraser.
- No phones, calculators, smart-watches or electronic devices of any kind allowed for any reason, including checking the time.
- If you have a question, raise your hand and one of the proctors will come to you. We will not answer any mathematical questions except possibly to clarify the wording of a problem.
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Spherical coordinates:

$$x = \rho \sin\phi \cos\theta$$
$$y = \rho \sin\phi \sin\theta$$
$$z = \rho \cos\phi$$

$$dxdydz = \rho^2 \sin\phi \, d\rho d\phi d\theta$$

Page:	1	2	3	4	5	Total
Points:	8	10	10	10	12	50
Score:						

You may use this page for scratch work.

1.

1 2

1. (8 points) (a) True or False? (circle one) $\int_1^4 \int_0^1 \sqrt{y}\,\sin(x^2y^2)\,dx\,dy \le 6$

True   False

$\boxed{4}$

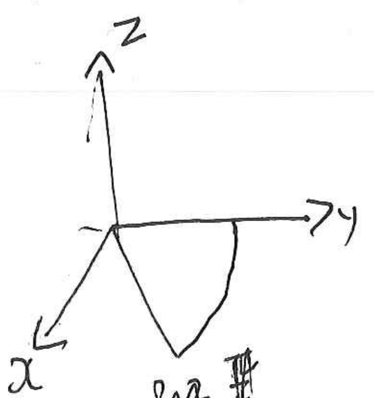
4

$z = \sqrt{y}\,\sin(x^2y^2)$

(b) Let D be the region in the positive octant $(x, y, z \ge 0)$ enclosed by the sphere $x^2+y^2+z^2 = 4$ and the planes $z = 0$, $x = 0$, and $x = y$. For **each** integral below, circle "yes" or "no" depending on whether or not it equals $\iiint_D x\,dV$.

$x = \rho \sin\phi \cos\theta$

ρ^3



$\sqrt{2}$ $\sqrt{2}$

$\frac{\pi}{4} \le \theta \le \frac{\pi}{2}$

$0 \le \phi \le \frac{\pi}{2}$

yes no	$\int_0^{\pi/2}\int_0^{\pi/4}\int_0^2 \rho^3 \cos\theta \sin^2\phi \,d\rho\,d\theta\,d\phi$
yes no (No)	$\int_0^{\sqrt{2}}\int_0^{\sqrt{4-x^2}}\int_0^{\sqrt{4-x^2-y^2}} x\,dz\,dy\,dx$
yes no	$\int_0^{\pi/2}\int_{\pi/4}^{\pi/2}\int_0^2 \rho^2 \cos\theta \sin\phi \,d\rho\,d\theta\,d\phi$
yes no	$\int_0^{\pi/2}\int_{\pi/4}^{\pi/2}\int_0^2 \rho^2 \cos\theta \sin\phi \,d\rho\,d\phi\,d\theta$
yes no	$\int_0^{2}\int_{\pi/4}^{\pi/2}\int_0^{\sqrt{4-r^2}} r^2 \cos\theta \,dz\,d\theta\,dr$
yes no	$\int_0^{\pi/2}\int_0^{2}\int_{\pi/4}^{\pi/2} \rho^3 \cos\theta \sin^2\phi \,d\theta\,d\rho\,d\phi$

2. (10 points) Let R be the region in $\mathbb{R}^2$ which lies above the x-axis and between the circles of radius 1 and 2 centered at $(0,0)$.

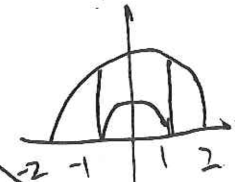
(a) Write the following integral as a sum of integrals in **rectangular coordinates:**

$$\iint_R 3y\,dA.$$

Do not evaluate these integrals. Box your answers.

$x^2+y^2=4$

$x^2+y^2=1$



$$\boxed{\int_{-2}^{-1}\int_{0}^{\sqrt{4-x^2}} 3y\,dy\,dx + \int_{-1}^{1}\int_{\sqrt{1-x^2}}^{\sqrt{4-x^2}} 3y\,dy\,dx + \int_{1}^{2}\int_{0}^{\sqrt{4-x^2}} 3y\,dy\,dx}$$

(b) Evaluate the integral in part (a) using **polar coordinates.** Box your answer.

$-7\cos\pi$

7

$$\int_0^{\pi}\int_1^{2} \cancel{3\sin}\ 3r^2\sin\theta\,dr\,d\theta$$

$$= \int_0^{\pi}\left[r^3\sin\theta\right]_1^2 d\theta$$

$$= \int_0^{\pi} 7\sin\theta\,d\theta$$

$$= \left[-7\cos\theta\right]_0^{\pi}$$

$$= 7+7$$

$$= \boxed{14}$$

$1+\frac{1}{2}+\frac{1}{2}, -1+1+1$ $\quad (1, 0.5)$ $\quad (1, 2) \quad (1,-1) \quad (0,0)$

$2, 1$

3. (10 points) Let $G : \mathbb{R}^2 \to \mathbb{R}^2$ be the non-linear transformation

$1+\frac{1}{2}+\frac{1}{2}, -1+1, +1$ $\quad (2,1)$

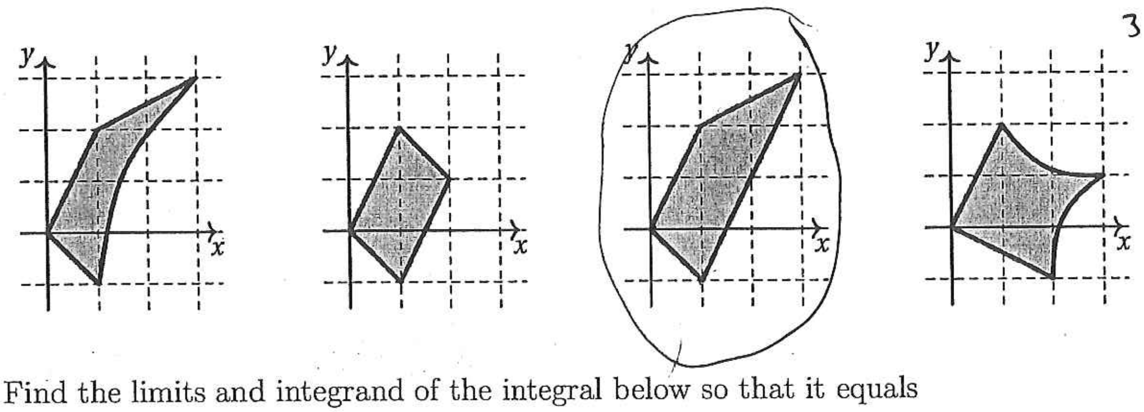
$$G(u, v) = (u + v + uv, -u + 2v + 2uv).$$

$(0,0) \quad (3,3) \quad (1,1)$

Let R be the unit square $[0, 1] \times [0, 1]$ in the uv-plane and let $D = G(R)$ in the xy-plane.

(a) Circle the picture of D below. The dashed grid consists of unit squares. $\quad 1+1+1, -1+2+2$

$3 \quad 3$



(b) Find the limits and integrand of the integral below so that it equals

GOAL:

$$\iint_D \sqrt{x}\, dA$$

as an integral over the square R. Do not evaluate the integral. Show your work.

$y - 2x = -3u$

$y = -3u + 2x$

$u = \frac{2}{3}x - \frac{y}{3}$

$$\mathrm{Jac}(G) = \begin{vmatrix} 1+v & 1+u \\ -1+2v & 2+2u \end{vmatrix}$$

det

$= (1+v)(2+2u) - (1+u)(-1+2v)$

$= 2 + 2u\cancel{v} + 2v + 2u + 1 + u - 2u\cancel{v} - 2v$

$= 3u + 3$

$\sqrt{x} = \sqrt{u+v+uv}$

$$\boxed{\iint_D \sqrt{x}\, dA = \int_0^1 \int_0^1 (3u+3)\sqrt{u+v+uv} \quad du\, dv}$$

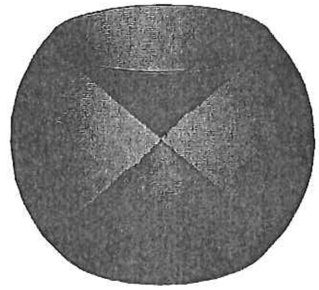
4. (10 points) (a) In **spherical coordinates**, describe the region outside the cone $x^2 + y^2 = z^2$ and inside the sphere $x^2 + y^2 + z^2 = 2$ (shown below – the sphere is translucent so you can see the cone inside).

$z = y$ $\quad z^2 + z^2 = 2$

$z^2 = 1$

$z = 1$

$(1,1)$ $\quad \frac{\pi}{4}$



$$0 \le \theta \le 2\pi$$
$$\frac{\pi}{4} \le \phi \le \frac{3\pi}{4}$$
$$0 \le \rho \le \sqrt{2}$$

(b) Fill in the limits and integrand of the double and triple integrals below so that they both equal the volume of the region in the first octant $(x, y, z \ge 0)$ below the plane $x + y + z = 1$. Be sure to follow the provided order of integration.

$1-x-z$ $\quad \left[x - \frac{1}{2}x^2 - zx\right]$

$\frac{1}{2}z^2 - \frac{1}{2} + 1 - z$

$\int_0^1 \int_0^{1-y} 1-x-y$ $\quad \left[x - \frac{1}{2}x^2 - yx\right]_0^{1-y}$ $\quad 1 - y - \frac{1}{2}(1 - 2y + y^2)$ $\quad \frac{1}{2}z^2 + \frac{1}{2} - z$

$-y + y^2$ $\quad \frac{1}{6}z^3 + \frac{1}{2}z - \frac{1}{2}z^2$

$1 - y - \frac{1}{2} + y - \frac{1}{2}y^2 + y^2 - y$

$\frac{1}{2}y^2 - y + \frac{1}{2}$ $\quad \frac{1}{6}y^3 - \frac{1}{2}y^2 + \frac{1}{2}y$

$$\boxed{\text{Vol} = \int_0^1 \int_0^{1-y} 1-x-y \, dx\, dy}$$

$\left[\frac{1}{6}z^3 - \frac{1}{2}z\right]_0^1$

$-x-z$

$\left[-\frac{1}{2}x^2 - zx\right]^{1-z}$

$(1-z)^2 = 1 + z^2 - 2z$

$-\frac{1}{2} - \frac{1}{2}z^2 + z - z + z^2 = \frac{1}{2}z^2 - \frac{1}{2}$

$-x-z$

$-\frac{1}{2}x^2 - zx$

$-\frac{1}{2}(1 + z^2 - 2z) - z + 2z$

$-2.5 - \frac{1}{2}z^2 + 3z$

$-2.5z - \frac{1}{6}z^3 + \frac{3}{2}z^2$

$$\boxed{\text{Vol} = \int_0^1 \int_0^{1-z} \int_0^{1-x-z} 1 \, dy\, dx\, dz}$$

$-2.5 - \frac{1}{6}$

$\rho \sin$

$\sin$

5. (12 points) Multiple choice. Circle the correct answer.

(a) In spherical coordinates the plane $y = x$ can be written as

$$\rho = \frac{1}{\cos\phi} \qquad \phi = \frac{\pi}{3} \qquad \rho = 1 \qquad \theta = \frac{\pi}{4} \qquad \rho = \frac{1}{\sin\phi}.$$

(b) The Jacobian of the map $G(u, v) = (u^2 - v^2, uv)$ is

$$2u^2 + 2v^2 \qquad 2u^2 - 2v^2 \qquad 4uv \qquad 2u + 2v \qquad -4uv$$

$2u \quad -2v$

$v \quad u$

$2u^2 - -2v^2$

$\cancel{u^2 - v^2}$

$2u \quad -2v$

$v \quad u$

$2u^2 + 2v^2$

(c) In cylindrical coordinates the plane $x = 1$ can be written as

$r\cos\theta = 1$

$$r = \frac{1}{\cos\theta} \qquad \theta = \frac{\pi}{3} \qquad r = 1 \qquad \theta = \frac{\pi}{4} \qquad r = \frac{1}{\sin\theta}$$

(d) The linear map which sends the unit square $[0, 1] \times [0, 1]$ to the parallelogram with vertices $(0, 0)$, $(6, 1)$, $(8, 5)$, and $(2, 4)$ is $G(u, v) =$

$$(6u + v, 2u + 4v) \qquad (6u + 2v, u + 4v) \qquad (6u + v, 4u + 2v)$$
$$(6u + 2v, 4u + v) \qquad (6u + 4v, u + 2v)$$

$(6,1)$

$(2,4)$