Math 32B Final Exam

Xiangyu Wan

TOTAL POINTS

95.5 / 100

QUESTION 1

1 Change order of integration 4 / 4

✓ **- 0 pts** **answer = 2 (limits are x=0,pi y=0,x)**

- 1 pts minor error

- 2 pts incorrect integration bounds

- 1 pts integration error

- 4 pts swapping the order of integration without changing the bounds

- 4 pts incorrect

- 2 pts major integration error

QUESTION 2

2 Spherical coords 6 / 8

- 0 pts (1 pt) theta 0 to 2pi
(2 pts) phi pi/6 to 5pi/6
(2 pts) rho lower bound 1/sin phi
(1 pt) rho upper bound 2
(2 pts) integrand rho^2 sin phi

- 1 pts 1 error

✓ **- 2 pts** **2 errors**

- 3 pts 3 errors

- 4 pts 4 errors

- 5 pts 5 errors

- 6 pts 6 errors

- 7 pts 7 errors

- 8 pts 8 errors

QUESTION 3

Vortex field 12 pts

3.1 line integral 4 / 4

✓ **- 0 pts** **2pi**

- 2 pts Incorrect integral setup

- 2 pts Integration error

- 1 pts Minor error

- 4 pts Incorrect

3.2 curlz(F) 3 / 3

✓ **- 0 pts** **0**

- 1 pts minor error

- 2 pts major error

- 3 pts completely incorrect

- 1 pts should be a scalar, not a vector

3.3 Fill in the blanks 2 / 2

✓ **- 0 pts** **0, simply connected**

- 1 pts one wrong

- 2 pts both wrong

3.4 Conservative? 3 / 3

✓ **- 0 pts** **No, because the integral in (a) is nonzero**

- 1 pts No (but partially correct reason)

- 2 pts No (but incorrect reason)

- 3 pts Yes

QUESTION 4

Surface integral w/ vector potential 12 pts

4.1 vector potential 2 / 2

✓ **- 0 pts** **Correct.**

- 1 pts Incorrect, but knew that they needed calculate the curl of A.

- 2 pts Incorrect.

4.2 Stokes' theorem 8 / 8

✓ **+ 2 pts** **Applying Stoke's Theorem**

✓ **+ 2 pts** **Correctly parameterising the boundary.**

✓ **+ 1 pts** **Correct Orientation on boundary**

✓ **+ 2 pts** **Correctly setting the boundary integral up.**

✓ **+ 1 pts** **Correct final answer. (-24\pi or 24\pi if orientation wrong.)**

+ 0 pts Incorrect.

4.3 Other orientation 2 / 2

✓ **- 0 pts** **Correct. (negative of answer in b)**

- 1 pts Almost Correct (same as answer in b)

- 2 pts Incorrect

QUESTION 5

5 Worksheet problem: line integral in a plane 12 / 12

✓ **+ 12 pts** **Correct**

+ 2 pts Stokes' Theorem

+ 2 pts Correct curl

+ 2 pts Correct normal vector / orientation

+ 1 pts normalized

+ 2 pts dot with curl

+ 2 pts Recognizing the surface area as the integral of 1

+ 1 pts Correct answer (15*sqrt(3) or 45/sqrt(3))

+ 0 pts Click here to replace this description.

QUESTION 6

Divergence theorem 12 pts

6.1 integral of bottom cap 4 / 4

✓ **- 0 pts** **Correct**

- 1 pts Incorrect integrand (r^2 instead of r^3)

- 1 pts Incorrect integrand (r^4 instead of r^3)

- 1 pts Sign error

- 1 pts Integration error

- 1 pts Incorrect integrand (r instead of r^3)

- 1 pts Incorrect integrand (should have r^3)

6.2 integral of hemisphere 6 / 8

- 0 pts Correct

- 0 pts Correct, given your answer to (a)

- 8 pts Incorrect

- 2 pts Incorrect divergence

- 0.5 pts Minor calculation error

- 2 pts Forgot to solve for flux at end

- 1 pts Incorrect integrand

- 1 pts Sign error

- 1 pts Small calculation error

✓ **- 2 pts** **Incorrect integral**

- 7 pts 1 point for attempting to write out the integral in terms of a parametrization

QUESTION 7

MC 15 pts

7.1 (a) 3 / 3

- 3 pts Incorrect

✓ **- 0 pts** **Correct (positive)**

7.2 (b) 3 / 3

- 3 pts Incorrect

✓ **- 0 pts** **Correct (positive)**

7.3 (c) 3 / 3

✓ **- 0 pts** **Correct (zero)**

- 3 pts Incorrect

7.4 (d) 3 / 3

- 3 pts Incorrect

✓ **- 0 pts** **Correct (0.2)**

7.5 (e) 3 / 3

✓ **- 0 pts** **Correct (div(curl F)=0)**

- 3 pts Incorrect

- 1.5 pts Click here to replace this description.

- 2 pts Click here to replace this description.

QUESTION 8

MC 15 pts

8.1 (a) 3 / 3

✓ **- 0 pts** **Correct**

- 3 pts Incorrect

8.2 (b) 3 / 3

✓ **- 0 pts** **Correct**

- 3 pts Incorrect

8.3 (c) 3 / 3

✓ **- 0 pts** **Correct**

- **3 pts** Incorrect similar)

8.4 (d) **3 / 3**

✓ - **0 pts** **Correct**

- **3 pts** Incorrect

8.5 (e) **3 / 3**

✓ - **0 pts** **Correct**

- **3 pts** Incorrect

QUESTION 9

Fill in the blanks 10 pts

9.1 counterclockwise **1 / 1**

✓ - **0 pts** **Correct**

- **1 pts** Incorrect.

9.2 curlz(F) **0.5 / 1**

- **0 pts** Correct
- **1 pts** Incorrect.

✓ - **0.5 pts** **Wrote curl(F) instead of curl_z(F), or got the order of derivatives wrong way.**

9.3 boundary of D **1 / 1**

✓ - **0 pts** **Correct**

- **1 pts** Incorrect

9.4 RHS of Stokes' thm **3 / 3**

✓ - **0 pts** **Correct**

- **1 pts** Incorrect integral bounds (\partial S)
- **1 pts** Did not put single integral.
- **1 pts** Incorrect integrand

9.5 outward **1 / 1**

✓ - **0 pts** **Correct**

- **1 pts** Incorrect

9.6 LHS of Div thm **3 / 3**

✓ - **0 pts** **Correct**

- **1 pts** Not triple integral
- **1 pts** Bounds wrong (W)
- **1 pts** Integrand wrong. (div(F)dV, div(F)dxdydz or

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Section	3	A

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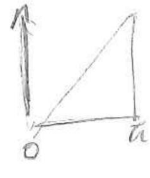
Spherical coordinates:

$$x = \rho \sin\phi \cos\theta$$
$$y = \rho \sin\phi \sin\theta$$
$$z = \rho \cos\phi$$

$$dxdydz = \rho^2 \sin\phi \, d\rho d\phi d\theta$$

Page:	1	2	3	4	5	6	7	8	Total
Points:	12	12	12	12	12	15	15	10	100
Score:									

1. (4 points) Evaluate the integral $\int_0^\pi \int_y^\pi \frac{\sin x}{x}\,dx\,dy$ by changing the order of integration.



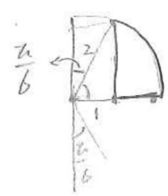
$$\int_0^\pi \int_0^x \frac{\sin x}{x}\,dy\,dx$$

$$= \int_0^\pi \sin x\,dx$$

$$= -\cos x\Big|_0^\pi$$

$$= 2$$

2. (8 points) Using spherical coordinates, set up *but do not evaluate* a triple integral that computes the volume of a sphere of radius 2 from which a central cylinder of radius 1 has been removed.



$$\int_{\theta=0}^{2\pi} \int_{\phi=\frac{\pi}{6}}^{\frac{5}{6}\pi} \int_{\rho=1}^{2\sin\phi-1} \rho^2 \sin\phi \, d\rho\, d\phi\, d\theta$$

$\frac{R\cos t}{R^2}$ $\quad x \cdot 2x \quad -y \cdot 2y \quad G$

3. (12 points) Let $\boldsymbol{F}$ denote the vortex field $\boldsymbol{F} = \left\langle \frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2} \right\rangle$.

(a) Suppose that C_R is the circle of radius R centered at $(0,0)$ oriented counterclockwise. **By parametrizing** C_R, compute $\oint_{C_R} \boldsymbol{F} \cdot d\boldsymbol{r}$. $\boxed{\text{Box your answer}}$

Note: you may not use the fundamental theorem of line integrals or anything about the winding number in this problem. Also, the answer is not zero.

$C_R: \vec{r}(t) = \langle R\cos t, R\sin t \rangle \quad \vec{r}'(t) = \langle -R\sin t, R\cos t \rangle \quad 0 \le t \le 2\pi$

$\vec{F}(\vec{r}(t)) = \langle \frac{-\sin t}{R}, \frac{\cos t}{R} \rangle$

$\oint_{C_R} \vec{F} \, d\vec{r} = \int_0^{2\pi} \frac{1}{R} \langle -\sin t, \cos t \rangle \cdot R \langle -\sin t, \cos t \rangle \, dt$

$= \int_0^{2\pi} \sin^2 t + \cos^2 t \, dt$

$= \int_0^{2\pi} 1 \, dt = \boxed{2\pi}$

(b) Compute $\text{curl}_z(\boldsymbol{F})$. Show your work. $\boxed{\text{Box your answer}}$

$\text{curl}_z(\vec{F}) = \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} = \frac{x^2+y^2-2x^2}{(x^2+y^2)^2} + \frac{+x^2+y^2-2y^2}{(x^2+y^2)^2}$

$= \frac{2x^2+2y^2-2x^2-2y^2}{(x^2+y^2)^2} = \boxed{0}$

(c) Fill in the blanks:

(i) If $\boldsymbol{F} = \nabla f$ on a domain D then $\oint_C \boldsymbol{F} \cdot d\boldsymbol{r} =$ __0__ for every closed curve C in D.

(ii) If $\text{curl}_z(\boldsymbol{F}) = 0$ on a __simple connected__ domain D then $\boldsymbol{F}$ is conservative.

(d) Is the vortex field $\boldsymbol{F}$ conservative on the domain $\mathbb{R}^2 \setminus \{(0,0)\}$? Explain your reasoning.

No, it's not since on this domain $\oint_C \vec{F} \, d\vec{r} \neq 0$, according to part (a), and the domain is not simple connected.

4. (12 points) Let $\boldsymbol{F} = \langle 2x, 0, -2z \rangle$.

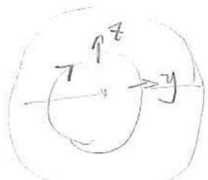
(a) Verify that $\boldsymbol{A} = \langle yz, -xz, yx \rangle$ is a vector potential for $\boldsymbol{F}$.

$$\text{curl } \vec{A} = \left\langle \frac{\partial A_3}{\partial y} - \frac{\partial A_2}{\partial z}, \frac{\partial A_1}{\partial z} - \frac{\partial A_3}{\partial x}, \frac{\partial A_2}{\partial x} - \frac{\partial A_1}{\partial y} \right\rangle$$

$$= \langle x - (-x), y - y, -z - z \rangle$$

$$= \langle 2x, 0, -2z \rangle$$

(b) Let S be the portion of the sphere $x^2 + y^2 + z^2 = 13$ where $x \leq 3$, oriented with outward-pointing normal vector. Find the flux of $\boldsymbol{F}$ through S. *Hint:* use the result of part (a). $\boxed{\text{Box your answer}}$



$$\iint_S \vec{F}\, d\vec{S} = \iint_S \text{curl } \vec{A}\, d\vec{S} = \oint_{\partial S} \vec{A}\, d\vec{r}$$

$$\partial S: 3^2 + y^2 + z^2 = 13$$

$$y^2 + z^2 = 4$$

$$\vec{r}(t) = \langle 3, 2\cos t, 2\sin t \rangle \quad t = [0, 2\pi]$$

$$\vec{r}\,(t) = \langle 0, -2\sin t, 2\cos t \rangle$$

$$\vec{A}(\vec{r}(t)) = \langle 4\cos t \sin t, -6\sin t, 6\cos t \rangle$$

$$\iint_S \vec{F}\, d\vec{S} = -\oint_{\partial S} \vec{A}\, d\vec{r} = -\int_0^{2\pi} \langle 4\cos t\sin t, -6\sin t, 6\cos t \rangle \langle 0, -2\sin t, 2\cos t \rangle\, dt$$

$$= -\int_0^{2\pi} 12\sin^2 t + 12\cos^2 t\, dt$$

$$= -\int_0^{2\pi} 12\, dt = \boxed{-24\pi}$$

(c) Let S' be the portion of the sphere $x^2 + y^2 + z^2 = 13$ where $x \geq 3$, oriented with outward-pointing normal vector. Find the flux of $\boldsymbol{F}$ through S'. $\boxed{\text{Box your answer}}$
You don't need to show your work for this part of the problem.

since $\vec{F}$ has a vector potential, $\oiint_S \vec{F}\, d\vec{S} = 0$

so $\iint_{S'} \vec{F}\, d\vec{S} = 0 - \iint_S \vec{F}\, d\vec{S} = \boxed{24\pi}$

5. (12 points) Given that C is a simple closed curve in the plane $x+y+z=1$ (oriented counterclockwise when viewed from above) that encloses a surface area of 5, compute $\int_C \boldsymbol{F}\cdot d\boldsymbol{r}$ for $\boldsymbol{F} = \langle 3z, 2x, 4y\rangle$.

$\boxed{\text{Box your answer}}$ *Hint:* it may be helpful to remember that $\iint_S \boldsymbol{G}\cdot d\boldsymbol{S} = \iint_S (\boldsymbol{G}\cdot\boldsymbol{n})dS$.

$$\oint_C \vec{F}\, d\vec{r} = \iint_{S_C} \text{curl}\,\vec{F}\, d\vec{S} = \iint_{S_C} \text{curl}\,\vec{F}\cdot\vec{n}\, dS$$

$$\text{curl}\,\vec{F} = \left\langle \frac{\partial}{\partial y}4y - \frac{\partial}{\partial z}2x,\ \frac{\partial}{\partial z}3z - \frac{\partial}{\partial x}4y,\ \frac{\partial}{\partial x}2x - \frac{\partial}{\partial y}3z \right\rangle$$

$$= \langle 4, 3, 2\rangle$$

$\vec{N}$ of plane is $\langle 1,1,1\rangle$ so $\vec{n} = \dfrac{\vec{N}}{\|\vec{N}\|} = \left\langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right\rangle$

$\text{curl}\,\vec{F}\cdot\vec{n} = \dfrac{4}{\sqrt{3}} + \dfrac{3}{\sqrt{3}} + \dfrac{2}{\sqrt{3}} = \dfrac{9}{\sqrt{3}} = 3\sqrt{3}$ it's constant

so, $\oint_C \vec{F}\, d\vec{r} = \iint_S \text{curl}(\vec{F})\cdot\vec{n}\, dS = \text{curl}(\vec{F})\cdot\vec{n}\cdot \text{Area}(S)$

$$= 3\sqrt{3}\cdot 5$$

$$= \boxed{15\sqrt{3}}$$

6. (12 points) Let $\boldsymbol{F} = \langle z^2x, \frac{1}{3}y^3 + \sin^2 z, x^2z + y^2\rangle$.

(a) Let $\mathcal{D}$ be the unit disk $x^2 + y^2 \leq 1$ in the xy-plane, oriented downward. Compute $\iint_{\mathcal{D}} \boldsymbol{F} \cdot d\boldsymbol{S}$.

It may be helpful to know that $\int_0^{2\pi} \sin^2\theta\, d\theta = \pi$. Box your answer
Hint: if $\mathcal{D}$ is parametrized via $G(r,\theta) = \langle r\cos\theta, r\sin\theta, 0\rangle$ then $\boldsymbol{N} = \pm\langle 0, 0, r\rangle$.

since the disk is on x-y. plane, z=0 constant

D: $G(\theta, r) = \langle r\cos\theta, r\sin\theta, 0\rangle$ $\quad \vec{N} = \langle 0, 0, -r\rangle$ downward

$F(G(\theta,r)) = \langle 0, \frac{1}{3}y^3, y^2\rangle = \langle 0, \frac{1}{3}r^3\sin^3\theta, r^2\sin^2\theta\rangle$

$\iint_D \vec{F}\, d\vec{S} = \int_0^{2\pi}\int_0^1 r^2\sin^2\theta \cdot (-r)\, dr\, d\theta = \int_0^{2\pi} \sin^2\theta\, d\theta \cdot \int_1^0 r^3\, dr$

$= \pi \cdot \left.\frac{r^4}{4}\right|_1^0 = \boxed{-\frac{\pi}{4}}$

(b) Let $\mathcal{S}$ be the top half of the sphere $x^2 + y^2 + z^2 = 1$, oriented upward. Compute $\iint_{\mathcal{S}} \boldsymbol{F} \cdot d\boldsymbol{S}$.

Box your answer *Hint:* you should use your answer to part (a). If you cannot do part (a), let A denote the value of the integral in part (a) and give your answer in terms of A.

let W be the whole semi sphere, and $\partial W = D + S$

$\iint_{\partial W} \vec{F}\, d\vec{S} = \iiint_W \operatorname{div}\vec{F}\, dV \qquad \operatorname{div}\vec{F} = z^2 + y^2 + x^2 = 1$

$\iiint_W \operatorname{div} F\, dV = \text{volume}(W) = \frac{1}{2} \cdot \frac{4}{3}\pi \cdot 1^3 = \frac{2}{3}\pi$

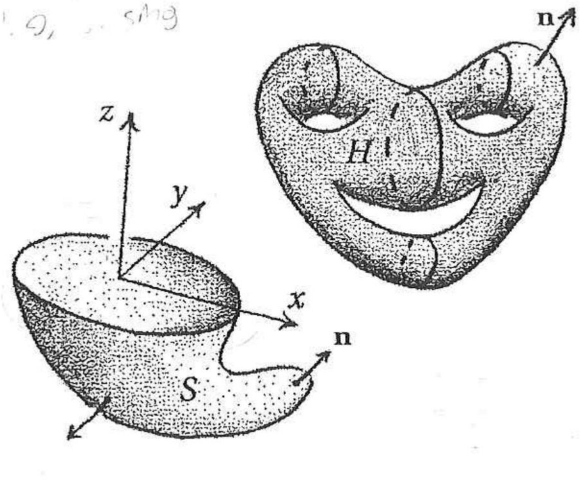
$\iint_S \vec{F}\, d\vec{S} = \iint_{\partial W} \vec{F}\, d\vec{S} - \iint_D \vec{F}\, d\vec{S} = \frac{2}{3}\pi - \left(-\frac{\pi}{4}\right)$

$= \left(\frac{2}{3} + \frac{1}{4}\right)\pi = \boxed{\frac{11}{12}\pi}$

7. (15 points) Multiple choice. Circle the correct answer.

Let S and H be the surfaces shown to the right. The boundary of S is the unit circle in the xy-plane, while H has no boundary. Let $\boldsymbol{G} = \langle x, y, z \rangle$.

$\operatorname{div}(\vec{G}) = 3$



(a) The flux $\iint_H \boldsymbol{G} \cdot d\boldsymbol{S}$ is

negative   zero   positive

(b) The flux $\iint_S \boldsymbol{G} \cdot d\boldsymbol{S}$ is

negative   zero   positive

Hint for (b): use the divergence theorem.

(c) The flux $\iint_S \operatorname{curl} \boldsymbol{G} \cdot d\boldsymbol{S}$ is

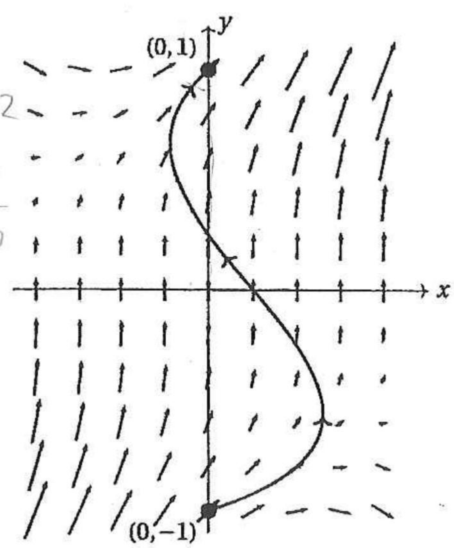
negative   zero   positive

curl G = 0

(d) A vector field is shown to the right. For scale, $\boldsymbol{F}(0,0) = \langle 0, 0.1 \rangle$.

$\frac{\sqrt{2}+1}{2} \cdot 0.2$

$\frac{\sqrt{2}+1}{10} = \frac{2.4}{10}$



Given that <u>$\boldsymbol{F}$ is conservative</u>, estimate $\int_C \boldsymbol{F} \cdot d\boldsymbol{r}$, where C is the curve shown from $(0,-1)$ to $(0,1)$.

−0.5   − 0.2   0   0.2   0.5

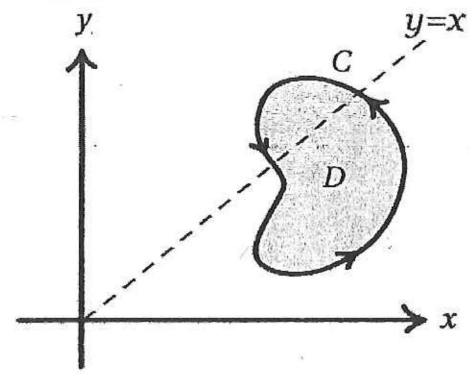
$\frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z}, \quad \frac{\partial F_1}{\partial z} - \frac{\partial F_3}{\partial x}, \quad \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}$

(e) Which of the following statements makes sense and is true for any vector field $\boldsymbol{F}$ in $\mathbb{R}^3$ whose components have continuous second-order partial derivatives?

$\nabla(\operatorname{curl} \boldsymbol{F}) = 0$ $\operatorname{div}(\operatorname{curl} \boldsymbol{F}) = 0$ $\operatorname{div}(\nabla \boldsymbol{F}) = 0$ $\operatorname{curl}(\operatorname{curl} \boldsymbol{F}) = 0$

8. (15 points) Multiple choice. Circle the correct answer.

Consider the region D in the plane bounded by the curve C as shown to the right. For parts (a)–(c), circle the best answer.



(a) For $\boldsymbol{F}(x, y) = \langle x+1, y^2 \rangle$, the integral $\int_C \boldsymbol{F} \cdot d\boldsymbol{r}$ is $\quad \frac{\partial y^2}{\partial x} - \frac{\partial (x+1)}{\partial y} = 0$

negative   zero   positive

(b) The integral $\int_C (-y\,dx + 2dy)$ is $\quad \frac{\partial 2}{\partial x} - \frac{\partial (-y)}{\partial y} = 1$

Area

negative   zero   positive

(c) The integral $\iint_D (y - x)\, dA$ is

negative   zero   positive

Hint for (c): look at the location of D in the plane.

(d) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation of the plane sending the triangle with vertices $(0,0)$, $(1,0)$, $(0,1)$ to the triangle with vertices $(0,0)$, $(1,2)$, $(-1,3)$, respectively. Find the Jacobian of T.

1   2   3   4   5

$Au \quad Bv \quad Cu \quad Dv$

$\begin{matrix} 1 & 2 \\ -1 & 3 \end{matrix} \quad 3+2=5$

$A = 1 \quad C = 2$

$B = -1 \quad D = 3$

(e) Let $\mathcal{R} = [1,2] \times [1,2]$ and let $\mathcal{D} = G(\mathcal{R})$, where G is the map $G(u, v) = (u^2/v, v^2/u)$. Compute the area of $\mathcal{D}$.

1   2   3   4   5 $\quad \frac{u^2}{v} \cdot \frac{v^2}{u}$

Area D

$= \iint_R \text{Jac}\, dudv = \underline{\text{Area } R} \cdot \underline{\text{Jac}}$

$\quad 3$

$\cancel{2 \times 2}$

$1 \times 1 = 1$

$\begin{matrix} \frac{2u}{v} & -\frac{u^2}{v^2} \\ -\frac{v^2}{u^2} & \frac{2v}{u} \end{matrix}$

$4 - 1 = 3$

$\iint_D dxdy = \iint_R \text{Jac}\, dudv$

9. (10 points) Fill in the blanks in the big theorems of vector calculus.

The fundamental theorem of line integrals. If $\mathcal{C}$ is an oriented curve from P to Q in $\mathcal{D}$ then

$$\int_{\mathcal{C}} \nabla f \cdot d\boldsymbol{r} = f(Q) - f(P).$$

Green's theorem. Let $\mathcal{D}$ be a domain whose boundary ∂D is a simple closed curve, oriented $\boxed{\text{with lefthand inside } D}$. Then

$$\iint_{\mathcal{D}} \boxed{\text{curl}(\vec{F})}\, dA = \oint_{\boxed{\partial D}} \boldsymbol{F} \cdot d\boldsymbol{r}.$$

Stokes' theorem. Let $\mathcal{S}$ be a "sufficiently nice" surface, and let $\boldsymbol{F}$ be a vector field whose components have continuous partial derivatives on an open region containing $\mathcal{S}$. Then

$$\iint_{\mathcal{S}} \text{curl}(\boldsymbol{F}) \cdot d\boldsymbol{S} = \boxed{\oint_{\partial S} \vec{F}\, d\vec{r}}.$$

The integral on the right-hand side is defined relative to the boundary orientation of ∂S.

The divergence theorem. Let $\mathcal{S}$ be a closed surface that encloses a region $\mathcal{W}$ in $\mathbb{R}^3$. Assume that $\mathcal{S}$ is piecewise smooth and is oriented by normal vectors pointing $\boxed{\text{outward}}$. Let $\boldsymbol{F}$ be a vector field whose domain contains $\mathcal{W}$. Then

$$\boxed{\iiint_{W} \text{div}(\vec{F})\, dV} = \iint_{\partial\mathcal{W}} \boldsymbol{F} \cdot d\boldsymbol{S}.$$

You may use this page for scratch work.