Math 32B Exam 1

Tian Yu Liu

TOTAL POINTS

47 / 50

QUESTION 1

TF 8 pts

1.1 TF **2 / 2**

✓ **- 0 pts** **True**

1.2 Yes/No Integrals **5 / 6**

✓ **- 1 pts** **1st wrong**

QUESTION 2

Worksheet Question 10 pts

2.1 Rectangular coords **4 / 4**

✓ **- 0 pts** **Correct**

2.2 Polar coords **6 / 6**

✓ **+ 6 pts** **Correct**

+ 1 pts Correct bound

+ 2 pts Correct bounds

+ 1 pts Correct integrand (excluding Jacobian)

+ 2 pts Jacobian

+ 1 pts Correct final answer

- 1 pts Minor Miscalculation/Incorrect final answer

+ 0 pts incorrect or nothing shown

QUESTION 3

Non-linear transformation 10 pts

3.1 Picture **4 / 4**

✓ **- 0 pts** **Correct (third picture)**

3.2 Integral **5 / 6**

✓ **- 1 pts** **Incorrect Jacobian.**

QUESTION 4

Q4 10 pts

4.1 Sphere/Cone **3 / 4**

✓ **- 1 pts** **Incorrect Phi bounds due to a minor mistake.**

4.2 Volume integrals **6 / 6**

✓ **- 0 pts** **Correct**

QUESTION 5

MC 12 pts

5.1 Spherical Plane **3 / 3**

✓ **- 0 pts** **theta=pi/4**

5.2 Jacobian **3 / 3**

✓ **- 0 pts** **2u^2+2v^2**

5.3 Cylindrical Plane **3 / 3**

✓ **- 0 pts** **r=1/cos\theta**

5.4 Linear Map **3 / 3**

✓ **- 0 pts** **(6u+2v, u+4v)**

MATH 32B Exam 1 Spring 2019

Full Name Tian Yu Liu UID 005-086-282

3A	Ben Szczesny	T	GEOLOGY 4645
3B		R	GEOLOGY 4645
3C	Talon Stark	T	PUB AFF 2242
3D		R	MS 6221
3E	Ryan Wallace	T	BUNCHE 3156
3F		R	DODD 78

Section	3	D

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- Fill out your name, section letter, and UID above.
- Do not open this exam packet until you are told that you may begin.
- Turn off all electronic devices and and put away all items except for a pen/pencil and an eraser.
- No phones, calculators, smart-watches or electronic devices of any kind allowed for any reason, including checking the time.
- If you have a question, raise your hand and one of the proctors will come to you. We will not answer any mathematical questions except possibly to clarify the wording of a problem.
- Quit working and close this packet when you are told to stop.

Spherical coordinates:

$$x = \rho \sin\phi \cos\theta$$
$$y = \rho \sin\phi \sin\theta$$
$$z = \rho \cos\phi$$

$$dxdydz = \rho^2 \sin\phi \, d\rho d\phi d\theta$$

Page:	1	2	3	4	5	Total
Points:	8	10	10	10	12	50
Score:						

You may use this page for scratch work.

1. (8 points) (a) True or False? (circle one) $\displaystyle\int_1^4 \int_0^1 \sqrt{y}\,\sin(x^2y^2)\,dx\,dy \le 6$

True False

(b) Let D be the region in the positive octant $(x, y, z \ge 0)$ enclosed by the sphere $x^2+y^2+z^2=4$ and the planes $z=0$, $x=0$, and $x=y$. For **each** integral below, circle "yes" or "no" depending on whether or not it equals $\displaystyle\iiint_D x\,dV$.

yes | no $\displaystyle\int_0^{\pi/2}\int_0^{\pi/4}\int_0^{2} \rho^3\cos\theta\sin^2\phi\,d\rho\,d\theta\,d\phi$

yes | no $\displaystyle\int_0^{\sqrt{2}}\int_0^{\sqrt{4-x^2}}\int_0^{\sqrt{4-x^2-y^2}} x\,dz\,dy\,dx$

yes | no $\displaystyle\int_0^{\pi/2}\int_{\pi/4}^{\pi/2}\int_0^{2} \rho^2\cos\theta\sin\phi\,d\rho\,d\theta\,d\phi$

yes | no $\displaystyle\int_0^{\pi/2}\int_{\pi/4}^{\pi/2}\int_0^{2} \rho^2\cos\theta\sin\phi\,d\rho\,d\phi\,d\theta$

yes | no $\displaystyle\int_0^{2}\int_{\pi/4}^{\pi/2}\int_0^{\sqrt{4-r^2}} r^2\cos\theta\,dz\,d\theta\,dr$

yes | no $\displaystyle\int_0^{\pi/2}\int_0^{2}\int_{\pi/4}^{\pi/2} \rho^3\cos\theta\sin^2\phi\,d\theta\,d\rho\,d\phi$

2. (10 points) Let R be the region in $\mathbb{R}^2$ which lies above the x-axis and between the circles of radius 1 and 2 centered at $(0,0)$.

(a) Write the following integral as a sum of integrals in **rectangular coordinates**:

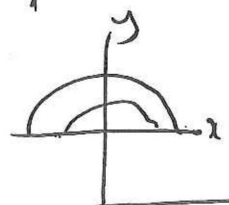
$$\iint_R 3y\, dA.$$

Do not evaluate these integrals. Box your answers.

$1 \le x^2+y^2 \le 4$

$\iint_R 3y\,dA = \iint_{R_1} 3y\,dA - \iint_{R_2} 3y\,dA$

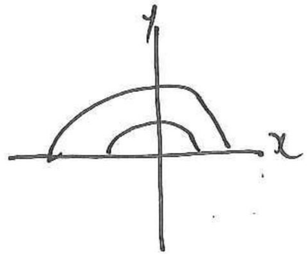
where $R_1 = \begin{cases} 0 \le y \le 2 \\ -\sqrt{4-y^2} \le x \le \sqrt{4-y^2} \end{cases}$



$R_2 = \begin{cases} 0 \le y \le 1 \\ -\sqrt{1-y^2} \le x \le \sqrt{1-y^2} \end{cases}$

$$\boxed{\int_0^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} 3y\,dx\,dy - \int_0^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} 3y\,dx\,dy}$$

(b) Evaluate the integral in part (a) using **polar coordinates**. Box your answer.



$D = \begin{cases} 1 \le r \le 2 \\ 0 \le \theta \le \pi \end{cases}$

$\iint_R 3y\,dA = \int_1^2 \int_0^\pi 3r\sin\theta\,(r)\,d\theta\,dr$

$= 3\int_1^2 r^2\,dr \int_0^\pi \sin\theta\,d\theta$

$= \left[r^3\right]_1^2 \cdot \left[-\cos\theta\right]_0^\pi$

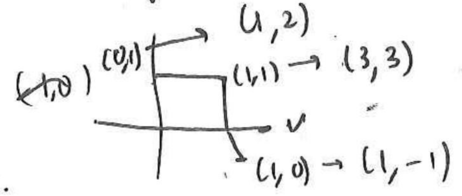
$= 7(2)$

$= 14$

$x = 1+2v, \quad 4v-1 = 1$

$v = \frac{x-1}{2} \quad 2x-2-1 = y$

$y = 2x-3$

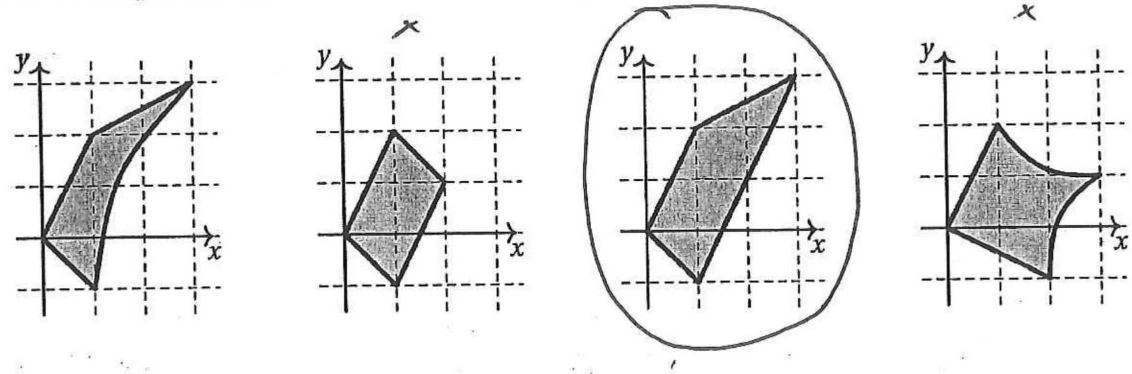


3. (10 points) Let $G : \mathbb{R}^2 \to \mathbb{R}^2$ be the non-linear transformation

$$G(u,v) = (u + v + uv, -u + 2v + 2uv).$$

Let R be the unit square $[0,1] \times [0,1]$ in the uv-plane and let $D = G(R)$ in the xy-plane.

(a) Circle the picture of D below. The dashed grid consists of unit squares.



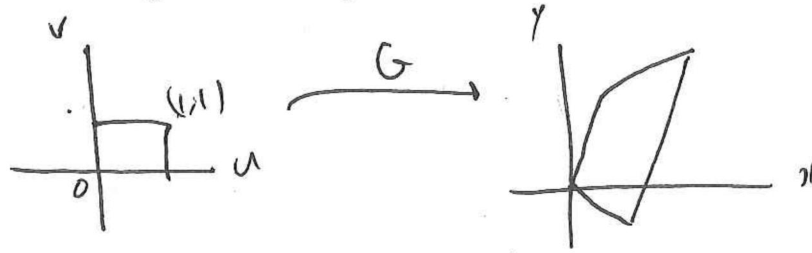
(b) Find the limits and integrand of the integral below so that it equals

$1+v \quad 1+u$

$-1+2v \quad 2+2u$

$$\iint_D \sqrt{x}\, dA$$

as an integral over the square R. Do not evaluate the integral. Show your work.



$$\text{Jac}(G) = \begin{vmatrix} 1+v & 1+u \\ -1+2v & 2+2u \end{vmatrix} = (1+v)(2+2u) + (1+2v)(1+u)$$

$$= 2(1+u+v+uv) + (1+2v+u+2vu)$$

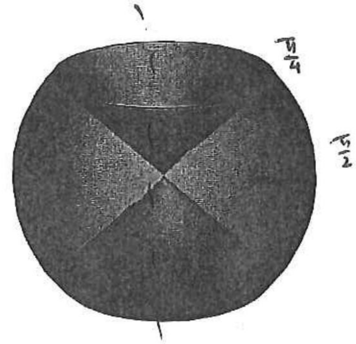
$$= 2+2u+2v+2vu + 1+2v+u+2vu$$

$$= 3+3u+4v+4vu$$

$$\iint_D \sqrt{x}\, dA = \int_0^1 \int_0^1 \sqrt{u+v+uv}\,(3+3u+4v+4uv)\, du\, dv$$

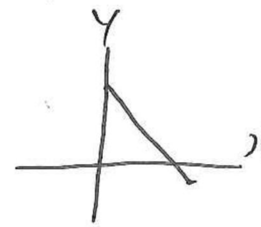
$$\boxed{\iint_D \sqrt{x}\, dA = \int_0^1 \int_0^1 \sqrt{u+v+uv}\,(3+3u+4v+4uv)\, du\, dv}$$

4. (10 points) (a) In **spherical coordinates**, describe the region outside the cone $x^2 + y^2 = z^2$ and inside the sphere $x^2 + y^2 + z^2 = 2$ (shown below – the sphere is translucent so you can see the cone inside).



$$0 \le \theta \le 2\pi$$
$$\frac{\pi}{4} \le \phi \le \frac{\pi}{2}$$
$$0 \le \rho \le \sqrt{2}$$

(b) Fill in the limits and integrand of the double and triple integrals below so that they both equal the volume of the region in the first octant ($x, y, z \ge 0$) below the plane $x + y + z = 1$. Be sure to follow the provided order of integration.



$$\text{Vol} = \int_0^1 \int_0^{1-y} 1-x-y \; dx\, dy$$

$$\text{Vol} = \int_0^1 \int_0^{1-z} \int_0^{1-x-z} 1 \; dy\, dx\, dz$$

5. (12 points) Multiple choice. Circle the correct answer.

(a) In spherical coordinates the plane $y = x$ can be written as

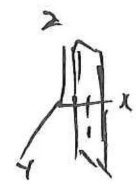
$$\rho = \frac{1}{\cos\phi} \qquad \phi = \frac{\pi}{3} \qquad \rho = 1 \qquad \theta = \frac{\pi}{4} \qquad \rho = \frac{1}{\sin\phi}.$$

(b) The Jacobian of the map $G(u, v) = (u^2 - v^2, uv)$ is

$$2u^2 + 2v^2 \qquad 2u^2 - 2v^2 \qquad 4uv \qquad 2u + 2v \qquad -4uv$$

(c) In cylindrical coordinates the plane $x = 1$ can be written as

$$r = \frac{1}{\cos\theta} \qquad \theta = \frac{\pi}{3} \qquad r = 1 \qquad \theta = \frac{\pi}{4} \qquad r = \frac{1}{\sin\theta}$$



(d) The linear map which sends the unit square $[0, 1] \times [0, 1]$ to the parallelogram with vertices $(0, 0)$, $(6, 1)$, $(8, 5)$, and $(2, 4)$ is $G(u, v) =$

$$(6u + v, 2u + 4v) \qquad (6u + 2v, u + 4v) \qquad (6u + v, 4u + 2v)$$
$$(6u + 2v, 4u + v) \qquad (6u + 4v, u + 2v)$$