Math 32B Final Exam

Tian Yu Liu

TOTAL POINTS

92 / 100

QUESTION 1

1 Change order of integration 4 / 4

✓ **- 0 pts** **answer = 2 (limits are x=0,pi y=0,x)**

- **- 1 pts** minor error
- **- 2 pts** incorrect integration bounds
- **- 1 pts** integration error
- **- 4 pts** swapping the order of integration without changing the bounds
- **- 4 pts** incorrect
- **- 2 pts** major integration error

QUESTION 2

2 Spherical coords 8 / 8

✓ **- 0 pts** **(1 pt) theta 0 to 2pi**
(2 pts) phi pi/6 to 5pi/6
(2 pts) rho lower bound 1/sin phi
(1 pt) rho upper bound 2
(2 pts) integrand rho^2 sin phi

- **- 1 pts** 1 error
- **- 2 pts** 2 errors
- **- 3 pts** 3 errors
- **- 4 pts** 4 errors
- **- 5 pts** 5 errors
- **- 6 pts** 6 errors
- **- 7 pts** 7 errors
- **- 8 pts** 8 errors

QUESTION 3

Vortex field 12 pts

3.1 line integral 4 / 4

✓ **- 0 pts** **2pi**

- **- 2 pts** Incorrect integral setup
- **- 2 pts** Integration error
- **- 1 pts** Minor error
- **- 4 pts** Incorrect

3.2 curlz(F) 3 / 3

✓ **- 0 pts** **0**

- **- 1 pts** minor error
- **- 2 pts** major error
- **- 3 pts** completely incorrect
- **- 1 pts** should be a scalar, not a vector

3.3 Fill in the blanks 2 / 2

✓ **- 0 pts** **0, simply connected**

- **- 1 pts** one wrong
- **- 2 pts** both wrong

3.4 Conservative? 1 / 3

- **- 0 pts** No, because the integral in (a) is nonzero
- **- 1 pts** No (but partially correct reason)

✓ **- 2 pts** **No (but incorrect reason)**

- **- 3 pts** Yes

QUESTION 4

Surface integral w/ vector potential 12 pts

4.1 vector potential 2 / 2

✓ **- 0 pts** **Correct.**

- **- 1 pts** Incorrect, but knew that they needed calculate the curl of A.
- **- 2 pts** Incorrect.

4.2 Stokes' theorem 6 / 8

✓ **+ 2 pts** **Applying Stoke's Theorem**

✓ **+ 2 pts** **Correctly parameterising the boundary.**

+ 1 pts Correct Orientation on boundary

✓ **+ 2 pts** **Correctly setting the boundary integral up.**

+ 1 pts Correct final answer. (-24\pi or 24\pi if orientation wrong.)

+ 0 pts Incorrect.

4.3 Other orientation 2 / 2

✓ - **0 pts** **Correct. (negative of answer in b)**

- **1 pts** Almost Correct (same as answer in b)

- **2 pts** Incorrect

QUESTION 5

5 Worksheet problem: line integral in a plane 10 / 12

+ **12 pts** Correct

✓ + **2 pts** **Stokes' Theorem**

✓ + **2 pts** **Correct curl**

✓ + **2 pts** **Correct normal vector / orientation**

+ **1 pts** normalized

✓ + **2 pts** **dot with curl**

✓ + **2 pts** **Recognizing the surface area as the integral of 1**

+ **1 pts** Correct answer (15*sqrt(3) or 45/sqrt(3))

+ **0 pts** Click here to replace this description.

QUESTION 6

Divergence theorem 12 pts

6.1 integral of bottom cap 4 / 4

✓ - **0 pts** **Correct**

- **1 pts** Incorrect integrand (r^2 instead of r^3)

- **1 pts** Incorrect integrand (r^4 instead of r^3)

- **1 pts** Sign error

- **1 pts** Integration error

- **1 pts** Incorrect integrand (r instead of r^3)

- **1 pts** Incorrect integrand (should have r^3)

6.2 integral of hemisphere 6 / 8

- **0 pts** Correct

- **0 pts** Correct, given your answer to (a)

- **8 pts** Incorrect

- **2 pts** Incorrect divergence

- **0.5 pts** Minor calculation error

- **2 pts** Forgot to solve for flux at end

- **1 pts** Incorrect integrand

- **1 pts** Sign error

- **1 pts** Small calculation error

✓ - **2 pts** **Incorrect integral**

- **7 pts** 1 point for attempting to write out the integral in terms of a parametrization

QUESTION 7

MC 15 pts

7.1 (a) 3 / 3

- **3 pts** Incorrect

✓ - **0 pts** **Correct (positive)**

7.2 (b) 3 / 3

- **3 pts** Incorrect

✓ - **0 pts** **Correct (positive)**

7.3 (c) 3 / 3

✓ - **0 pts** **Correct (zero)**

- **3 pts** Incorrect

7.4 (d) 3 / 3

- **3 pts** Incorrect

✓ - **0 pts** **Correct (0.2)**

7.5 (e) 3 / 3

✓ - **0 pts** **Correct (div(curl F)=0)**

- **3 pts** Incorrect

- **1.5 pts** Click here to replace this description.

- **2 pts** Click here to replace this description.

QUESTION 8

MC 15 pts

8.1 (a) 3 / 3

✓ - **0 pts** **Correct**

- **3 pts** Incorrect

8.2 (b) 3 / 3

✓ - **0 pts** **Correct**

- **3 pts** Incorrect

8.3 (c) 3 / 3

✓ - **0 pts** **Correct**

- **3 pts** Incorrect similar)

8.4 (d) **3 / 3**

✓ - **0 pts Correct**

- **3 pts** Incorrect

8.5 (e) **3 / 3**

✓ - **0 pts Correct**

- **3 pts** Incorrect

QUESTION 9

Fill in the blanks 10 pts

9.1 counterclockwise **1 / 1**

✓ - **0 pts Correct**

- **1 pts** Incorrect.

9.2 curlz(F) **1 / 1**

✓ - **0 pts Correct**

- **1 pts** Incorrect.
- **0.5 pts** Wrote curl(F) instead of curl_z(F), or got the order of derivatives wrong way.

9.3 boundary of D **1 / 1**

✓ - **0 pts Correct**

- **1 pts** Incorrect

9.4 RHS of Stokes' thm **3 / 3**

✓ - **0 pts Correct**

- **1 pts** Incorrect integral bounds (\partial S)
- **1 pts** Did not put single integral.
- **1 pts** Incorrect integrand

9.5 outward **1 / 1**

✓ - **0 pts Correct**

- **1 pts** Incorrect

9.6 LHS of Div thm **3 / 3**

✓ - **0 pts Correct**

- **1 pts** Not triple integral
- **1 pts** Bounds wrong (W)
- **1 pts** Integrand wrong. (div(F)dV, div(F)dxdydz or

MATH 32B Final Exam Spring 2019

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3B		R	GEOLOGY 4645
3C	Talon Stark	T	PUB AFF 2242
3D		R	MS 6221
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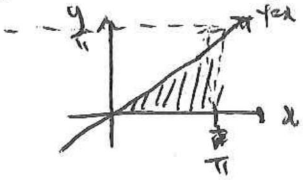
Spherical coordinates:

$$x = \rho \sin\phi \cos\theta$$
$$y = \rho \sin\phi \sin\theta$$
$$z = \rho \cos\phi$$

$$dxdydz = \rho^2 \sin\phi \, d\rho d\phi d\theta$$

Page:	1	2	3	4	5	6	7	8	Total
Points:	12	12	12	12	12	15	15	10	100
Score:									

1. (4 points) Evaluate the integral $\int_0^{\pi}\int_y^{\pi} \frac{\sin x}{x}\,dx\,dy$ by changing the order of integration.



$$\int_0^{\pi}\int_y^{\pi} \frac{\sin x}{x}\,dx\,dy = \int_0^{\pi}\int_0^{x} \frac{\sin x}{x}\,dy\,dx$$

$0 \le x \le \pi$

$0 \le y \le x$

$$= \int_0^{\pi} \left[\frac{\sin x}{x} y \right]_0^{x} dx$$

$$= \int_0^{\pi} \sin x \, dx$$

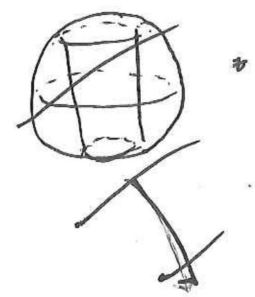
$$= \left[-\cos x\right]_0^{\pi}$$

$$= -(-1) - (-\cos 0)$$

$$= 2$$

2. (8 points) Using spherical coordinates, set up *but do not evaluate* a triple integral that computes the volume of a sphere of radius 2 from which a central cylinder of radius 1 has been removed.

$\int$



intersection of cylinder with sphere:

$x^2+y^2+z^2 = 4$ -- sphere

$x^2+y^2 = 1$ --- cylinder

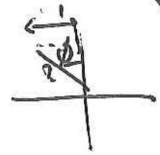
intersection: $z^2 = 3$

$z = \pm\sqrt{3}$

~~integral:~~ $\cancel{\underbrace{\int_0^{2}\int_0^{\pi}\int_0^{2\pi} \rho^2 \sin\phi \, d\theta\, d\phi\, d\rho}_{\text{area of sphere}} - \overbrace{\int_0^{1}\int_0^{2\pi}\int_{-\sqrt{3}}^{\sqrt{3}} r\, dz\, d\theta\, dr}^{\text{area of cylinder}}}$

$\frac{1}{\sin\phi} \le \rho \le 2$

$0 \le \theta \le 2\pi$



intersection: $z = \pm\sqrt{3}$

$\phi = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$

$\frac{\pi}{6} \le \phi \le \frac{5\pi}{6}$

$$\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}}\int_0^{2\pi}\int_{\frac{1}{\sin\phi}}^{2} \rho^2 \sin\phi \, d\rho\, d\theta\, d\phi$$

$\sin\phi = \frac{1}{\rho_2}$

$\rho_2 = \frac{1}{\sin\phi}$

3. (12 points) Let $\boldsymbol{F}$ denote the vortex field $\boldsymbol{F} = \left\langle \frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2} \right\rangle$.

(a) Suppose that C_R is the circle of radius R centered at $(0,0)$ oriented counterclockwise. **By parametrizing** C_R, compute $\oint_{C_R} \boldsymbol{F} \cdot d\boldsymbol{r}$. Box your answer

Note: you may not use the fundamental theorem of line integrals or anything about the winding number in this problem. Also, the answer is not zero.

C_R: $r(t) = \langle R\cos\theta, R\sin\theta \rangle$, $0 \le \theta \le 2\pi$, $r'(t) = \langle -R\sin\theta, R\cos\theta \rangle$

$$\oint_{C_R} \vec{F} \cdot d\vec{r} = \int_0^{2\pi} F(r(t)) \cdot r'(t)\,dt$$

$$= \int_0^{2\pi} \left\langle \frac{-R\sin\theta}{R^2\cos^2\theta + R^2\sin^2\theta}, \frac{R\cos\theta}{R^2\cos^2\theta + R^2\sin^2\theta} \right\rangle \cdot \langle -R\sin\theta, R\cos\theta \rangle\,d\theta$$

$$= \int_0^{2\pi} \frac{\cancel{R^2}\sin^2\theta}{\cancel{R^2}} + \frac{\cancel{R^2}\cos^2\theta}{\cancel{R^2}}\,dt$$

$$= \int_0^{2\pi} 1\,dt = \boxed{2\pi}$$

(b) Compute $\text{curl}_z(\boldsymbol{F})$. Show your work. Box your answer

$$\text{curl}_z(F) = \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}$$

$$= \frac{(x^2+y^2)(1) - x(2x)}{(x^2+y^2)^2} + \frac{(x^2+y^2)(1) - y(2y)}{(x^2+y^2)^2}$$

$$= \frac{2x^2+2y^2-2x^2-2y^2}{(x^2+y^2)^2}$$

$$= \boxed{0}$$

(c) Fill in the blanks:

(i) If $\boldsymbol{F} = \nabla f$ on a domain D then $\oint_C \boldsymbol{F} \cdot d\boldsymbol{r} =$ 0 for every closed curve C in D.

(ii) If $\text{curl}_z(\boldsymbol{F}) = 0$ on a simply connected domain D then $\boldsymbol{F}$ is conservative.

(d) Is the vortex field $\boldsymbol{F}$ conservative on the domain $\mathbb{R}^2 \setminus \{(0,0)\}$? Explain your reasoning.

No. The domain is not simply connected.

4. (12 points) Let $\boldsymbol{F} = \langle 2x, 0, -2z\rangle$.

(a) Verify that $\boldsymbol{A} = \langle yz, -xz, yx\rangle$ is a vector potential for $\boldsymbol{F}$.

$$\operatorname{curl}(\vec{A}) = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & -xz & yx \end{vmatrix} = \langle x+x, -(y-y), \cancel{-x} -z-z \rangle$$

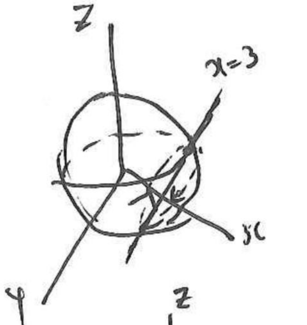
$$= \langle 2x, 0, -2z\rangle$$

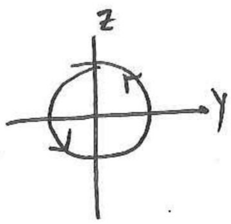
$$= \vec{F}$$

(b) Let S be the portion of the sphere $x^2 + y^2 + z^2 = 13$ where $x \le 3$, oriented with outward-pointing normal vector. Find the flux of $\boldsymbol{F}$ through S. *Hint:* use the result of part (a). Box your answer

$$\cancel{\iint_S \vec{F}\cdot d\vec{S} \equiv} \quad \oint_C \vec{A}\cdot d\vec{r} = \iint_D \operatorname{curl}(A)\, dS$$

$$= \iint_D \vec{F}\cdot dS$$





$\cancel{\oint_C \vec{A}\cdot d\vec{r} = r'(\theta)}$ $\quad r'(\theta) = \langle 0, -\sin\theta, \cos\theta\rangle$

$C = \partial D$: $r(\theta) = \langle 3, \cos\theta, \sin\theta\rangle$, $0 \le \theta \le 2\pi$

$$\oint_C \vec{A}\cdot d\vec{r} = \int_0^{2\pi} A(r(\theta))\cdot r'(\theta)\, d\theta$$

$$= \int_0^{2\pi} \langle \cos\theta\sin\theta, -3\sin\theta, 3\cos\theta\rangle \cdot \langle 0, -\sin\theta, \cos\theta\rangle\, d\theta$$

$$= \int_0^{2\pi} 3\sin^2\theta + 3\cos^2\theta\, d\theta$$

$$= \boxed{6\pi}$$

(c) Let S' be the portion of the sphere $x^2 + y^2 + z^2 = 13$ where $x \ge 3$, oriented with outward-pointing normal vector. Find the flux of $\boldsymbol{F}$ through S'. Box your answer
You don't need to show your work for this part of the problem.

$$\boxed{-6\pi}$$

5. (12 points) Given that C is a simple closed curve in the plane $x+y+z=1$ (oriented counterclockwise when viewed from above) that encloses a surface area of 5, compute $\int_C \boldsymbol{F} \cdot d\boldsymbol{r}$ for $\boldsymbol{F} = \langle 3z, 2x, 4y \rangle$.

Box your answer *Hint:* it may be helpful to remember that $\iint_S \boldsymbol{G} \cdot d\boldsymbol{S} = \iint_S (\boldsymbol{G} \cdot \boldsymbol{n}) dS$.

$$\int_C \vec{F} \cdot d\vec{r} = \iint_D \text{curl}(\vec{F})\, dA \qquad \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3z & 2x & 4y \end{vmatrix} = \langle 4,3,2 \rangle$$

$$= \iint_D \langle 4,3,2 \rangle \, dA$$

$$= \cancel{\iint_D} \quad G(x,y) = \langle x, y, 1-x-y \rangle$$

$$G_x = \langle 1, 0, -1 \rangle$$

$$G_y = \langle 0, 1, -1 \rangle$$

$$N = G_x \times G_y = \begin{vmatrix} i & j & k \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{vmatrix}$$

$$= \langle 1, 1, 1 \rangle$$

$$\iint_D \langle 4,3,2 \rangle \, dA = \iint_D \langle 4,3,2 \rangle \cdot \langle 1,1,1 \rangle \, dA$$

$$= \iint_D 9 \, dA$$

$$= \cancel{9(A)}\ 9(5)$$

$$= \boxed{45}$$

$\frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z}$ $\frac{\partial}{\partial x}$ $\frac{\partial}{\partial y}$ $\frac{\partial}{\partial z}$

F_1 F_2 F_3

6. (12 points) Let $\boldsymbol{F} = \langle z^2x, \frac{1}{3}y^3 + \sin^2 z, x^2z + y^2\rangle$. z^2x $\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} = x^2z - y^2$

(a) Let $\mathcal{D}$ be the unit disk $x^2+y^2 \le 1$ in the xy-plane, ~~oriented downward~~. Compute $\iint_{\mathcal{D}} \boldsymbol{F}\cdot dS$.

It may be helpful to know that $\int_0^{2\pi} \sin^2\theta\, d\theta = \pi$. $\boxed{\text{Box your answer}}$
Hint: if $\mathcal{D}$ is parametrized via $G(r,\theta) = \langle r\cos\theta, r\sin\theta, 0\rangle$ then $\boldsymbol{N} = \pm\langle 0,0,r\rangle$.

D: $G(r,\theta) = \langle r\cos\theta, r\sin\theta, 0\rangle$

$N = \langle 0, 0, -r\rangle$ (oriented downward)

$$\iint_D \vec{F}\cdot d\vec{S} = \iint_D F(G(r,\theta))\cdot N\, dA$$
$$= \int_0^{2\pi}\int_0^1 \langle 0, \tfrac{1}{3}(r\sin\theta)^3, (r\sin\theta)^2\rangle \cdot \langle 0,0,-r\rangle\, dr\, d\theta$$
$$= \int_0^{2\pi}\int_0^1 -r^3\sin^2\theta\, dr\, d\theta = \int_0^{2\pi} \sin^2\theta\, d\theta \int_0^1 -r^3\, dr$$
$$= \pi\left[-\tfrac{1}{4}r^4\right]_0^1 = \boxed{-\tfrac{1}{4}\pi}$$

(b) Let $\mathcal{S}$ be the top half of the sphere $x^2+y^2+z^2=1$, oriented upward. Compute $\iint_{\mathcal{S}} \boldsymbol{F}\cdot dS$.

$\boxed{\text{Box your answer}}$ *Hint:* you should use your answer to part (a). If you cannot do part (a), let A denote the value of the integral in part (a) and give your answer in terms of A.

$\iint_D \vec{F}\cdot d\vec{S} = -\frac{1}{4}\pi$ from (a) for a unit disk D oriented downward.

$$\iint_{S+D} \vec{F}\cdot d\vec{S} = \iiint_W \operatorname{div}(\vec{F})\, dV$$
$$= \iiint_W 3z^2+y^2+x^2\, dV$$
$$= \iiint_W 1\, dV$$

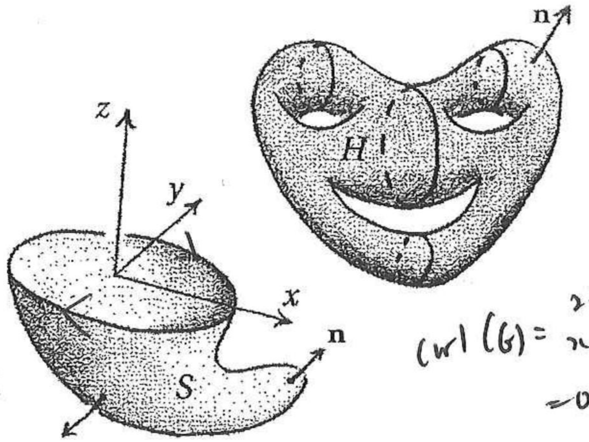
volume of W $= \frac{\frac{4}{3}\pi}{2} = \frac{2}{3}\pi$

$$\iint_{S+D} \vec{F}\cdot d\vec{S} = \iint_S \vec{F}\cdot d\vec{S} + \iint_D \vec{F}\cdot d\vec{S} = \frac{2}{3}\pi$$

$$\iint_S \vec{F}\cdot d\vec{S} = \frac{2}{3}\pi - \left(-\frac{1}{4}\pi\right) = \boxed{\frac{11}{12}\pi}$$

7. (15 points) Multiple choice. Circle the correct answer.

Let S and H be the surfaces shown to the right. The boundary of S is the unit circle in the xy-plane, while H has no boundary. Let $\boldsymbol{G} = \langle x, y, z \rangle$.



$\text{curl}(G) = \begin{vmatrix} \partial x & \partial y & \partial z \\ x & y & z \end{vmatrix} = 0$

(a) The flux $\iint_H \boldsymbol{G} \cdot d\boldsymbol{S}$ is

negative zero positive

(b) The flux $\iint_S \boldsymbol{G} \cdot d\boldsymbol{S}$ is

negative zero positive

$\iiint \text{div}(G)\, dV = \iiint_W 3\, dV$

Hint for (b): use the divergence theorem.

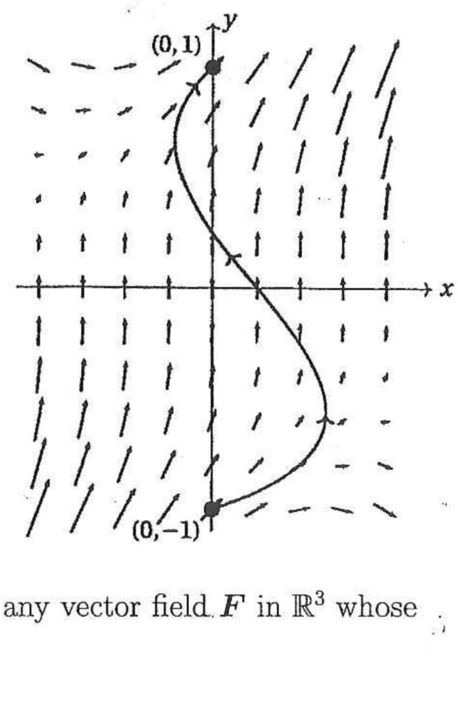
(c) The flux $\iint_S \text{curl}\, \boldsymbol{G} \cdot d\boldsymbol{S}$ is

negative zero positive

(d) A vector field is shown to the right. For scale, $\boldsymbol{F}(0,0) = \langle 0, 0.1 \rangle$.

Given that <u>$\boldsymbol{F}$ is conservative</u>, estimate $\int_C \boldsymbol{F} \cdot d\boldsymbol{r}$, where C is the curve shown from $(0,-1)$ to $(0,1)$.

-0.5 -0.2 0 0.2 0.5



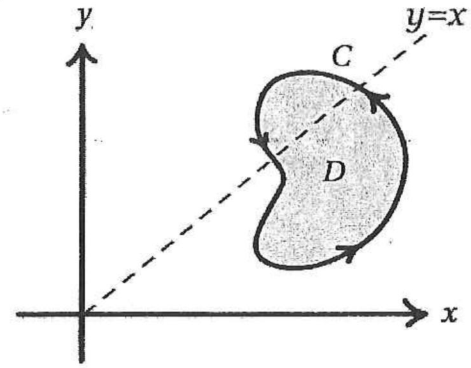
(e) Which of the following statements makes sense and is true for any vector field $\boldsymbol{F}$ in $\mathbb{R}^3$ whose components have continuous second-order partial derivatives?

$\nabla(\text{curl}\, \boldsymbol{F}) = 0$ $\text{div}(\text{curl}\, \boldsymbol{F}) = 0$ $\text{div}(\nabla \boldsymbol{F}) = 0$ $\text{curl}(\text{curl}\, \boldsymbol{F}) = 0$

8. (15 points) Multiple choice. Circle the correct answer.

Consider the region D in the plane bounded by the curve C as shown to the right. For parts (a)–(c), circle the best answer.

$\iint_D \mathrm{curl}_z(F) = \iint 0$



(a) For $\boldsymbol{F}(x,y) = \langle x+1, y^2\rangle$, the integral $\int_C \boldsymbol{F}\cdot d\boldsymbol{r}$ is

negative   zero   positive

(b) The integral $\int_C (-y\,dx + 2dy)$ is $\iint 1$

negative   zero   positive

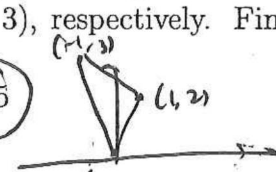
(c) The integral $\iint_D (y-x)\,dA$ is

negative   zero   positive

Hint for (c): look at the location of D in the plane.

(d) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation of the plane sending the triangle with vertices $(0,0)$, $(1,0)$, $(0,1)$ to the triangle with vertices $(0,0)$, $(1,2)$, $(-1,3)$, respectively. Find the Jacobian of T.

1   2   3   4   5



Δ area $= \frac{5}{2}$

$\frac{5/2}{1/2} = 5$

$\sqrt{5}\ \sqrt{5}$

$5 = \frac{5}{2}$

$(1,2)$

$(-2,1) = 0$

(e) Let $\mathcal{R} = [1,2]\times[1,2]$ and let $\mathcal{D} = G(\mathcal{R})$, where G is the map $G(u,v) = (u^2/v, v^2/u)$. Compute the area of $\mathcal{D}$.

1   2   3   4   5

area$(R) = 1\times 1 = 1$

$$\mathrm{Jac}(G) = \begin{vmatrix} \frac{2u}{v} & -\frac{u^2}{v^2} \\ -\frac{v^2}{u^2} & \frac{2v}{u} \end{vmatrix} = 4 - \frac{u^2v^2}{v^2u^2}$$

$= 4-1 = 3$

9. (10 points) Fill in the blanks in the big theorems of vector calculus.

The fundamental theorem of line integrals. If C is an oriented curve from P to Q in $\mathcal{D}$ then

$$\int_C \nabla f \cdot dr = f(Q) - f(P).$$

Green's theorem. Let $\mathcal{D}$ be a domain whose boundary ∂D is a simple closed curve, oriented $\boxed{\text{counterclockwise}}$. Then

$$\iint_{\mathcal{D}} \boxed{\mathrm{curl}_z(F)}\, dA = \oint_{\boxed{\partial D}} \boldsymbol{F} \cdot d\boldsymbol{r}.$$

Stokes' theorem. Let $\mathcal{S}$ be a "sufficiently nice" surface, and let $\boldsymbol{F}$ be a vector field whose components have continuous partial derivatives on an open region containing $\mathcal{S}$. Then

$$\iint_S \mathrm{curl}(\boldsymbol{F}) \cdot d\boldsymbol{S} = \boxed{\oint_{\partial S} \vec{F} \cdot d\vec{r}}.$$

The integral on the right-hand side is defined relative to the boundary orientation of ∂S.

The divergence theorem. Let $\mathcal{S}$ be a closed surface that encloses a region $\mathcal{W}$ in $\mathbb{R}^3$. Assume that $\mathcal{S}$ is piecewise smooth and is oriented by normal vectors pointing $\boxed{\text{outwards}}$. Let $\boldsymbol{F}$ be a vector field whose domain contains $\mathcal{W}$. Then

$$\boxed{\iiint_W \mathrm{div}(\vec{F})\, dV} = \iint_{\partial W} \boldsymbol{F} \cdot d\boldsymbol{S}.$$

You may use this page for scratch work.