Math 32A – Midterm 1

February 5th, 2014

Your name: [redacted]
Your SID:

There are four problems on this exam, each worth ten points. You have fifty minutes. Good luck!

1	10
2	6
3	10
4	10
Total	36

Problem 1: Find the area of the parallelogram spanned by the vectors $\mathbf{u} = \langle 2, 3, 2 \rangle$ and $\mathbf{v} = \langle 2, 0, 1 \rangle$.

[redacted]

$\|v \times u\|$

$$v \times u = \begin{bmatrix} i & j & k \\ 2 & 0 & 1 \\ 2 & 3 & 2 \end{bmatrix} = \det\begin{bmatrix} 0 & 1 \\ 3 & 2 \end{bmatrix}\hat{\imath} - \det\begin{bmatrix} 2 & 1 \\ 2 & 2 \end{bmatrix}\hat{\jmath} + \det\begin{bmatrix} 2 & 0 \\ 2 & 3 \end{bmatrix}\hat{k}$$

10

$$v \times u = \langle -3\hat{\imath} - 2\hat{\jmath} + 6\hat{k}$$

$$\text{Area} = \|v \times u\| = \| \langle -3, -2, 6 \rangle \|$$

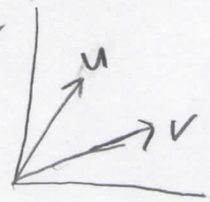
$$\|v \times u\| = \sqrt{9 + 4 + 36}$$

$$\|v \times u\| = \sqrt{49}$$

$$\boxed{\|v \times u\| = 7}$$

6/10

in plane



Problem 2: Let P be a plane in space with normal vector $\mathbf{n}$, and let $\mathbf{u}$ and $\mathbf{v}$ both be vectors, based at the origin, with end points that lie in the plane P. Find $(\mathbf{u}-\mathbf{v}) \cdot \mathbf{n}$, and explain your reasoning.

$$(u-v)\cdot n$$

$$u\cdot n - v\cdot n$$

$$= \boxed{0}$$

This is because the normal vector is orthogonal to the plane. Since u & v both lie in the plane n is orthogonal to both u & v, There fore u·n & v·n both equal 0.

10

Problem 3: Consider an object moving in the plane along the path $c(t) = \langle 3\cos(t), \sin(t)\rangle$. What is the object's maximum speed? What is its minimum speed? Express your answers as numbers, no units are necessary.

$$C'(t) = \langle -3\sin t, \cos t \rangle$$

$$S = \|C'(t)\| = \sqrt{9\sin^2 t + \cos^2 t}$$

note
S=Speed $\rightarrow$ $S_{(t)} = \sqrt{9\sin^2 t + \cos^2 t}$ $\quad (9\sin^2 t + \cos^2 t)^{1/2}$

$$\frac{dS}{dt} = \frac{1}{2}(9\sin^2 t + \cos^2 t)^{3/2}(18\sin^2 t \cdot \cos t)(-2\cos^2 t \cdot \sin t)$$

Values of t that make this function 0 are:

$$t = 0, \frac{\pi}{2}$$

$$S_{(0)} = \sqrt{9\sin^2(0) + \cos^2 0}$$

$$\boxed{S(0) = 1 \leftarrow \text{min}}$$

$$S\left(\frac{\pi}{2}\right) = \sqrt{9\sin^2\left(\frac{\pi}{2}\right) + \cos^2\frac{\pi}{2}}$$

$$\boxed{S\left(\frac{\pi}{2}\right) = 3 \leftarrow \text{max}}$$

10/10

Problem 4: Let $c(t)$ be the parametrized path $c(t) = \langle \cos(t), \sin(t), t \rangle$. Find an arc length parametrization for this curve, and use it to calculate the curvature at the point $(1, 0, 0)$.

$$c'(t) = \langle -\sin(t), \cos(t), 1 \rangle$$

$$\|c'(t)\| = \sqrt{\sin^2 t + \cos^2 t + 1}$$

$$\|c'(t)\| = \sqrt{2}$$ ✓

$$s(t) = \int_0^t \sqrt{2}\, du$$

$$s(t) = \sqrt{2}\, t$$

$$\frac{s}{\sqrt{2}} = t(s)$$

$$c_1(s) = \left\langle \cos\left(\frac{s}{\sqrt{2}}\right), \sin\left(\frac{s}{\sqrt{2}}\right), \frac{s}{\sqrt{2}} \right\rangle$$ ✓

$$c_1'(s) = \frac{1}{\sqrt{2}} \left\langle -\sin\left(\frac{s}{\sqrt{2}}\right), \cos\left(\frac{s}{\sqrt{2}}\right), 1 \right\rangle$$

$$c_1''(s) = \frac{-1}{2} \left\langle -\cos\left(\frac{s}{\sqrt{2}}\right), \sin\left(\frac{s}{\sqrt{2}}\right), 0 \right\rangle$$

$$\|c''(s)\| = \sqrt{\frac{1}{4}\cos^2\left(\frac{s}{\sqrt{2}}\right) + \frac{1}{4}\sin^2\left(\frac{s}{\sqrt{2}}\right)}$$

$$= \boxed{\frac{1}{2}}$$ ✓

curvature everywhere so therefore (1, 0, 0)