3. (10pt) Let $f(x,y) = \frac{1}{\sqrt{2+xy+y}}$.

(a) (5pt) Find the tangent plane to the function f at $P = (1,1)$.

$$z = d_x f(a,b)(x-a) + d_y f(a,b)(y-b)$$

$$f(x,y) = (2+xy+y)^{-1/2}$$

$$d_x f = -\tfrac{1}{2}(2+xy+y)^{-3/2} \cdot (y)$$

$$d_x f(1,1) = -\tfrac{1}{2}(2+1+1)^{-3/2} \cdot 1 = -\tfrac{1}{2}(4)^{-3/2} \cdot 1 = -\frac{1}{2^4} = -\frac{1}{16}$$

$$d_y f = -\tfrac{1}{2}(2+xy+y)^{-3/2} \cdot (x+1)$$

$$d_y f(1,1) = -\tfrac{1}{2}(2+1+1)^{-3/2} \cdot (1+1) = -\tfrac{1}{2}(4)^{-3/2} \cdot 2 = -\frac{1}{2^3} = -\tfrac{1}{8}$$

$$\Rightarrow \quad z = -\tfrac{1}{16}(x-1) - \tfrac{1}{8}(y-1) + \frac{1}{2}$$

4

(5pt) Find the **Linear approximation** $L(x, y)$ for $f(x, y)$ at $P = (1, 1)$. Use it to approximate the value $f(1.2, 0.9)$.

$L(x,y) \approx f(x_0,y_0) + d_x f(x_0,y_0)(x-x_0) + d_y f(x_0,y_0)(y-y_0)$

$d_x f(x_0,y_0) = -1/16$

$d_y f(x_0,y_0) = -1/8$

$f(x_0,y_0) = \frac{1}{\sqrt{2+xy+y}} = \frac{1}{\sqrt{2+1+1}} = \frac{1}{\sqrt{4}} = \frac{1}{2}$

$L(1.2, 0.9) \approx \frac{1}{2} - \frac{1}{16}(0.2) - \frac{1}{8}(-0.1)$ OE

5

Exam continues...

4. (10pt)

(a) (5pt) Let $f_x(x,y) = 2x + y$ and $f_y(x,y) = y^2 + x$. **Check the condition from Clairaut's theorem.** Solve for $f(x,y)$ when $f(1,1) = 4$.

$f_{xy} = 1$
$f_{yx} = 1$ $> 1 = 1$ ✓

5

$f(x,y) = x^2 + yx + g(u)$

$f_y(x,y) = x + g'(u) \rightarrow g'(u) = y^2 \rightarrow g(u) = \frac{y^3}{3} + C$

$f(x,y) = x^2 + xy + \frac{y^3}{3} + C$

$f(1,1) = 4 = 1^2 + 1 + \frac{1^3}{3} + C = 1 + 1 + \frac{1}{3} + C = 2 + \frac{1}{3} + C \rightarrow C = \frac{5}{3}$

$\rightarrow f(x,y) = x^2 + xy + \frac{y^3}{3} + \frac{5}{3}$

(b) (5pt) (Harder, do last). Find all the values b, c, d such that

$$v(x, y) = x^4 + cx^3y + bx^2y^2 + dxy^3 + y^4,$$

satisfies the equation

$$v_{xx} + v_{yy} = 0.$$

3

$V_x = 4x^3 + 3cx^2y + 2bxy^2 + dy^3$

$V_{xx} = 12x^2 + 6cxy + 2by^2$

$V_y = cx^3 + 2bx^2y + 3dxy^2 + 4y^3$

$V_{yy} = 2bx^2 + 6dxy + 12y^2$

$V_{yy} = 12y^2 + 6dxy + 2bx^2$

$V_{xx} + V_{yy} = 0 \rightarrow V_{xx} = -V_{yy}$

$-V_{yy} = -12y^2 - 6dxy - 2bx^2$

$\rightarrow$ $V_{xx} = -V_{yy}$

$12x^2 + 6cxy + 2by^2 = -12y^2 - 6dxy - 2bx^2$

$12x^2 - 2bx^2 + 12cxy = -12y^2 + 2by^2$

$(12-2b)x^2 + 12cxy = (-12+2b)y^2$

$y = \sqrt{\dfrac{(12-2b)x^2 + 12cxy}{-12+2b\,x^2}}$

End of exam

Math 32 A: Multi-Variable Calculus
Fall 2018
University of California, Los Angeles
Exam: Midterm 2
Date: Friday 16/11/2018
Time Limit:
Instructor: Sylves

Problem	Max Points	Points
1	10	10
2	10	[illegible]
3	10	9
4	10	

33

Student (

Please circle your discussion

	Bon-Soon	Eric	Tianqi
Tuesday	1E	1C	1A
Thursday	1F	1D	1B

Policies/Instructions:

- **Remember to write your name and student ID!!**
- No calculators, or books.
- Fit your answer in the space provided. Use the scratch sheets to do calculations before writing.
- Mobile phones turned off and in bags.
- Be considerate; Must remain seated until end of exam.
- Write clearly. You may use the backs of the pages, and the additional last page if needed.
- Check your work. Answer all questions. Make sure you chose between True and False.
- All problems graded on correctness and demonstrating work. **Must show steps and explain to receive full credit.**
- **Good luck!**

"Do not let what you cannot do interfere with what you can do."

Coach John Wooden

10

1. (10pt) For each of the following answer **briefly**. No need to justify unless specifically asked. Each worth two points.

   (a) (2pt) If the scalar normal component of the acceleration vanishes, that is $a_N = 0$, then what can you say about the motion of $\mathbf{r}$? Justify briefly.

Straight line

the velocity never changes direction as any a would all be in a_T

so if velocity doesn't change direction then the path has the same direction throughout

   (b) (2pt) If $\mathbf{r}''' = 0$, then indicate all of the following that remain constant: the speed $v(t)$, the curvature $\kappa(t)$ and/or the torsion $\tau(t)$, or none of these. **For the ones that are constant, explain why?** (Do not have to explain why not constant.) You can think about the curve $\mathbf{r}(t) = \langle 1+t+t^2, t, t^2+1 \rangle$, but do not do detailed computations as that would waste time.

torsion is constant

$\tau = \frac{r' \cdot (r'' \times r''')}{\|r' \times r''\|^2}$

$r'' \times r''' = 0$ since r''' is 0

$\to r' \cdot 0 = 0$

$\to \tau$ is constant

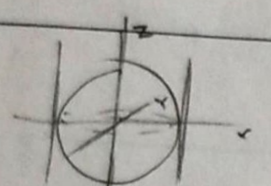
(c) (1pt) Express in one sentence, what assumption on the function f is required for the directional derivative formula $\mathbf{D}_{\mathbf{u}} f = \nabla f \bullet \mathbf{u}$ to hold.

f must be differentiable in direction of u

ok

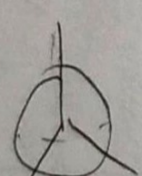
(d) (3pt) Let $x^2 + y^2 + z^2 = 1$ describe a unit sphere. At how many, and which, points (x, y, z) is the tangent plane parallel to the yz-plane, that is, has a normal parallel to $\mathbf{i} = \langle 1, 0, 0 \rangle$. Hint: Draw a picture.

2



(1, 0, 0)

(-1, 0, 0)



(e) (2pt) Fill in the sentence in two different ways: The gradient $\nabla f(x, y, z)$ is a vector that ______

is normal to the level surface

is in the direction of greatest increase at a given point on ~~the~~ level surface

2. (10pt)

(a) (5pt) Let $\mathbf{r}(t) = \langle \sin(4t), \cos(4t), 3t \rangle$. Find a unit speed parametrization of this curve.

$r'(t) = \langle 4\cos(4t), -4\sin(4t), 3 \rangle$

$\|r'(t)\| = \sqrt{16\cos^2(4t) + 16\sin^2(4t) + 9} = \sqrt{16+9} = \sqrt{25} = 5$

$s = \int_0^t \|r'(t)\|\, dt = \int_0^t 5\, dt = 5t$

$t = \frac{s}{5}$

$\Rightarrow r(s) = \langle \sin\left(\frac{4s}{5}\right), \cos\left(\frac{4s}{5}\right), \frac{3s}{5}$

(b) (5pt) Let $f(x, y, z) = x^2 + xy + y^2 + xz$, and consider the curve from before. Find

$$\frac{d}{dt} f(\mathbf{r}(t))\Big|_{t=\pi}.$$

Also, find the **directional derivative**

$$\mathbf{D_u} f(P)$$

at $P = \mathbf{r}(\pi)$ in the direction of the unit vector $\mathbf{u}$ parallel to $\mathbf{r}'(t)$. That is, find the directional derivative of f at $\mathbf{r}(\pi)$ in the direction of $\mathbf{r}'$.

$\frac{d}{dt} f(r(t))\Big|_{t=\pi} = f'(r(\pi))$

$r(\pi) = \langle \sin(4\pi), \cos(4\pi), 3\pi \rangle = \langle 0, 1, 3\pi \rangle$

$r'(\pi) = \langle 4\cos(4\pi), -4\sin(4\pi), 3 \rangle = \langle 4, 0, 3 \rangle$

$f'(r(\pi)) = \nabla f(r(\pi))$

$\nabla f = \langle f_x, f_y, f_z \rangle$

$f_x = 2x + y + \cancel{z}$

$f_x(4,0,3) = 8 + 0 + 3 = 11$

$f_y = x + 2y$

$f_y(4,0,3) = 4 + 2(0) = 4$

$f_z = x$

$f_z(4,0,3) = 4$

$\rightarrow \nabla f(4,0,3) = f'(r(\pi)) = \langle 11, 4, 4 \rangle$

$\frac{d}{dt} f(r(t))\Big|_{t=\pi} = \langle 11, 4, 4 \rangle$

X −2

−1

$u = T(\pi) = \frac{r'(\pi)}{\|r'(\pi)\|} = \langle \frac{4}{5}, 0, \frac{3}{5} \rangle$

$D_u f(P) = \nabla f(P) \cdot u = \langle 11, 4, 4 \rangle \cdot \langle 4/5, 0, 3/5 \rangle = \frac{44}{5} + 0 + 12/5 = \boxed{\frac{56}{5}}$ X −1

Exam continues...