1. (a) (3 points) Let $f(x,y) = x^2 + y^2$. Compute the partial derivative f_{xx}.

$f_x = 2x$

$\boxed{f_{xx} = 2}$

(b) (5 points) Let $f(u,v,w,x,y,z) = u^2/v + vxyz + \sin(xwv)$. Compute the partial derivative f_{xvz}.

$f_x = vyz + (\sin(x\,wv))(wv)$

$f_{xv} = yz + (\sin(xwv))(w) + (wv)(\sin(xwv))(xw)$

$f_{xvz} = y + 0$

$\boxed{f_{xvz} = y}$

(c) (5 points) Compute the following limit:

$$\lim_{(x,y)\to(0,2)} (1+x)^{y/x}.$$

$\lim_{(x,y)\to(0,2)} \left((1+x)^{\frac{1}{x}}\right)^y$

$\lim_{y\to 2} e^y = \boxed{e^2}$

2. (10 points) Let $f(x,y) = x^2y^3$. Compute the gradient $\nabla f(x,y)$. Then, find the tangent plane to the surface $z = f(x,y)$ at the point $(a,b) = (2,3)$.

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$f_x = 2xy^3$

$f_y = 3x^2y^2$

$\nabla f = (f_x, f_y) = (2xy^3, 3x^2y^2)$

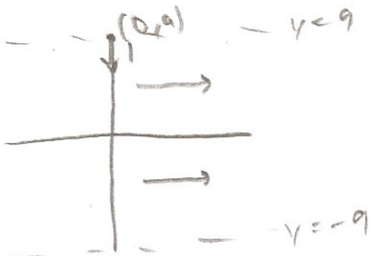
$f(a,b) = (2)^2(3)^3 = (4)(27) = 108$

$\nabla f(a,b) = (2(2)(3)^3, 3(2)^2(3)^2) = (108, 108)$

$z = f(a,b) + ((x,y) - (a,b)) \cdot \nabla f(a,b)$

$\boxed{z = 108 + (x-2)(108) + (y-3)(108)}$ ✓

3. (10 points) Suppose your initial position in the plane is $(0, 9)$. Between the lines $y = 9$ and $y = -9$ is a river. The river's speed at the point (x, y) is $81 - y^2$, where the river runs in the direction of the positive x-axis. Suppose you are in a boat which begins with a constant speed of 1, in the negative y-direction. (So, the velocity of the boat in the y-direction will always be -1.) What will be your position when you reach the bottom of the river? That is, what is your position when you reach the line $y = -9$?



$\frac{dy}{dt} = -1$ +2

$\frac{dx}{dt} = 81 - y^2$

$dt = \frac{dy}{-1}$

$\int dx = \int 81 - y^2 \, dt$

$x = -\int_{-9}^{9} 81 - y^2 \, dy$

$x = -81y + \frac{y^3}{3} \Big]_{-9}^{9}$

$x = 729 - \frac{729}{3} - \left(-729 + \frac{729}{3}\right)$

$729 + 729 - \frac{729}{3} - \frac{729}{3}$

$1458 - \frac{1458}{3}$

$x = \frac{2}{3}(1458)$

$\boxed{(x, y) = \left(\frac{2}{3}(1458), -9\right)}$

4. (10 points) Find a function $f(x, y)$ such that

$$\frac{\partial f}{\partial x} = 1 + e^x \cos y,$$

$$\frac{\partial f}{\partial y} = 14y - e^x \sin y,$$

and such that $f(\ln 2, 0) = \ln 2$. (As usual, you must show your work to receive full credit.)

$\frac{\partial F}{\partial x} = 1 + e^x \cos y$ $\qquad$ $\frac{\partial f}{\partial y} = 14y - e^x \sin y$

$x + e^x \cos y$ $\qquad$ $7y^2 + e^x \cos y$

why? $\quad -1$

$f(x,y) = x + 7y^2 + e^x \cos y + C$

$f(\ln 2, 0) = \ln 2 = \ln 2 + 0 + e^{\ln 2} \cos 0 + C$

$\ln 2 + 2 + C = \ln 2$

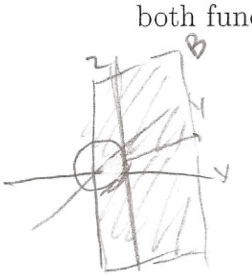
$C = -2$

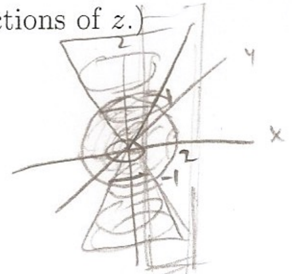
$$\boxed{f(x,y) = x + 7y^2 + (e^x \cos(y)) - 2}$$

5. (15 points) Let D be the solid region in Euclidean space $\mathbf{R}^3$ defined as the set of all (x, y, z) such that $x^2 + y^2 + z^2 \leq 4$, $x^2 + y^2 - 3z^2 \geq 0$ and $x^2 + y^2 \geq 1$. Note that D is a solid region, so its boundary is a surface. Let E denote the boundary surface of D. (If the solid region D were dipped in paint, then the boundary of D is the outer part of D that is covered in paint.)

Let B be the region in Euclidean space $\mathbf{R}^3$ defined as the set of all (x, y, z) such that $y = x$, $x \geq 0$ and $y \geq 0$. Then E and B are surfaces.

Parametrize the intersection of E and B. (Make sure to parametrize the entire intersection. You **MUST** specify the domain of your parameter for any parametrization you give.) (Any parametrization that you write **MUST** use z as a parameter. That is, any parametrization you write must be of the form $r(z) = (x(z), y(z), z)$, where x and y are both functions of z.)





$x^2 + y^2 + z^2 = 4$

$x^2 + y^2 = 3z^2$

$x^2 + y^2 \geq 1$

$4z^2 = 4$

$z^2 = 1$

$z = \pm 1$

$x = \sqrt{\frac{3}{2} z^2} = |z|\sqrt{\frac{3}{2}}$

$y = x \Rightarrow 2x^2 = 3z^2$

$2y^2 = 3z^2$

$y = |z|\sqrt{\frac{3}{2}}$

$r(z) = \left(z\sqrt{\frac{3}{2}}, z\sqrt{\frac{3}{2}}, z\right)$ $z \in [\cancel{1}, \cancel{\infty})$

$r(z) = \cancel{\left(-z\sqrt{\frac{3}{2}}, -z\sqrt{\frac{3}{2}}, z\right)}$ $z \in (-\infty, -1]$

$2x^2 + z^2 = 4$

$2x^2 = 4 - z^2$

$x = \sqrt{\frac{4 - z^2}{2}}$

$y = \sqrt{\frac{4 - z^2}{2}}$

$$\boxed{r(z) = \left(\sqrt{\frac{4-z^2}{2}}, \sqrt{\frac{4-z^2}{2}}, z\right) \ \cancel{z \in (-1, 1)}}$$

$4 - z^2 = 1$

$-z^2 = -3$

$z^2 = 3$

$z = \pm\sqrt{3}$

$\sqrt{\frac{4 - z^2}{2}} \geq 1$

$\frac{4 - z^2}{2} = 1$

$4 - z^2 = 2$

$2 = z^2$

$z = \pm\sqrt{2}$

N = 2