1. (a) (3 points) Find the unit vector which points in the same direction as $(1,2,3)$.

$$e_v = \frac{v}{\|v\|} = \frac{(1,2,3)}{\|(1,2,3)\|} = \frac{(1,2,3)}{\sqrt{14}} = \left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right)$$

✓

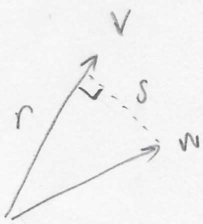
(b) (5 points) Let v, w be vectors in $\mathbf{R}^3$ such that $\|v\| = 2$, $\|w\| = 3$ and such that $v \cdot w = 0$. Find $\|v + w\|$.

$\|v\|^2 = v \cdot v$ $\quad \|v\| = \sqrt{v \cdot v}$

$$\|v+w\| = \sqrt{(v+w)\cdot(v+w)}$$
$$= \sqrt{(v\cdot v + \cancel{2v\cdot w} + w \cdot w)}$$
$$= \sqrt{v\cdot v + w \cdot w}$$
$$= \sqrt{\|v\|^2 + \|w\|^2} = \sqrt{4+9} = \sqrt{13}$$

✓

(c) (5 points) Let $v = (1,2,0)$ and let $w = (2,1,3)$. Write w as the sum of two vectors r, s, so that $w = r + s$, and such that r is parallel to v, and s is perpendicular to v.



$r \parallel v \quad s \perp v$

$$w = \mathrm{proj}_v w + (w - \mathrm{proj}_v w)$$
$$= \frac{w\cdot v}{v \cdot v} v + \left(w - \frac{w \cdot v}{v\cdot v} v\right)$$

$(2,1,3)\cdot(1,2,0) = 4$

$(1,2,0)\cdot(1,2,0) = 5$

$$= \frac{4}{5}(1,2,0) + \left((2,1,3) - \frac{4}{5}(1,2,0)\right) = \left(\frac{4}{5}, \frac{8}{5}, 0\right) + \left((2,1,3) - \left(\frac{4}{5}, \frac{8}{5}, 0\right.\right.$$

$$w = \underbrace{\left(\frac{4}{5}, \frac{8}{5}, 0\right)}_{r} + \underbrace{\left(\frac{6}{5}, -\frac{3}{5}, 3\right)}_{s}$$

✓

$(1,2,1)$ $(1,-1,-2)$

2. (10 points) Find the angle between the planes $x+2y+z=1$ and $x-y-2z=0$. Then, find a parametrization for the line of intersection of these planes.

$x+2y+z=1 \Rightarrow \text{normal} = (1,2,1) = n_1$

$x-y-2z=0 \Rightarrow \text{normal} = (1,-1,-2) = n_2$

Angle betw. planes = angle. betw. norms.

$1+(-1)(2)+(1)(-2) = -3$

$$\theta = \cos^{-1}\left(\frac{n_1 \cdot n_2}{\|n_1\| \cdot \|n_2\|}\right) = \cos^{-1}\left(\frac{(1,2,1)\cdot(1,-1,-2)}{\|(1,2,1)\| \cdot \|(1,-1,-2)\|}\right)$$

$$= \cos^{-1}\left(\frac{-3}{\sqrt{6}\cdot\sqrt{6}}\right) = \cos^{-1}\left(\frac{-3}{6}\right) = \cos^{-1}\left(\frac{-1}{2}\right)$$

$\theta = \cos^{-1}\left(\frac{-1}{2}\right)$

$\theta = \frac{2\pi}{3}$

line of intersection:

$$n_1 \times n_2 = \begin{pmatrix} (1,0,0) & (0,1,0) & (0,0,1) \\ 1 & 2 & 1 \\ 1 & -1 & -2 \end{pmatrix}$$

$= (1,0,0)(-4+1) - (0,1,0)(-2-1) + (0,0,1)(-1-2)$

$= (-3, 3, -3)$ is parallel to line

intersection of planes: $x = y+2z$ $\quad x+2y+z=1$

$y+2z+2y+z=1$

$3y+3z=1$ $\quad$ A point on both planes $= \cancel{(0,0,\frac{1}{3})}$ -1

$s(t) = \cancel{(0,0,\frac{1}{3})} + t(-3,3,-3)$

3. In each of the following items, an equation is listed. This equation describes a quadric surface in $\mathbf{R}^3$. That is, each equation describes a set of points (x, y, z) in three-dimensional Euclidean space. For each such equation, identify the type of surface to which the equation corresponds. You can choose from the following labels: Paraboloid, Hyperboloid, Cone, Ellipsoid, Cylinder. You do not need to show any work in this section. No partial credit will be given in this section.

(a) (2 points) $2x^2 + 3y^2 = 1 - z^2$ $\quad 2x^2+3y^2+z^2=1$

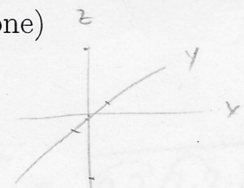
PARABOLOID HYPERBOLOID CONE ELLIPSOID CYLINDER
(circle one)

(b) (2 points) $z^2 - 2y^2 + 10z^2 = 0$ $\quad 11z^2-2y^2=0 \quad 11z^2=2y^2$

~~PARABOLOID~~ HYPERBOLOID CONE ~~ELLIPSOID~~ ~~CYLINDER~~
(circle one)

(c) (2 points) $2x^2 + 3y^2 = z^2 - 1$

PARABOLOID HYPERBOLOID CONE ELLIPSOID CYLINDER
(circle one)



$x^2+y^2+z^2=1$

(d) (2 points) $z^2 + 100y^2 = 100$

PARABOLOID HYPERBOLOID CONE ELLIPSOID CYLINDER
(circle one)

(e) (2 points) $y^2 + 4z^2 = x^2 + 1$

PARABOLOID HYPERBOLOID CONE ELLIPSOID CYLINDER
(circle one)

$4x = y^2 + \frac{z^2}{4}$ Hyperbolic Paraboloid

(f) (2 points) $4x = y^2 + z^2/4$

PARABOLOID HYPERBOLOID CONE ELLIPSOID CYLINDER
(circle one)

4. (10 points) Consider the following three points in $\mathbf{R}^5$:

$$(5,3,2,3,1), \quad (5,3,0,1,2), \quad (5,3,0,1,0).$$

$-5\ 3\ 0\ 1\ 0 \quad -5\ 3\ 0\ 1\ 0 \quad -5\ 3\ 0\ 1\ 0$

Consider the triangle which has these three points as its vertices. Find the area of this triangle.

$A = \frac{1}{2}\|v \times w\|$

Translate 3 points to origin $\Rightarrow$ subtract $(5,3,0,1,0)$

$\Rightarrow (0,0,2,2,1),\ (0,0,0,0,2),\ (0,0,0,0,0)$

$$v \times w = \begin{pmatrix} (1,0,0) & (0,1,0) & (0,0,1) \\ 2 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix}$$

$= (1,0,0)(4-0) - (0,1,0)(4-0) + (0,0,1)(0)$

$= (4,-4,0)$

$\|v \times w\| = \sqrt{(4)^2+(-4)^2+0^2} = \sqrt{32} = \sqrt{2\cdot 2\cdot 2\cdot 2\cdot 2}$

$= 4\sqrt{2}$

$\frac{1}{2}\|v \times w\| = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$

5. (10 points) Compute:

$$\overbrace{\left[\underbrace{(1,2,1) \times \left(\overbrace{\overset{1}{[(1,2,1) \times (3,4,1)]} \times \overset{2}{[(1,2,1) \times (5,7,9)]}}^{3}\right)}_{4}\right] \times (4,10,3)}^{5}.$$

Hint: There is a way to do this problem without doing any computations.

Make sure to justify your answer. (If you compute all of these cross products explicitly, and you get the wrong answer, you will get at most half credit.)

① $(1,2,1) \times (3,4,1) = \begin{pmatrix} (1,0,0) & (0,1,0) & (0,0,1) \\ 1 & 2 & 1 \\ 3 & 4 & 1 \end{pmatrix} = (-2,2,-2)$

② $(1,2,1) \times (5,7,9) = \begin{pmatrix} (1,0,0) & (0,1,0) & (0,0,1) \\ 1 & 2 & 1 \\ 5 & 7 & 9 \end{pmatrix} = (11,-4,-3)$

③ $(-2,2,-2) \times (11,-4,-3) = \begin{pmatrix} (1,0,0) & (0,1,0) & (0,0,1) \\ -2 & 2 & -2 \\ 11 & -4 & -3 \end{pmatrix} = (-14,-28,-14)$

④ $(1,2,1) \times (-14,-28,-14) = \begin{pmatrix} (1,0,0) & (0,1,0) & (0,0,1) \\ 1 & 2 & 1 \\ -14 & -28 & -14 \end{pmatrix}$

$= (0,0,0).$

⑤ $(0,0,0) \times (4,10,3) = (0,0,0) = 0$