Math 32A
Multivariable Calculus

Midterm 2

Instructions: You have 50 minutes to complete this exam. You may bring one page of handwritten notes (one side of one standard sheet). No calculators. The score will be out of 40. Write legibly.

For full credit, show all work and justify your answers. **Box your answers.** Write your solutions in the space below the questions; INDICATE if you go over the page and/or use scrap paper.

Write your name and UID clearly in the space below, and circle your section. Circle your surname (family name), as it is listed on MyUCLA.

Name: [redacted]

Student ID number:

Section:

Question	Points	Score
1	10	
2	12	
3	8	
4	10	
5	2	
Total:	42	

There are 42 available points, but the score is out of 40.

Problem 1.

Consider a particle travelling along the path $2t^{1/2}$

$$\mathbf{r}(t) = \langle t, 2t^2, 2\sqrt{t}+1 \rangle.$$

(a) [2pts.] Find the velocity of the particle at time $t = 1$.

$v(t) = r'(t) = \langle 1, 4t, t^{-1/2} \rangle$

$v(1) = \boxed{\langle 1, 4, 1 \rangle}$

(b) [1pts.] Find the speed of the particle at time $t = 1$.

Speed $= \|v(1)\| = \sqrt{1+16+1} = \sqrt{18} = \boxed{3\sqrt{2}}$

(c) [3pts.] Find the unit tangent vector to the path, at $t = 1$, that is, find $\mathbf{T}(1)$.

$$T(1) = \frac{v(1)}{\|v(1)\|} = \frac{1}{3\sqrt{2}} \langle 1, 4, 1 \rangle$$

$$= \boxed{\left\langle \frac{1}{3\sqrt{2}}, \frac{4}{3\sqrt{2}}, \frac{1}{3\sqrt{2}} \right\rangle}$$

(d) [1pts.] Find the acceleration of the particle at $t = 1$.

$a(t) = v'(t) = \langle 0, 4, -\frac{1}{2} t^{-3/2} \rangle$

$a(1) = \boxed{\langle 0, 4, -\frac{1}{2} \rangle}$

This problem is continued from the previous page.

(e) [3pts.] Find a parametric equation for the tangent line to the path at $t = 1$.

$r(1) + t r'(1)$

$r(1) = \langle 1, 2, 3 \rangle$ $\quad$ $r'(1) = \langle 1, 4, 1 \rangle$

$\langle 1, 2, 3 \rangle + t \langle 1, 4, 1 \rangle$

$\boxed{\langle 1+t, 2+4t, 3+t \rangle}$

$x = 1+t$
$y = 2+4t$
$z = 3+t$

Problem 2.

Match the graphs to the functions **on the other side of your formula sheet**. Don't forget to box your answer. You don't need to show work for full credit, but correct, clearly labelled level curves for f may get partial credit.

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(a) [2pts.] $f(x, y) = |x| + \cos(y)$

$x = a$

$z = |a| + \cos(y)$

a cosine curve

$y = b$

$z = |x| + \cos(b)$

absolute value

$\boxed{B}$

(b) [2pts.] $f(x, y) = x^2 + 2y^2$

$x = a$

$z = 2y^2 + a^2$

upward parabola

$y = b$

$z = x^2 + 2b^2$

upward parabola

$\boxed{E}$

—

(c) [2pts.] $\cos(x + y)$

$x = a$

$z = \cos(a+y)$

cosine curve

between $-1 < \cos(a+y) < 1$

$y = b$

$z = \cos(x+b)$

cosine curve

between $-1 < \cos(a+y) < 1$

$\boxed{C}$

— (d) [2pts.] $\cos(x) + \cos(y)$

$x = a$

$z = \cos(a) + \cos(y)$

a cosine curve with height varying

$\boxed{D}$

$y = b$

$z = \cos(x) + \cos(b)$

a cosine curve

(e) [2pts.] $x^2 - y^2$

$x = a$

$z = -y^2 + a^2$

downward parabola

$\boxed{A}$

$y = b$

$z = x^2 - b^2$

upward parabola

(f) [2pts.] $\cos(|x| + |y|)$.

$\boxed{F}$

$4t^2\cos^2 t - 4t\sin t\cos t - 4t\sin t\cos t + 4\sin^2 t$

Problem 3. *8pts.*

Consider the path $\mathbf{r}(t) = \langle t, t^2, \cos(t) \rangle$. Find the curvature of the path at $t = \pi$.

$r'(t) = \langle 1, 2t, -\sin t \rangle$

$r''(t) = \langle 0, 2, -\cos t \rangle$

$r'(\pi) = \langle 1, 2\pi, 0 \rangle$ $\qquad$ $r''(\pi) = \langle 0, 2, 1 \rangle$

$$\begin{array}{cccccc} 1 & 2\pi & 0 & 1 & 2\pi & 0 \\ 0 & 2 & 1 & 0 & 2 & 1 \end{array}$$

$r'(\pi) \times r''(\pi) = \langle 2\pi, -1, 2 \rangle$

$\| r'(\pi) \times r''(\pi) \| = \sqrt{4\pi^2 + 1 + 4} = \sqrt{4\pi^2 + 5}$

$\| r'(\pi) \| = \sqrt{1 + 4\pi^2}$

$\| r'(\pi) \|^3 = (1 + 4\pi^2)^{3/2}$

$$\boxed{K(t) = \frac{(4\pi^2 + 5)^{1/2}}{(1 + 4\pi^2)^{3/2}}}$$

$15t^{1/2}$ $\qquad \frac{t^{3/2}}{3/2} = \frac{2}{3} t^{3/2}$

Problem 4.

Suppose a particle travels along a path, $\mathbf{r}(t)$, such that the acceleration of the particle is given by $\mathbf{a}(t) = (\sin(t), 2, 15\sqrt{t})$, and such that at time $t = 0$, the initial velocity is $(1, -1, 1)$ and the initial position is $(1, 0, -1)$.

(a) [2pts.] Find the velocity in terms of time.

$v(t) = \langle -\cos t, 2t, 10t^{3/2} \rangle + c$

$v(0) = \langle -1, 0, 0 \rangle + c \qquad v(0) = \langle 1, -1, 1 \rangle$

$c = \langle 2, -1, 1 \rangle$

$v(t) = \boxed{\langle -\cos t + 2, 2t - 1, 10t^{3/2} + 1 \rangle}$

(b) [2pts.] We can write the acceleration at time $t = 0$ as

$$\mathbf{a} = a_T\mathbf{T} + a_N\mathbf{N}$$

for real number a_T and non-negative number a_N with $\mathbf{T}$ the unit tangent vector at ~~π~~ 0, and $\mathbf{N}$ a vector normal to $\mathbf{T}$. In this situation, find a_T.

$T = \frac{v}{\|v\|} \qquad v(0) = \langle 1, -1, 1 \rangle$

$\|v(0)\| = \sqrt{1+1+1} = \sqrt{3}$

$= \langle \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \rangle$

$a(0) = \langle 0, 2, 0 \rangle$

$a_T = a \cdot T = 0 - \frac{2}{\sqrt{3}} + 0 = \boxed{-\frac{2}{\sqrt{3}}}$

This problem is continued from the previous page.

(c) [2pts.] Is the particle speeding up or slowing down at $t = 0$? (Explain your answer.)

$$\cancel{a \cdot T = \|a\| \, \|T\| \cos\theta = \|a\| \cos\theta}$$

$$\cancel{a \cdot T < 0 \quad \text{so} \quad \cos\theta < 0}$$

$a_T = v' =$ change in speed

because $a_T < 0$, $v' < 0$, so change in speed is negative

$\boxed{\text{Particle is slowing down.}}$

(d) [4pts.] Continuing the above situation, find a_N and $\mathbf{N}$ at $t = 0$.

$$T = \left\langle \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right\rangle \qquad a = \langle 0, 2, 0 \rangle$$

$$a_T = -\frac{2}{\sqrt{3}}$$

$$a_N N = a - a_T T = \langle 0, 2, 0 \rangle - \left(-\frac{2}{\sqrt{3}}\right) \left\langle \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right\rangle$$

$$= \langle 0, 2, 0 \rangle - \left\langle -\frac{2}{3}, \frac{2}{3}, -\frac{2}{3} \right\rangle$$

$$= \left\langle \frac{2}{3}, \frac{4}{3}, \frac{2}{3} \right\rangle$$

$$a_N = \|a_N N\| = \sqrt{4/9 + 16/9 + 4/9} = \sqrt{\frac{24}{9}} = \boxed{\frac{2\sqrt{6}}{3}}$$

$$N = \frac{a_N N}{a_N} = \left(\frac{3}{2\sqrt{6}}\right) \left\langle 2/3, 4/3, 2/3 \right\rangle$$

$$= \left\langle \frac{6}{6\sqrt{6}}, \frac{12}{6\sqrt{6}}, \frac{6}{6\sqrt{6}} \right\rangle$$

$$= \boxed{\left\langle \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}} \right\rangle}$$

Problem 5. *2pts.*

(This is a **bonus** problem. Partial credit will only be given for full solutions with only minor arithmetic mistakes.) Consider a particle going along a path

$$\mathbf{r}(t) = (t^3, \sin(t^3) + \cos(t^4), \sin(t) - t).$$

Find the tangent line to the underlying curve at $t = 0$.

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