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ESTD. 2003

Math 32A - Midterm 2

1. a. I certify on my honor that I have neither given nor received any help and that I have not used any non-permitted resources while completing this assignment.

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6. $\vec{r}(t) = \int \vec{v}(t)\,dt = \langle 2t, t^3, -2t^2 \rangle + \vec{c}$

$\vec{r}(0) = \langle 0,0,8 \rangle = \langle 2\cdot 0, 0^3, -2\cdot 0^2 \rangle + \vec{c}$

$\vec{c} = \langle 0,0,8 \rangle$

$\vec{r}(t) = \langle 2t, t^3, -2t^2 + 8 \rangle$

c. $\vec{r}'(t) = \langle 2, 3t^2, -4t \rangle$

$\cancel{\|\vec{r}'(t)\|} =$

$\vec{a}(t) = \langle 0, 6t, -4 \rangle$

$\vec{a}(1) = \langle 0, 6, -4 \rangle$

$\|\vec{r}'(t)\| = \sqrt{2^2 + 9t^4 + 16t^2}$

$\vec{T}(t) = \frac{1}{\sqrt{16t^2+4+9t^4}} \langle 2, 3t^2, -4t \rangle$ { get $T'(t)$ using product of scalar & vector rule }

$t = 1$

$\cancel{= \frac{1}{\sqrt{29}} \langle 2, 3, -4 \rangle}$ $\qquad \frac{d}{dt} \frac{1}{\sqrt{9t^4+16t^2+4}} = \frac{-(36t^2+32t)}{2(9t^4+16t^2+4)^{3/2}}$

$t = 1 \qquad = \cancel{\frac{-68}{}}\ \frac{-34}{29\sqrt{29}}$

$\cancel{\vec{a} \| \vec{T} = \frac{0 + \frac{18}{\sqrt{29}} + \frac{16}{\sqrt{29}}}{\frac{23}{29}}}$ at $t=1$

$\frac{d}{dt} \langle 2, 3t^2, -4t \rangle = \langle 0, 6t, -4 \rangle = \langle 0, 6, -4 \rangle$ at $t = 1$

$\langle 2, 3t^2, -4t \rangle$ at $t = 1$ $= \langle 2, 3, -4 \rangle$

$\frac{1}{\sqrt{9t^4+16t^2+4}} = \frac{1}{\sqrt{29}}$ at $t = 1$

$\vec{T}'(1) = \frac{-34}{29\sqrt{29}} \cdot \langle 2, 3, -4 \rangle + \frac{1}{\sqrt{29}} \langle 0, 6, -4 \rangle$

$= \frac{1}{\sqrt{29}} \langle \frac{-68}{29}, \frac{72}{29}, \frac{20}{29} \rangle$

$\|\vec{T}'(1)\| = \frac{1}{\sqrt{29}} \cdot \sqrt{\left(\frac{68}{29}\right)^2 + \left(\frac{72}{29}\right)^2 + \left(\frac{20}{29}\right)^2} = \frac{1}{\sqrt{29}} \cdot \frac{4\sqrt{638}}{29} = \frac{4\sqrt{22}}{29}$

$\vec{N}(1) = \frac{29}{4\sqrt{22}} \cdot \frac{1}{\sqrt{29}} \langle \frac{-68}{29}, \frac{72}{29}, \frac{20}{29} \rangle$

$= \langle \frac{-17}{\sqrt{22}\cdot\sqrt{29}}, \frac{18}{\sqrt{22}\cdot\sqrt{29}}, \frac{5}{\sqrt{22}\cdot\sqrt{29}} \rangle = \langle \frac{-17}{\sqrt{638}}, \frac{18}{\sqrt{638}}, \frac{5}{\sqrt{638}} \rangle$

2.a. Consider path $y=0$

$$\lim_{x\to 0} \frac{\sin(x)\cdot 0}{x^2} = 0$$

Consider path $y=x$

$$\lim_{x\to 0} \frac{\sin x \cdot \sin x}{x^2+x^2} = \lim_{x\to 0} \frac{\sin^2 x}{2x^2} = \frac{1}{2}\cdot \lim_{x\to 0}\frac{\sin x}{x}\cdot \lim_{x\to 0}\frac{\sin x}{x}$$

$$= \frac{1}{2}\times 1\times 1$$

$$= \frac{1}{2}$$

$\Rightarrow$ Inconsistent limits, $\therefore$ the limit does not exist.

b. $$\frac{x^4y^4}{x^4+y^4} \le \left|\frac{x^4y^4}{x^4+y^4}\right| \le \left|\frac{x^4y^4}{x^4}\right| \le |y^4|$$

$$\therefore -|y^4| \le \frac{x^4y^4}{x^4+y^4} \le |y^4|$$

$$\lim_{y\to 0} -|y^4| \le \lim_{x,y\to 0,0} \frac{x^4y^4}{x^4+y^4} \le \lim_{y\to 0} |y^4|$$

$$0 \le \lim_{x,y\to 0,0} \frac{x^4y^4}{x^4+y^4} \le 0$$

$$\therefore \lim_{x,y\to 0,0} \frac{x^4y^4}{x^4+y^4} = 0$$ by Squeeze theorem

3a. $f(x,y) = y^2 e^x$ | $f(0,2) = 4$

$f_x = y^2 e^x$ | $f_x(0,2) = 4$

$f_y = 2ye^x$ | $f_y(0,2) = 4$

$$z = f(0,2) + f_x(0,2)(x-0) + f_y(0,2)(y - 2)$$

$$z = 4 + 4x + 4y - 8$$

$$z = 4x + 4y - 4$$

b. $2.08^2 e^{-0.1} \approx (0,2)$ in $y^2 e^x$

$\cancel{L(x,y) = 4x + 4y - 4 = 4 + (}$

$= L(x,y) = 4 + (x-0)\cdot 4 + (y-2)\cdot 4$

$L(2.08, -0.1) = 4 + 4 \cdot -0.1 + 4 \cdot 0.08$

$= 4 - 0.4 + 0.32$

$= \cancel{4.08}\ 3.92$

c. $\cancel{z = 4x + 4y - 4}$ Normal of $x - z = 3$ is $\langle 1, 0, -1 \rangle$

$\cancel{x - z = 3}$ Normal of tangent plane if it exists is $\langle f_x, f_y, -1 \rangle$

$\cancel{z = 3 - x}$

$f_x = 1$ & $f_y = 0$ must be true for parallel

$y^2 e^x = 1$

$2ye^x = 0 \longrightarrow y = 0$

$\longrightarrow 0^2 \cdot e^x \neq 1$

~~No~~ ~~values~~ No real values of x and y can satisfy this set of equations therefore no such points exist.

4a. $f(x, y) = (xy^2)^{1/3} = x^{1/3} \cdot y^{2/3}$

$\cancel{f_x = \frac{1}{3} y^{2/3} \cdot x^{-2/3} = \frac{1}{3}\left(\frac{y}{x}\right)^{2/3}}$

$\cancel{f_y = \frac{2}{3} x^{1/3} y^{-1/3} = \frac{2}{3}\left(\frac{x}{y}\right)^{1/3}}$

$\cancel{f_x = \lim_{h \to 0} f(x+h)}$

$$f_x = \lim_{h \to 0} \frac{f(a+h, b) - f(a, b)}{h} \qquad (a, b) = (0, 0)$$

$$f_x = \lim_{h \to 0} \frac{(a+h)^{1/3} \cdot b^{2/3} - a^{1/3} \cdot b^{2/3}}{h} = \lim_{h \to 0} \frac{0}{h} = 0$$

$\cancel{= b^{2/3} \lim_{h \to 0}}$

$$f_y = \lim_{h \to 0} \frac{f(a, b+h) - f(a, b)}{h}$$

$$= \lim_{h \to 0} \frac{(a)^{1/3} (b+h)^{2/3} - a^{1/3} \cdot b^{2/3}}{h} = \lim_{h \to 0} \frac{0}{h} = 0$$

$$f_x(0,0) = 0 = f_y(0,0)$$

b. $$D_{\vec{v}} f(0,0) = \lim_{t \to 0} \frac{f(a+th, b+tk) - f(a, b)}{t}$$

$(a, b) = 0, 0$
$\vec{v} = \langle h, k \rangle = \langle 1, 1 \rangle$

$$= \lim_{t \to 0} \frac{f(t, t) - f(0, 0)}{t}$$

$$= \lim_{t \to 0} \frac{(t^3)^{1/3} - 0}{t} = \lim_{t \to 0} \frac{t}{t} = 1$$

c. For $f(x, y)$ to be differentiable ~~everyt~~ everywhere f_x and f_y must be continuous for all $x, y \in \mathbb{R}$. However this is not true along $x = 0$ for $f_x = \frac{1}{3}\left(\frac{y}{x}\right)^{2/3}$ and $y = 0$ for $f_y = \frac{2}{3}\left(\frac{x}{y}\right)^{1/3}$ hence it is not differentiable along these axes.

5a. $\nabla f(x, y) = \langle 3x^2 + y, x \rangle$

b. Yes. $3x^2+y$ and x are polynomial and therefore f_x & f_y are always continuous and therefore $f(x, y)$ is differentiable everywhere.

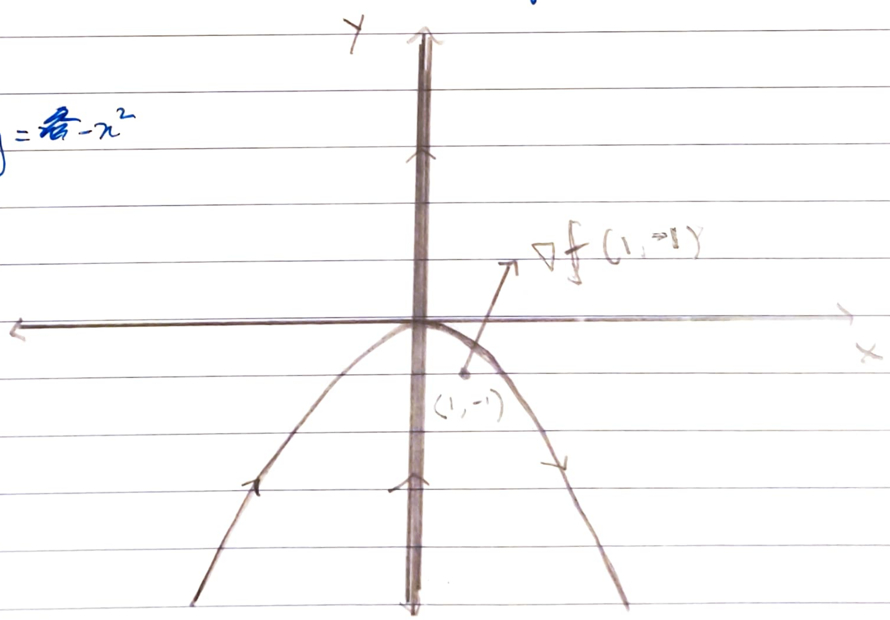
c. $\nabla f(1, -1) = \langle 2, 1 \rangle$

$\langle 2, 1 \rangle$ is the direction of fastest increase

$\therefore \langle -2, -1 \rangle$ is the direction of fastest decrease.

d.

$x = 0 \lor y = -x^2$



6a. ~~$D_{\vec{x}}$ $D_{\vec{x}}$~~ ~~$e_{\vec{v}}$ is a unit vector in direction $\vec{v}$~~.

$$e_{\vec{v}} = \frac{1}{3}\vec{v}$$

$$D_{e_{\vec{v}}} \nabla f(3,5) = \|\nabla f(3,5)\| \cos\frac{3\pi}{4}$$

$$= \sqrt{1+6^2} \cdot -\frac{\sqrt{2}}{2}$$

$$= \frac{-\sqrt{74}}{2}$$

b. Tangent line $= f_x \cdot (x-2) + f_y \cdot (y-1) = 0$

$$\Leftrightarrow f_x \cdot x - 2f_x + f_y \cdot y - f_y = 0$$

$f_x = -2 \longrightarrow -2f_x - f_y$

$f_y = 3$

$= 4 - 3$

$= 1 \longrightarrow 1$

~~$\nabla f(2,1) = \langle -2, 3 \rangle$~~

~~$\|\nabla f(2,1)\| = \sqrt{13}$~~

$\nabla f(2,1) = \langle 2, -3 \rangle$

$\|\nabla f(2,1)\| = \sqrt{13}$

we get

$-2x + 3y + 1 = 0$

but $f_x > 0 \therefore$ equation is

$2x - 3y - 1 = 0$

$f_x = -2 \longrightarrow f_x = 2$

$f_y = 3 \longrightarrow f_y = -3$

Unit vector $= \left\langle \frac{2}{\sqrt{13}}, \frac{-3}{\sqrt{13}} \right\rangle$