Midterm 2

Instructions:

- This is a take-home exam, open book and open notes. You have 24 hours to complete it. You have to submit your solutions **on gradescope by 8am on Thursday November 26, 2020**.
- Your exam should be **hand-written** (electronic pencil on a tablet is allowed).
- Please start a **new page for each question**. When uploading to gradescope, you will be asked to indicate for each question on which page(s) it can be found.
- To keep answers consistent, if you have questions while working on this exam, you should direct them by email to me: `pspaas@math.ucla.edu`. In particular, your TA will just forward any emails he gets to me, hence there is no need to email them.
  Only questions about clarification of questions will be considered, no hints will be given.
- We refer to the course syllabus for exam policies regarding academic integrity, and want to remind you that any violations will be taken seriously. Note that you have to **copy and sign** the academic integrity statement contained in question 1. Failing to do so may result in your exam receiving a failing grade.
- As usual, you must **show all your work** to receive credit. Correct answers without justification will not be awarded any points.
- Good luck!!!

Question	Points	Score
1	7	
2	8	
3	7	
4	6	
5	6	
6	6	
Total:	40	

1. (a) Copy the following statement on your solution, and sign it with your **full name**, **UID**, and **signature**.

   I certify on my honor that I have neither given nor received any help, and that I have not used any non-permitted resources, while completing this assignment.

   At time $t = 0$ a skydiver jumps out of a plane. Gravity together with some heavy winds make his velocity vector at time t equal to

   $$\mathbf{v}(t) = \langle 2, 3t^2, -4t \rangle.$$

   (b) (3 points) Assuming the skydiver jumped out of the plane at the initial point $(0, 0, 8)$, find the parametrization $\mathbf{r}(t)$ of the curve our skydiver traverses.

   (c) (4 points) Find the unit normal vector $\mathbf{N}$ to the curve at time $t = 1$.

2. For each of the following limits, calculate it, or show that it doesn't exist.

   (a) (4 points) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{\sin(x)\sin(y)}{x^2 + y^2}$.

   (b) (4 points) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^4 y^4}{x^4 + y^4}$.

3. Consider the differentiable function $f(x, y) = y^2 e^x$.

   (a) (3 points) Find an equation of the tangent plane to the graph of f at $(0, 2, f(0, 2))$.

   (b) (2 points) Use an appropriate linear approximation to estimate the value $2.08^2 e^{-0.1}$.

   (c) (2 points) Find all points on the graph of f where the tangent plane is parallel to the plane $x - z = 3$, or explain why no such points exist.

4. Consider the function defined by $f(x, y) = (xy^2)^{1/3}$.

   (a) (2 points) Calculate $f_x(0, 0)$ and $f_y(0, 0)$.

   (b) (3 points) Calculate the directional derivative of f at $(0, 0)$ in the direction of $\mathbf{v} = \langle 1, 1 \rangle$ using the limit definition of the directional derivative.

   (c) (1 point) Is $f(x, y)$ everywhere differentiable? Explain.

5. Consider the function $f(x, y) = x^3 + xy$.

   (a) (1 point) Calculate $\nabla f(x, y)$.

   (b) (1 point) Is $f(x, y)$ everywhere differentiable? Explain.

   (c) (1 point) Standing at the point $(1, -1)$ in the xy-plane, in which direction do we have to walk in order to see the value of $f(x, y)$ decrease the fastest?

   (d) (3 points) Sketch the level curve $f(x, y) = 0$ in the xy-plane, and draw the vector $\nabla f(1, -1)$ based at $(1, -1)$ in the same sketch.

Question 6 continues on the next page.

6. Suppose $f(x, y)$ is a differentiable function.

   (a) (3 points) Suppose that $\nabla f(3, 5) = \langle -1, 6 \rangle$, and that $\mathbf{v}$ is a vector with magnitude 3 that makes an angle of $3\pi/4$ with $\nabla f(3, 5)$. Find the directional derivative of f at $(3, 5)$ in the direction of $\mathbf{v}$.

   (b) (3 points) Suppose that at the point $(2, 1)$, the tangent line to the level curve of $f(x, y)$ that goes through $(2, 1)$ has equation $-2x + 3y + 1 = 0$. Furthermore, assume that $f_x(2, 1) > 0$. Find the unit vector that points in the same direction as $\nabla f(2, 1)$.