19F-MATH32A-1 Midterm 2

JEFFREY MA

TOTAL POINTS

39.5 / 40

QUESTION 1

1 Question 1 10 / 10

✓ **- 0 pts Correct.**

- 1 pts (a) Calculational mistake.

- 2 pts (a) Wrong equation of tangent plane.

- 1 pts (a) Mistake or inaccuracy, see comment in text.

- 1.5 pts (a) A function in x,y cannot be equal to that function with a specific point plugged in. One needs to separate those.

- 1 pts (a) f_x(0,3) should be specified. Now there is "f_x is equal to f_x evaluated at (0,3)"

- 0.5 pts (b) Didn't calculate the actual number.

- 0.5 pts (b) Calculational mistake

- 1 pts (b) L(0.1,2.9) is exactly equal to 9.4, not approximately.

- 1 pts (b) f(x,y) is approximately L(x,y), not exactly.

- 0.5 pts (b) f(0.1,2.9) is approximately L(0.1,2.9), not exactly.

- 0.5 pts (b) Once we are calculating the value of L(0.1,2.9) we have equalities, not approximate equalities.

- 1 pts (b) A function in x,y cannot be equal to that function with a specific point plugged in. One needs to separate those.

- 1.5 pts (b) A function in x,y cannot be equal to that function with a specific point plugged in. One needs to separate those.

- 1.5 pts (b) Wrong equation.

QUESTION 2

2 Question 2 10 / 10

✓ **+ 5 pts (a) Correct**

✓ **+ 5 pts (b) Correct**

+ 1 pts (a) Set up limit of f(x, y) as (x, y) goes to (0, 0)

+ 1 pts (a) Apply squeeze theorem to computing the limit of f(x, y). No credit if incorrectly split the limit as a product of limits, or only calculated the limit along a path.

+ 1 pts (a) Found appropriate bounding functions for f(x, y), using that the value of sine is between -1 and 1. No credit if incorrectly split the limit as a product of limits, or only calculated the limit along a path.

+ 1 pts (a) Compute limit of f(x, y) as (x, y) goes to (0, 0) correctly.

+ 1 pts (a) Deduce from limit of f(x, y) as (x, y) goes to (0, 0) exists that function can be extended continuously to (0, 0)

+ 1 pts (b) Basic approach is finding two paths with different limits. No credit if got the same limit for all paths given (whether correctly or incorrectly), or incorrectly factored the limit as a product of limits, or gave an incorrect example of path that has no limit (it's technically possible to give a correct example of this and will solve the problem but it's much harder).

+ 1 pts (b) Found a path and correctly computed its limit

+ 2 pts (b) Found another path with a different limit and correctly computed its limit (no credit if the path doesn't have a different limit, minus 1 if computed the limit wrong)

+ 1 pts (b) Conclude from paths with different limits that limit does not exist

+ 0 pts Incorrect

QUESTION 3

3 Question 3 9.5 / 10

✓ **- 0 pts a: Correct**

✓ **- 0 pts b: Correct**

- 0 pts c: Correct

✓ **- 0 pts d: Correct**

- **0.5 pts** a: Missing level curve
- **2 pts** a: Incorrect sketch of contours
- **3 pts** b: Missing
- **1 pts** b: Incorrect gradient computation
- **1 pts** b: Incorrect or missing drawing

✓ **- 0.5 pts c: Missing condition of continuity of higher partial derivatives**

- **0.5 pts** c: Missing conclusion that derivatives commute
- **2 pts** d: Incorrect computation of higher partial derivatives.
- **0.5 pts** d: Minor computational error
- **1 pts** a: Missing labels for contours
- **1 pts** a: Minor error in drawing contours
- **1 pts** d: Computational error
- **3 pts** a: Missing
- **1.5 pts** d: Computational error
- **1 pts** (a): Missing contours
- **0.5 pts** (b): Minor error in drawing

QUESTION 4

4 Question 4 **10 / 10**

- **3 pts** (a)incorrect v(t) and a(t) , half point for each coordinate
- **0.5 pts** (a)calculation mistake
- **1 pts** (a)calculation mistake
- **1.5 pts** (a) calculation mistake
- **1 pts** (b)incorrect unit tangent vector
- **0.5 pts** (b)minor calculation error of unit tangent vector
- **1 pts** (b)incorrect unit normal vector
- **0.5 pts** (b)minor calculation error of unit normal vector
- **1 pts** (b) incorrect tangential component
- **1 pts** (b) incorrect normal component
- **3 pts** (c) not correct
- **1 pts** (c)did not state that normal component is 0
- **1 pts** (c)did not get the relation a= -2T from the decomposition
- **1 pts** (c)miscalculation of T or didn't calculate T

✓ **- 0 pts all correct**

Midterm 2

Name: Jeffrey Ma

UID: 305309403

Section:

	Tuesday:	Thursday:	
	1A	1B	TA: Yurun Ge
	1C	1D	TA: Benjamin Johnsrude
	(1E)	1F	TA: Tianqi Wu

Instructions: Do not open this exam until instructed to do so. You will have 50 minutes to complete the exam. Please print your name and student ID number above, and circle the number of your discussion section. You may not use calculators, books, notes, or any other material to help you. Please make sure your phone is silenced and stowed where you cannot see it. You may use the front and back of the page for your answers. **Do not write answers for one question on the page of another question**. If you need scratch paper, please ask one of the proctors. You must **show all your work** to receive credit. Please circle or box your final answers.

Please do not write below this line.

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
Total:	40	

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1. Consider the function $f(x, y) = \sin x + y^2 e^x$.

   (a) (6 points) Find an equation of the tangent plane to the graph of f at $(0, 3, f(0, 3))$.

   (b) (4 points) Estimate the value of $f(0.1, 2.9)$ using linear approximation.

a) $f_x(x,y) = \cos x + y^2 e^x$ cont. for $D \to \mathbb{R}^2$

$f_y(x,y) = 2ye^x$ cont. for $D \to \mathbb{R}^2$

Has tang. plane — differentiable — f_x and f_y exist and are continuous at all points

$L(x,y) = z = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$(a,b) = (0,3)$ $f(0,3) = 9 = \sin 0 + 3^2 e^0$

$f_x(0,3) = 1 + 3^2 e^0 = 10$

$f_y(0,3) = 2(3)e^0 = 6$

$\boxed{L(x,y) = z = 9 + 10(x) + 6(y-3)}$

b) $f(x,y) \approx L(x,y)$

$10 \cdot \frac{1}{10} = 1$

$f(0.1, 2.9) \approx L(0.1, 2.9) = 9 + 10(0.1) + 6(2.9-3)$

$= 9 + 1 + 6(-0.1)$

$= 9 + 1 - 0.6$

$= 9 + 0.4 = \boxed{9.4}$

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2. (a) (5 points) Consider the function $f(x, y)$ defined for $(x, y) \neq (0, 0)$ by

$$f(x,y) = x \sin\left(\frac{xy}{x^4 + y^4}\right).$$

Show that one can assign a value to $f(0,0)$ to make this function continuous.

(b) (5 points) Evaluate the following limit or show it doesn't exist:

$$\lim_{(x,y)\to(0,0)} \frac{x^2y^4}{x^4 + y^8}.$$

a) $f(x,y)$ is continuous at (a,b) if and only if

$\lim_{(x,y)\to(a,b)} f(x,y) = f(a,b)$

Currently not continuous at $(0,0)$ – undefined

Simply set $f(x,y)$ to the limit at $(0,0)$ to make it continuous

$\lim_{(x,y)\to(0,0)} x \sin\left(\frac{xy}{x^4+y^4}\right)$ Squeeze The.

Is bounded between $-1 \leq \sin(\cdots) \leq 1$

$\therefore -|x| \leq x \sin\left(\frac{xy}{x^4+y^4}\right) \leq |x|$

Squeeze The. : $\lim_{x\to 0} -|x| = \lim_{(x,y)\to(0,0)} x \sin\left(\frac{xy}{x^4+y^4}\right) = \lim_{x\to 0} |x|$

$= \boxed{0}$

to be continuous $\therefore f(x,y) = \begin{cases} x\sin\left(\frac{xy}{x^4+y^4}\right) & \text{at } (x,y)\neq(0,0) \\ 0 & \text{at } (x,y)=(0,0) \end{cases}$

b) $\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{(x,y)\to(0,0)} \frac{x^2y^4}{x^4+y^8}$

$\lim_{x\to 0} f(x,0) = \lim_{x\to 0} \frac{x^2(0)^4}{x^4+0^8} = \lim_{x\to 0} \frac{0}{x^4} = 0$

$\lim_{x\to 0} f(x,\sqrt{x}) = \lim_{x\to 0} \frac{x^2(\sqrt{x})^4}{x^4+(\sqrt{x})^8} = \lim_{x\to 0} \frac{x^2x^2}{x^4+x^4} = \lim_{x\to 0} \frac{x^4}{2x^4} = \frac{1}{2}$

Differing paths do not converge

$\therefore$ Limit Does Not Exist

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3. (a) (3 points) In the xy-plane, sketch the contour diagram of $f(x, y) = x^2 - y^2$ containing the level curves corresponding to $c = -1, 0, 1, 2$.

(b) (3 points) Calculate $\nabla f(x, y)$. Draw $\nabla f(0, 1)$ based at the point $(0, 1)$ in the contour diagram of part (a).

(c) (1 point) State Clairaut's Theorem on higher order partial derivatives.

(d) (3 points) Consider the function $f(x, y, z) = x^3 e^y \sin z + \frac{z^2 e^{2y}}{2+\cos z}$. Calculate $\frac{\partial^3 f}{\partial x \partial z \partial y}$.

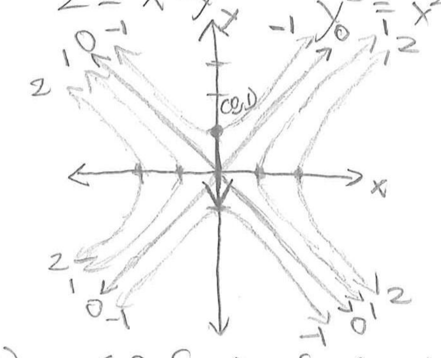
a) $f(x,y) = c = x^2 - y^2$

$c = -1$ $-1 = x^2 - y^2$ $y^2 = x^2 + 1$ $y = \pm\sqrt{x^2+1}$

$c = 0$ $0 = x^2 - y^2$ $x^2 = y^2$ $\pm y = \pm x$

$c = 1$ $1 = x^2 - y^2$ $y^2 = x^2 - 1$ $y = \pm\sqrt{x^2-1}$

$c = 2$ $2 = x^2 - y^2$ $y^2 = x^2 - 2$ $y = \pm\sqrt{x^2-2}$



b) $\nabla f(x,y) = \langle f_x(x,y), f_y(x,y) \rangle = \langle 2x, -2y \rangle$

$\nabla f(0,1) = \langle 0, -2 \rangle$ $\perp$ to curve

c) If f_x and f_y exist and are continuous in a disk surrounding (a,b)

Then $f_{xy} = f_{yx}$

$\therefore f_{xyx} = f_{yxx} = f_{xxy}$

Given f_z exists and is cont., $f_{xyz} = f_{yxz} = f_{zyx} = f_{yzx} \ldots$

d) $\frac{\partial^3 f}{\partial x \partial z \partial y} = \frac{\partial}{\partial x}\frac{\partial}{\partial z}\frac{\partial}{\partial y} f(x,y,z)$

f_x, f_y and f_z are continuous and exist (No Divide by 0's in any of the partial derivatives)

$f_x = 3x^2 e^y \sin z$

$f_{xy} = 3x^2 e^y \sin z$

Clairaut's The.

$f_{xyz} = \boxed{3x^2 e^y \cos z = f_{yzx} = \frac{\partial^3 f}{\partial x \partial z \partial y}}$

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4. (a) (3 points) Suppose $\mathbf{r}(t) = \langle 3\sin t, 3\cos t, 4t\rangle$. Find the acceleration vector $\mathbf{a}(t)$.

   (b) (4 points) For $\mathbf{r}(t)$ as in (a), calculate the unit tangent and unit normal vector at $t = 0$ and find the tangential component and normal component of $\mathbf{a}(t)$ at $t = 0$.

   (c) (3 points) A car is moving along a path $\mathbf{r}(t)$ which satisfies $\mathbf{r}(0) = \langle 1, 2, 0\rangle$ and $\mathbf{r}'(0) = \langle 2, 3, -6\rangle$. At time $t = 0$ the unit vector $\mathbf{N} = \langle 0, \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\rangle$ is normal to the curve. Suppose that at time $t = 0$, the car is slowing down at a rate of 2m/s^2 and is not changing direction. Find the acceleration vector at time $t = 0$.

a) $\vec{r}(t) = \langle 3\sin t, 3\cos t, 4t\rangle$

$\vec{r}'(t) = \vec{v}(t) = \langle 3\cos t, -3\sin t, 4\rangle$

$\vec{r}''(t) = \vec{a}(t) = \boxed{\langle -3\sin t, -3\cos t, 0\rangle}$

b) $\vec{T}(t) = \frac{\vec{r}'(t)}{\|\vec{r}'(t)\|} = \frac{\vec{v}(t)}{\|\vec{v}(t)\|} = \frac{\langle 3\cos t, -3\sin t, 4\rangle}{\sqrt{9\cos^2 t + 9\sin^2 t + 4^2}} =$ $9+16$

$= \boxed{\frac{1}{5}\langle 3\cos t, -3\sin t, 4\rangle}$

speed $= \nu(t) = \|\vec{v}(t)\| = 5$

$\nu'(t) = 0$

$a_T = \nu'(t) = \boxed{0}$

$\vec{N} = \frac{a - a_T\vec{T}(t)}{\|a - a_T\vec{T}(t)\|}$ $\quad a - a_T\vec{T}(t) = a = \langle -3\sin t, -3\cos t, 0\rangle$

$\vec{N}(t) = \frac{\langle -3\sin t, -3\cos t, 0\rangle}{\sqrt{(-3)^2\sin^2 t + (-3)^2\cos^2 t}} = \boxed{\langle -\sin t, -\cos t, 0\rangle}$

$a_N = \frac{a_N\vec{N}(t)}{\vec{N}(t)} = \frac{\|a - a_T\vec{T}(t)\|}{\|\vec{N}(t)\|} = \frac{\langle -3\sin t, -3\cos t, 0\rangle}{\langle -\sin t, -\cos t, 0\rangle} = \boxed{3}$

c) $r(0) = \langle 1, 2, 0\rangle$

$r'(0) = \langle 2, 3, -6\rangle$

$t=0$ $N = \langle 0, \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\rangle$ normal to curve

$t=0$ $\nu'(t) = -2\,m/s^2$ $\quad$ Not changing direction — no curving

$K(0) = 0$

$\vec{a}(0) = a_T T + a_N N$ $\quad a_T = \nu'(0) = -2$

$= -2T$ $\quad a_N = K(t)\nu^2 = 0$

$\vec{a}(0) = -2T$ $\quad\quad$ $\vec{T}(0) = \frac{r'(0)}{\|r'(0)\|} = \frac{\langle 2,3,-6\rangle}{\sqrt{4+9+36}}$

$40+9$
49

$\vec{a}(0) = -\frac{2}{7}\langle 2,3,-6\rangle$ $\quad\quad$ $= \frac{\langle 2,3,-6\rangle}{7}$

$= \boxed{\left\langle \frac{-4}{7}, \frac{-6}{7}, \frac{12}{7} \right\rangle}$

$\begin{array}{r} 36 \\ +\ 4 \\ \hline 40 \end{array}$