Jai Pareera 29/10/2020 PROF. P SPAAS

DHIRUBHAI AMBANI INTERNATIONAL SCHOOL
ESTD. 2003

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Midterm 1 - Math 32A

1a. I certify on my honor that I have neither given nor received any help, and that I have not used any non-permitted resources, while completing this assignment.

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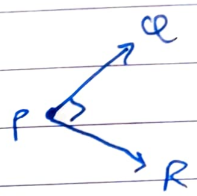
b. $\vec{PQ} = \langle 1, 2, 6 \rangle - \langle 1, 0, 0 \rangle$

$= \langle 0, 2, 6 \rangle$

$\vec{PR} = \langle 1, -3, 1 \rangle - \langle 1, 0, 1 \rangle$

$= \langle 0, -3, 1 \rangle$

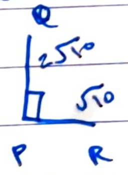


$\vec{PQ} \cdot \vec{PR} = 0 \times 0 + 2 \times -3 + 6 \times 1$

$= 0$

Therefore $\vec{PQ} \perp \vec{PR}$ and a right angle exists at P.

c. Since P is a right angle, $\frac{1}{2} \times$ base $\times$ height will suffice.



$\|\vec{PR}\| = \sqrt{3^2 + 1^2} = \sqrt{10}$

$\|\vec{PQ}\| = \sqrt{2^2 + 6^2} = \sqrt{40} = 2\sqrt{10}$

$\therefore$ Area $= \frac{1}{2} \times \sqrt{10} \times 2\sqrt{10} = 10$

d. Line L: direction vector $= \vec{PQ} = \langle 0, 2, 6 \rangle$

position vector $=$ P or Q $= \langle 1, 0, 0 \rangle$

$\vec{r}(t) = \langle 0, 2, 6 \rangle t + \langle 1, 0, 0 \rangle$

$\vec{r}(t) = \langle 1, 2t, 6t \rangle$

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2a. $\overrightarrow{PQ} = \langle -1, 2, 3\rangle - \langle 1, 2, 0\rangle$

$= \langle -2, 0, 3\rangle$

$\overrightarrow{PR} = \langle 0, -2, 5\rangle - \langle 1, 2, 0\rangle$

$= \langle -1, -4, 5\rangle$

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & 0 & 3 \\ -1 & -4 & 5 \end{vmatrix} = \vec{i}\begin{vmatrix} 0 & 3 \\ -4 & 5 \end{vmatrix} - \vec{j}\begin{vmatrix} -2 & 3 \\ -1 & 5 \end{vmatrix} + \vec{k}\begin{vmatrix} -2 & 0 \\ -1 & -4 \end{vmatrix}$$

$= \langle 12, 7, 8\rangle \neq \vec{0}$ $\therefore$ $\overrightarrow{PQ}$ is not parallel to $\overrightarrow{PR}$ and $\therefore$ PQR are not collinear.

b.
$$\begin{array}{c} \\ r_1 \\ r_2 \end{array}\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 2 & 0 \\ 0 & 2 & -4 \end{vmatrix} \Rightarrow \vec{n} = \langle -8, -4, -2\rangle$$

Plane P_1 has point $\langle 0, 1, 2\rangle$

$\therefore$ Eq is:

$\langle -8, -4, -2\rangle \cdot \langle x, y, z\rangle = \langle -8, -4, -2\rangle \cdot \langle 0, 1, 2\rangle$

$-8x - 4y - 2z = -4 - 4$

$\Rightarrow -8x - 4y - 2z = -8$

$\Rightarrow -4x - 2y - z = -4$

$\Rightarrow 4x + 2y + z = 4$

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3a. $\vec{r}_1(t) = \langle t^2, 3e^t, \sin(\pi t) \rangle$

$\vec{r}_1{}'(t) = \langle 2t, 3e^t, \pi \sin(\pi t) \rangle$

$\vec{r}_1(3) = \langle 9, 3e^3, 0 \rangle$

$\vec{r}_1{}'(3) = \langle 6, 3e^3, 0 \rangle$

Tangent line $= \langle 6, 3e^3, 0 \rangle t + \langle 9, 3e^3, 0 \rangle$

$= \langle 3(2t+3), 3e^3(t+1), 0 \rangle$

b. $\int \vec{r}_2{}'(t)\, dt = \langle 2t^2, 0, \frac{t^3}{3} \rangle + \vec{c}$

$\vec{r}_2(3)$: $\langle 1, 3, 0 \rangle = \vec{c} + \langle 18, 0, 9 \rangle$

$\vec{c} = \langle -17, 3, -9 \rangle$

$\vec{r}_2(t) = \langle 2t^2 - 17, 3, \frac{t^3}{3} - 9 \rangle$

$\frac{d}{dt}\left(\vec{r}_1(t) \cdot \vec{r}_2(t)\right) = \vec{r}_1(t) \cdot \vec{r}_2{}'(t) + \vec{r}_2(t) \cdot \vec{r}_1{}'(t)$

$\vec{r}_2(3) = \langle 1, 3, 0 \rangle$

$\vec{r}_2{}'(3) = \langle 12, 0\ 9 \rangle$

$\vec{r}_1(3) \cdot \vec{r}_2{}'(3) + \vec{r}_2(3) \cdot \vec{r}_1{}'(3)$

$\langle 9, 3e^3, 0 \rangle \cdot \langle 12, 0, 9 \rangle + \langle 1, 3, 0 \rangle \cdot \langle 6, 3e^3, 0 \rangle$

$= 9 \times 12 + \cancel{3e^3}\ 6 + 9e^3$

$= 114 + 9e^3$

4a. If $\vec{r}(t)$ is the arc length parametrization of $\vec{C}$

~~then $r(2) - r(1)$ must equal 1 : then the leng~~

then $\|r'(t)\|$ must equal 1 for all t.

~~$r(x) =$~~

$$\vec{r}'(t) = \langle 3t^2, 0, 6t^2 \rangle$$

$$\|\vec{r}'(t)\| = \sqrt{(3t^2)^2 + (6t^2)^2} = \sqrt{9t^4 + 36t^4}$$

$$= 3\sqrt{5}\,t^2$$

~~Be~~ $3\sqrt{5}\,t^2 \neq 1$ for all t $\therefore$ $r(t)$ is not the arc length parametrization.

b. $$s = \int_1^5 \|r'(t)\|\,dt = 3\sqrt{5}\int_1^5 t^2\,dt$$

$$= 3\sqrt{5}\left[\frac{t^3}{3}\right]_1^5 = \sqrt{5}\left[t^3\right]_1^5 = \sqrt{5}\left[125 - 1\right]$$

$$= 124\sqrt{5}$$

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5a. $\vec{r}(t) = \langle t\cos t - \sin t,\ t\sin t + \cos t,\ t^2 \rangle$

$\vec{T}(t) = \dfrac{\vec{r}\,'(t)}{\|\vec{r}\,'(t)\|}$

$\vec{r}\,'(t) = \langle -t\sin t + \cos t - \cos t,\ t\cos t + \sin t - \sin t,\ 2t \rangle$

$= \langle -t\sin t,\ t\cos t,\ 2t \rangle$

$\|\vec{r}\,'(t)\| = \sqrt{t^2\sin^2 t + t^2\cos^2 t + 4t^2}$

$= \sqrt{t^2 + 4t^2}$

$= t\sqrt{5}$

$\therefore\ \vec{T}(t) = \left\langle \dfrac{-\sin t}{\sqrt{5}},\ \dfrac{\cos t}{\sqrt{5}},\ \dfrac{2}{\sqrt{5}} \right\rangle$

$\vec{N}(t) = \dfrac{\vec{T}\,'(t)}{\|\vec{T}\,'(t)\|}$

$\vec{T}\,'(t) = \left\langle \dfrac{-\cos t}{\sqrt{5}},\ \dfrac{-\sin t}{\sqrt{5}},\ 0 \right\rangle$

$\|\vec{T}\,'(t)\| = \sqrt{\dfrac{\cos^2 t}{5} + \dfrac{\sin^2 t}{5}}$

$= \sqrt{\dfrac{1}{5}} = \dfrac{1}{\sqrt{5}} \qquad \dfrac{1}{\|\vec{T}\,'(t)\|} = \sqrt{5}$

$\therefore\ \vec{N}(t) = \left\langle \dfrac{-\cos t}{\sqrt{5}},\ \dfrac{-\sin t}{\sqrt{5}},\ 0 \right\rangle \times \sqrt{5}$

$\vec{N}(t) = \langle -\cos t,\ -\sin t,\ 0 \rangle$

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6.

6. $K(t) = \frac{1}{v(t)} \cdot \|\vec{T}'(t)\|$

$K(2) = \frac{1}{\sqrt{5}} \cdot \frac{1}{v(2)}$.

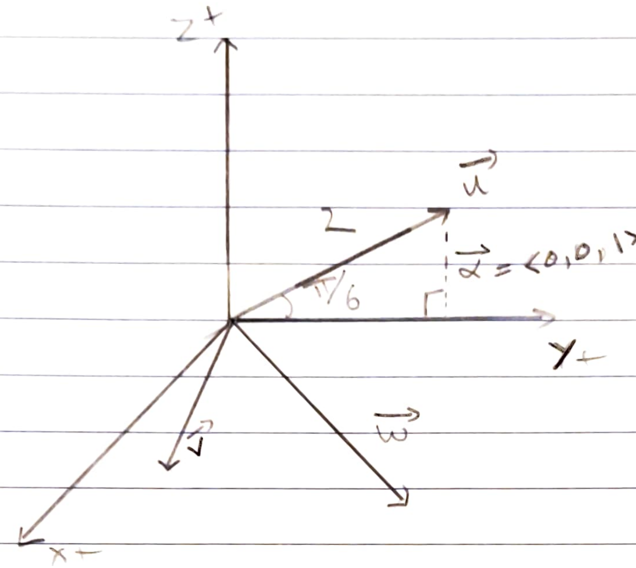
$v = \|\vec{r}'(t)\|$, $\|\vec{T}'(t)\| = \frac{1}{\sqrt{5}}$

$= t\sqrt{5}$

$v(2) = 2\sqrt{5}$

$K(2) = \frac{1}{2\sqrt{5}} \times \frac{1}{\sqrt{5}} = \frac{1}{10}$

6 a.



~~The~~ $\|\vec{v} \times \vec{w}\|$ gives the cross sectional area of one side of the parallelepiped. $\|\vec{\alpha}\|$ is the effective height of the parallelepiped since $\vec{v}$ and $\vec{w}$ are contained in the xy plane, which $\vec{\alpha}$ is normal to.

∴ Volume equals $\|\vec{\alpha}\| \|\vec{v} \times \vec{w}\|$

$\|\vec{\alpha}\| = 2 \sin \pi/6 = 1$

∴ the volume is 1×3

$= 3$ units^3

$k_1 = \frac{2n_1+1}{2}\pi$

b. $v \cdot w = 0$ for angles k_1 ~~$(\frac{\pi}{2})$~~ for ~~$k_1 \in \mathbb{Z}$~~ $n_1 \in \mathbb{Z}$

$v \times w = 0$ for angles ~~k_2~~ for ~~$k_2 \in \mathbb{Z}$~~ $n_2 \in \mathbb{Z}$

$k_2 = 2n_2$

Since n_1 and n_2 belong to $\mathbb{Z}$ and have constant discrete values, there is never any overlap between the set of angles contained by k_1 ~~$(\frac{\pi}{2})$~~ and k_2 ~~(π)~~. Therefore, while $\vec{v}$ and $\vec{w}$ are non zero vectors such a case cannot exist.