Midterm 1

Instructions: (READ CAREFULLY—YOU HAVE THE TIME FOR IT.)

- This is a take-home exam, open book and open notes. You have 24 hours to complete it. You have to submit your solutions **on gradescope by 8am on Thursday October 29, 2020**.
- Your exam should be **hand-written** (electronic pencil on a tablet is allowed).
- Please start a **new page for each question**. When uploading to gradescope, you will be asked to indicate for each question on which page(s) it can be found.
- To keep answers consistent, if you have questions while working on this exam, you should direct them by email to me: `pspaas@math.ucla.edu`. In particular, your TA will just forward any emails he gets to me, hence there is no need to email them.
  Only questions about clarification of questions will be considered, no hints will be given.
- We refer to the course syllabus for exam policies regarding academic integrity, and want to remind you that any violations will be taken seriously. Note that you have to copy and sign the academic integrity statement contained in question 1. Failing to do so may result in your exam receiving a failing grade.
- As usual, you must **show all your work** to receive credit. Correct answers without justification will not be awarded any points.
- Good luck!!!

Question	Points	Score
1	8	
2	6	
3	7	
4	6	
5	7	
6	6	
Total:	40	

1. (a) Copy the following statement on your solution, and sign it with your **full name**, **UID**, and **signature**.

   I certify on my honor that I have neither given nor received any help, and that I have not used any non-permitted resources, while completing this assignment.

   Consider the triangle formed by the points $P = (1, 0, 0)$, $Q = (1, 2, 6)$, and $R = (1, -3, 1)$.

   (b) (2 points) Show that this triangle has a right angle at P.

   (c) (3 points) Find the area of this triangle.

   (d) (3 points) Find a parametrization of the line through P and Q.

2. (a) (2 points) Are the points $P = (1, 2, 0)$, $Q = (-1, 2, 3)$ and $R = (0, -2, 5)$ collinear? Explain.

   (b) (4 points) Find the equation of the plane that contains the lines given by $\mathbf{r}_1(t) = \langle 0, 1, 2\rangle + t\langle -1, 2, 0\rangle$ and $\mathbf{r}_2(t) = \langle -1, 3, 2\rangle + t\langle 0, 2, -4\rangle$.

3. Suppose $\mathbf{r}_1(t) = \langle t^2, 3e^t, \sin(\pi t)\rangle$.

   (a) (4 points) Find a parametrization of the tangent line to this curve at $t = 3$.

   (b) (3 points) Suppose $\mathbf{r}_2(t)$ is the parametrization of another curve which satisfies $\mathbf{r}_2(3) = \langle 1, 3, 0\rangle$ and $\mathbf{r}_2'(t) = \langle 4t, 0, t^2\rangle$. Find $\dfrac{d}{dt}(\mathbf{r}_1(t) \bullet \mathbf{r}_2(t))$ at $t = 3$.

4. Consider the curve $\mathcal{C}$ parametrized by $\mathbf{r}(t) = \langle t^3, 5, 2t^3\rangle$ for $1 \le t \le 5$.

   (a) (3 points) Is $\mathbf{r}(t)$ the arc length parametrization of $\mathcal{C}$? Explain why/why not.

   (b) (3 points) Find the arc length of the curve for $1 \le t \le 5$.

5. Consider the curve parametrized by $\mathbf{r}(t) = \langle t\cos(t) - \sin(t), t\sin(t) + \cos(t), t^2\rangle$ for $t \ge 1$.

   (a) (5 points) Find expressions for the unit tangent vector $\mathbf{T}(t)$ and the unit normal vector $\mathbf{N}(t)$.

   (b) (2 points) Find the curvature of $\mathbf{r}(t)$ at $t = 2$.

6. (a) (3 points) Suppose $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ are vectors based at the origin in $\mathbb{R}^3$ such that $\mathbf{v}$ and $\mathbf{w}$ are contained in the xy-plane, $\|\mathbf{v} \times \mathbf{w}\| = 3$, and $\mathbf{u}$ is a vector of length 2 contained in the first quadrant of the yz-plane that makes an angle of $\dfrac{\pi}{6}$ with the y-axis. Find the volume of the parallelepiped spanned by $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$.

   (b) (3 points) Do there exist nonzero vectors $\mathbf{v}$ and $\mathbf{w}$ in $\mathbb{R}^3$ such that $\mathbf{v} \bullet \mathbf{w} = 0$ and $\mathbf{v} \times \mathbf{w} = \mathbf{0}$? If so, give an example. If not, explain why not.