Midterm 2
Calculus of Several Variables
(Math 32A-002)

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| Question: | 1 | 2 | 3 | 4 | 5 | Total |
|---|---|---|---|---|---|---|
| Points: | 20 | 25 | 25 | 15 | 15 | 100 |
| Score: | 7 | 11 | 8 | 12 | 15 | 53 |

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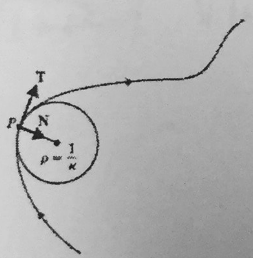


Figure 1: Osculating Circle

1. Consider the following parametrization of a curve:

$$x = \frac{1+t}{\sqrt{1+t^2}}, \quad y = \frac{1-t}{\sqrt{1+t^2}}.$$

(a) $\boxed{\text{10 points}}$ Find a relation between $x$ and $y$ by eliminating $t$.

(b) $\boxed{\text{10 points}}$ Find $\frac{dy}{dx}$ in terms of $x$ and $y$ only (your answer can not contain $t$).

[**Hint:** For Part (b), differentiate the equation in $x$-$y$ obtained in Part (a) with respect to $x$.]

$-yx = x^2 - 2$

$y = -x + \frac{2}{x}$

$(x\sqrt{1+t^2}) - 1 = t$

$(y\sqrt{1+t^2}) - 1 = -t$

$\frac{1+t}{\sqrt{1+t^2}} \quad \frac{1-t}{\sqrt{1+t^2}}$

$-x\sqrt{1+t^2} + 1 = y\sqrt{1+t^2} - 1$

$(x+y) = \frac{2}{\sqrt{1+t^2}}$

$\frac{1-t^2}{1+t^2}$

b) $\cancel{y' = -2 + 1 \cdot 2}$

$\boxed{y' = -1 - \frac{2}{x^2}}$

$\cancel{y = \frac{yx}{(1+t^2)}}$

$\cancel{2y = \frac{2yx}{1+t^2}}$

$y = \frac{1-t\sqrt{1+t^2}}{1+t^2}$

$2y = \frac{2}{1+t^2}(yx$

$\cancel{y = \frac{1-t}{x}}$

$2y = (x+y)\,yx$

a) $\boxed{2 = x^2 + yx}$

b) $0 = 2x + y'x + y$

$y' = \frac{-2 - (-x + \frac{2}{x})}{x}$

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2. The equation of the osculating circle to a curve parametrized by $\vec{r}(t) = \langle f(t), g(t)\rangle$ at the point $(2,1)$ is:

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$$x^2 + y^2 - 4x + 2y + 1 = 0.$$

(a) 5 points Find the curvature of $\vec{r}(t)$ at the point $(2,1)$.

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(b) 10 points Find the unit normal $\vec{N}$ to $\vec{r}(t)$ at the point $(2,1)$.

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(c) 10 points Find a parametrization of the tangent line to $\vec{r}(t)$ at the point $(2,1)$. (This is not the same thing as finding the unit tangent vector $\vec{T}$ at $(2,1)$, you need to find a parametrization of the whole tangent line).

[**Hint:** First rewrite the equation as $(x-a)^2 + (y-b)^2 = r^2$. Then look at the Figure 1 on Page 1 and use it's geometry (the relation of the osculating circle to the curve at the point $(2,1)$) to solve this problem.]

$$(x^2 - 4x + 4) + (y^2 + 2y + 1) = 4$$

$$(x - 2)^2 + (y - (-1))^2 = 4$$

a) $k = \frac{1}{r}$ $\quad r = 2 \quad k = \frac{1}{2}$

$$\boxed{\text{Curvature} = \frac{1}{2}}$$

b) $r(t) = \langle 2 + 4\cos t, -1 + 4\sin t\rangle$

$$T = \frac{\langle -4\sin t, 4\cos t\rangle}{\sqrt{32}}$$

$$\sqrt{(-4\sin t)^2 + (4\cos t)^2} = \sqrt{32}$$

$$\boxed{N = \left\langle \frac{-4\cos t}{\sqrt{32}}, \frac{-4\sin t}{\sqrt{32}} \right\rangle}$$

c) $-~~4\sin t, 4\cos t~~$

$y_1 - y_0 = m(x - x_0)$

$2x + 2y y' - 4 + 2y' = 0$

$y - 1 = 0(x - 2)$

$4 + 2y' - 4 + 2y'$

$\boxed{y = 1}$

$y' = 0$

3. (a) $\boxed{\text{15 points}}$ Using the Squeeze Theorem prove that $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2y}{\sqrt{x^4+y^2}} = 0.$

(b) $\boxed{\text{10 points}}$ Use Part (a) to show that the function $f$ defined by

$$f(x,y) = \begin{cases} \frac{x^4y^2}{\sqrt{x^4+y^2}} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{at } (0,0) \end{cases}$$

is continuous at $(0,0)$.

[**Hint:** A function $f$ is called continuous at $(a,b)$ if $\lim_{(x,y)\to(a,b)} f(x,y) = f(a,b)$.]

$$\frac{x^2y}{\sqrt{x^4+y^2}}$$

$x^2 \nleq 0$

~~$0 \le |x|$~~

~~$\sqrt{x^4+y^2} <$~~

$0 \le \sqrt{x^4+y^2}$

?

$$\frac{x^2|y|\sqrt{x^4+y^2}}{\sqrt{x^4+y^2}} \ge 0$$

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$$-\frac{x^2|y|\sqrt{\cancel{x^4+y^2}}}{\sqrt{\cancel{x^4}+y^2}} \le \frac{x^2y}{\sqrt{x^4+y^2}} \le \frac{x^2|y|\sqrt{\cancel{x^4}+y^2}}{\sqrt{\cancel{x^4}+y^2}}$$

$$-x^2|y| \le \frac{x^2y}{\sqrt{x^4+y^2}} \le x^2|y|$$

$$\lim_{(x,y)\to(0,0)} -x^2|y| = \lim_{(x,y)\to(0,0)} x^2|y| = 0$$

$$0 \le \frac{x^2y}{\sqrt{x^4+y^2}} \le 0$$



there fore ~~lim~~ $\displaystyle\lim \frac{x^2y}{\sqrt{x^4+y^2}} = 0$

4. $\boxed{\text{15 points}}$ Compute $f_{xyxzy}$, where

$$f(x,y,z) = y\sin(xz)\sin(x+z) + (x+z^2)\tan y + x\tan\left(\frac{z+z^{-1}}{y-y^{-1}}\right).$$

$$zy\cos(xz)\sin(x+z) + y\sin(xz)\cos(x+z)$$

no y $\cancel{z\cos(xz)\sin(x+z)} + \cancel{\sin(xz)\cos(x+z)}$

$$(x+z^2)\tan y = \overset{\text{no } z}{x\cancel{\tan y}} + \overset{\text{no } x}{z^2\cancel{\tan y}}$$

$$x\tan\left(\frac{z+z^{-1}}{y-y^{-1}}\right)$$

$$f_x = \tan\left(\frac{z+z^{-1}}{y-y^{-1}}\right)$$

organize your work

$$f_{xx} = 0$$

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$$\boxed{f(x,y,z) = 0}$$

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5. $\boxed{\text{15 points}}$ Let $L$ be the tangent plane to the graph of $z = f(x, y)$ at $(-2, 7, 3)$, such that it is also the tangent plane to graph of $h(x, y) = x^2 - y$ at $(-1, 1, 0)$, i.e., $L$ is a common tangent plane to the both of the graphs. Find the equation of $L$.

$h$

$\cancel{x}^2$

$$z = (a^2 - b) + 2a(x - a) + (-1)(y - b)$$

$$z0 = ((-1)^2 - 1) + 2(-1)(x+1) + (-1)(y-1)$$

$$z\cancel{0 = 0}$$

$$z0 = -2x - 2 - y + 1$$

$$z0 = -2$$

$$z = -2x - y - 1$$

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