NAME: Solution Set + Rubric ID: ______________

SIGNATURE: ______________

To get credit for a problem, you must show all of your reasoning and calculations. No calculators may be used. $\boxed{\textit{Box your final answer.}}$
If you cannot find a vector that you need for a later part of a problem, you may use the vector $\langle 1, 2, 3 \rangle$.

Circle your section:

Section:	**Tuesday:**	**Thursday:**	**TA:**
	2A	*2B*	*Frederick Vu*
	2C	*2D*	*Nicholas Boschert*
	2E	*2F*	*Victoria Kala*

Problem	**Possible**	**Points**
1	*10*	
2	*10*	
3	*10*	
4	*15*	
5	*15*	
6	*15*	
Total	*75*	

1. *(10 points) Find the area of the triangle with vertices at points*

$$P = (1,0,0), Q = (0,1,0), R = (0,0,1).$$

2 pts

2 pts

$$\left.\begin{aligned}\overrightarrow{PQ} &= \langle -1, 1, 0\rangle \\ \overrightarrow{PR} &= \langle -1, 0, 1\rangle\end{aligned}\right\} \qquad \overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}\vec{i} - \begin{vmatrix} -1 & 0 \\ -1 & 1 \end{vmatrix}\vec{j} + \begin{vmatrix} -1 & 1 \\ -1 & 0 \end{vmatrix}\vec{k} = \vec{i} + \vec{j} + \vec{k}$$

2 pts

$$\Rightarrow \|\overrightarrow{PQ} \times \overrightarrow{PR}\| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$$

2 pts

$\Rightarrow$ area of the triangle is $\boxed{\dfrac{\sqrt{3}}{2}}$

2 pts

2. *(2 points each) True/False! Circle the appropriate answer.* ***No justification is needed here.***

(1) For any two vectors $\vec{v}$ and $\vec{u}$, $\vec{v} \cdot \vec{u} = \vec{u} \cdot \vec{v}$.	*True* *False*
(2) For any two vectors $\vec{v}$ and $\vec{u}$, $\vec{v} \times \vec{u} = \vec{u} \times \vec{v}$.	*True* *False*
(3) If $\vec{u}$ and $\vec{v}$ are both orthogonal to $\vec{w}$, then $\vec{u} + \vec{v}$ is othogonal to $\vec{w}$.	*True* *False*
(4) The projection of $\vec{v}$ onto $\vec{w}$ can have a greater length than $\vec{v}$.	*True* *False*
(5) If $\vec{u}$ and $\vec{v}$ are orthogonal, then $$\|\|\vec{u}+\vec{v}\|\|^2 = \|\|\vec{u}\|\|^2 + \|\|\vec{v}\|\|^2.$$	*True* *False*

3. *(10 points)*

(a) *(5 points) Consider the vector*

$$\vec{v} = \langle -1, -1, 1 \rangle.$$

Find the equations of the lines parallel to $\vec{v}$

i. L_1 *passing through* $(0, -1, -2)$ $\longleftarrow$ $\langle x_0, y_0, z_0 \rangle = \langle 0, -1, -2 \rangle$

ii. L_2 *passing through* $(1, 2, -4)$ $\longleftarrow$ $\langle x_0, y_0, z_0 \rangle = \langle 1, 2, -4 \rangle$ } 1 pt each

1 pt $\longrightarrow$ $\vec{r}(t) = \vec{r}_0 + t\vec{v}$

i. $\vec{r}(t) = \langle 0, -1, -2 \rangle + t\langle -1, -1, 1 \rangle = \boxed{\langle -t, -1-t, -2+t \rangle}$

1pt each

ii. $\vec{r}(t) = \langle 1, 2, -4 \rangle + t\langle -1, -1, 1 \rangle = \boxed{\langle 1-t, 2-t, -4+t \rangle}$

(b) *(5 points) Find a vector perpendicular to* L_1 *that starts at a point on* L_1 *and ends at a point on* L_2.

Vector: $\vec{u} = \langle 1, 2, -4 \rangle - \langle 0, -1, -2 \rangle = \langle 1, 3, -2 \rangle$ – 1pt

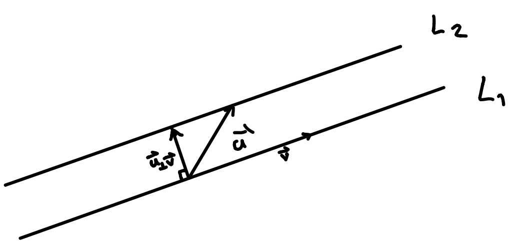
Want $\vec{u}_{\perp \vec{v}} = \vec{u} - \vec{u}_{\parallel \vec{v}}$

$\vec{u}_{\parallel \vec{v}} = \dfrac{\vec{u} \cdot \vec{v}}{\vec{v} \cdot \vec{v}} \cdot \vec{v}$ – 1pt

$\vec{u} \cdot \vec{v} = -1 - 3 - 2 = -6$ } 2p

$\vec{v} \cdot \vec{v} = 3$

$= -2\vec{v} = \langle 2, 2, -2 \rangle$



$\Rightarrow$ $u_{\perp \vec{v}} = \langle 1, 3, -2 \rangle - \langle 2, 2, -2 \rangle$

$= \langle -1, 1, 0 \rangle$ – 1pt

4. Describe the traces of

Only one trace: -6

$$x^2 - y^2 + z = 0,$$

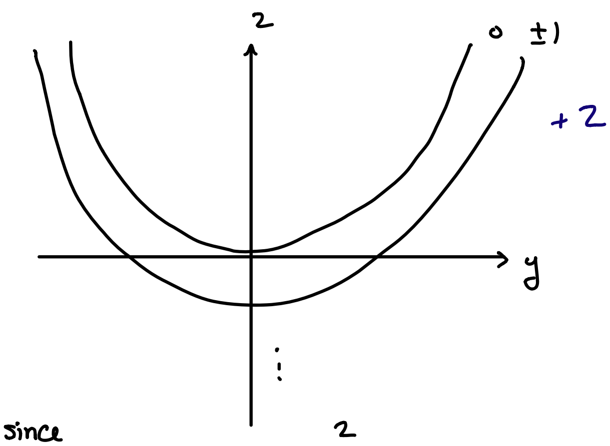
clearly labeling your plot and pointing out any places where the geometry changes (if there are any).

x-traces: (5 points) Sign of x doesn't matter.

$x=0$: $-y^2+z=0$: $z=y^2$

$x=\pm 1$: $-y^2+1+z=0$: $z=y^2-1$

So all $=k$ have $z=y^2-k^2$, which are parabolas opening up

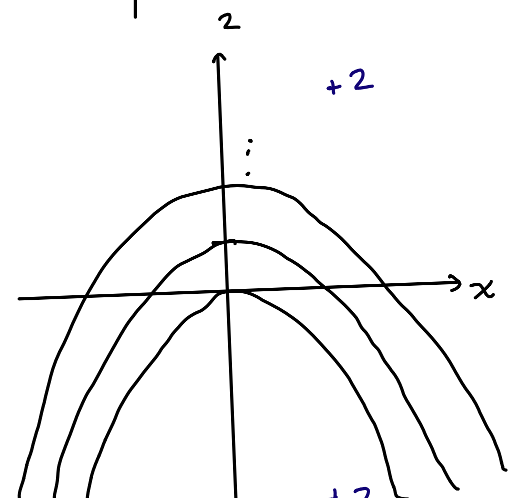


y-traces: (5 points) Sign of y doesn't matter, since y is squared :

$y=0$: $x^2+z=0$. $z=-x^2$

$y=\pm 1$: $-1+x^2+z=0$: $z=1-x^2$

for any $y=k$, always get $z=k^2-x^2$, which is a parabola opening down.



z-traces: (5 points)

+1

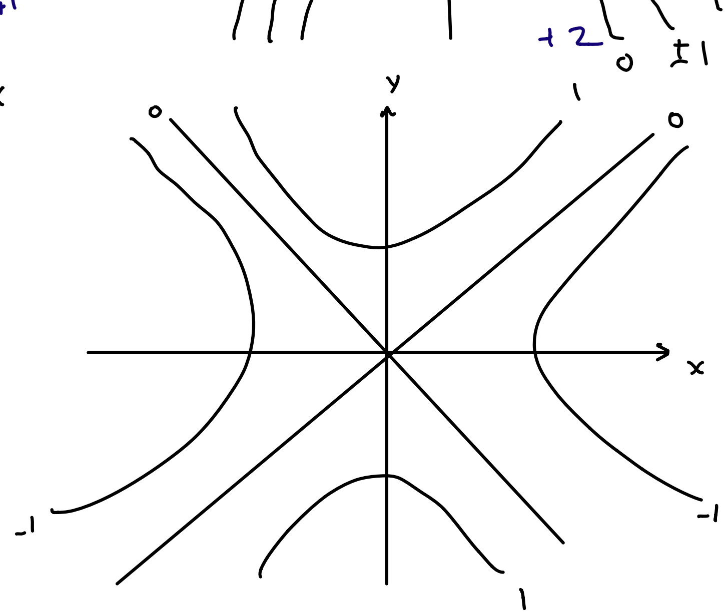
$z=0$: $x^2-y^2=0$: $y^2=x^2$: $y=\pm x$

$z=1$: $x^2-y^2+1=0$: $x^2-y^2=-1$ or $y^2-x^2=1$

for all $z>0$, get hyperbolas opening along y

$z=-1$: $x^2-y^2-1=0$: $x^2-y^2=1$

so for all $z<0$, get hyperbolas opening along x



No $z<0$: -2

5. *(15 points) Consider a particle moving according to*

$$\vec{r}(t) = \langle \cos(t), \sin(t), \cos(2t) \rangle.$$

(a) *(8 points) Find the velocity $\vec{v}(t)$ and acceleration $\vec{a}(t)$ as a function of t. Find the velocity and acceleration at $t = \pi/2$.*

$\vec{v}(t) = \vec{r}\,'(t) = \langle -\sin(t), \cos(t), -2\sin(2t) \rangle$ 2 pts

$\vec{a}(t) = \vec{r}\,''(t) = \langle -\cos(t), -\sin(t), -4\cos(2t) \rangle$ 2 pts

@ $\pi/2$: $\vec{v}(\pi/2) = \langle -1, 0, 0 \rangle$ 2 pts

$\vec{a}(\pi/2) = \langle 0, -1, 4 \rangle$ 2 pts

(b) *(7 points) Find the equation of the tangent line to the curve at $t = \pi/2$.*

2pts

$\vec{r}(\pi/2) = \langle 0, 1, -1 \rangle$, so our point is $(0,1,-1)$

& vector $= \vec{r}\,'(\pi/2) = \langle -1, 0, 0 \rangle$ 2 pts

Line:

$\vec{r}(t) = \langle 0,1,-1 \rangle + t \cdot \langle -1,0,0 \rangle$ 2 pts

$= \boxed{\langle -t, 1, -1 \rangle}$ 1 pt

6. *(15 points) Consider the plane P_1*

$$2x + y + 3z = 12.$$

(a) *(3 points) Find a normal vector $\vec{n}$ to P_1.*

$\vec{n} = \langle 2, 1, 3 \rangle$ (from our formula)

(b) *(5 points) Find an equation of the plane P_2 parallel to our given plane and passing through the point $R = (0, -1, 0)$.*

2 pts

$\vec{n} \cdot \langle x-0, y-{-1}, z-0 \rangle = 0$

1 pt

$2 \cdot (x-0) + (y+1) + 3z = 0$

$\boxed{2x + y + 3z = -1}$ 2 pts

(c) *(7 points) Choose a point Q on P_1, and let $\overrightarrow{QR}$ be the vector from Q to R. Find the projection of $\overrightarrow{QR}$ onto $\vec{n}$.*

1 pt

$Q = (0, 12, 0)$ is on P_1, so $\overrightarrow{QR} = \langle 0, -13, 0 \rangle$.

1 pt

$\overrightarrow{QR} \cdot \vec{n} = -13$

$\vec{n} \cdot \vec{n} = 2^2 + 1^2 + 3^2 = 14$ 1 pt

$\dfrac{\overrightarrow{QR} \cdot \vec{n}}{\vec{n} \cdot \vec{n}} \cdot \vec{n}$ 2 pt

$= \dfrac{-13}{14} \langle 2, 1, 3 \rangle = \boxed{\left\langle \dfrac{-13}{7}, \dfrac{-13}{14}, \dfrac{-39}{14} \right\rangle}$

2 pt