Midterm 2

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Instructions: Do not open this exam until instructed to do so. You will have 50 minutes to complete the exam. Please print your name and student ID number above. **You may use a calculator, as long as it is not a graphing calculator.** You may not use books, notes, or any other material to help you. Please make sure **your phone is silenced** and stowed where you cannot see it. If you have a smartwatch, please put this away too. You may use any available space on the exam for scratch work. If you need more scratch paper, please ask one of the proctors. You must **show your work** to receive credit. Please circle or box your final answers.

Please do not write below this line.

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total:	50	

[redacted]

1. (a) (5 points) Consider the limit $\lim_{(x,y)\to(0,0)} \dfrac{xy+y^3}{x^2+y^2}$. Either evaluate this limit or show that it does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{xy+y^3}{x^2+y^2}$$

Plugging in $\Rightarrow \frac{0}{0}$ X

$$\lim_{(x,0)\to(0,0)} \frac{0}{x^2} \quad 0 \quad \neq 0$$

$$\lim_{(0,y)\to(0,0)} \frac{y^3}{y^2}$$

$y = x$

$$\lim_{(x,x)\to(0,0)} \frac{x^2+x^3}{2x^2}$$

$$\lim_{(x,x)\to(0,0)} \frac{x^2(1+x)}{2x^2}$$

$$\lim_{(x,x)\to(0,0)} \frac{(1+x)}{2} = \frac{1}{2}$$

$y = mx$

$$\lim_{(x,y)\to(0,0)} \frac{mx^2+m^3x^3}{x^2+m^2x^2}$$

$$\lim_{(x,y)\to(0,0)} \frac{mx^2(1+mx)}{x^2(1+m^2)}$$

$$\lim_{(x,mx)\to(0,0)} \frac{m(1+mx)}{(1+m^2)}$$

$$= \frac{0}{1+m^2}$$

Limit DNE

(b) (5 points) Consider the limit $\lim_{(x,y)\to(1,2)} \dfrac{xy}{\sqrt{x^2+y^2}}$. Either evaluate this limit or show that it does not exist.

$$\lim_{(x,y)\to(1,2)} \frac{xy}{\sqrt{x^2+y^2}} = \frac{2}{\sqrt{1^2+2^2}} = \frac{2}{\sqrt{1+4}} = \boxed{\frac{2}{\sqrt{5}}}$$

2. (10 points) Consider the partial differential equation $u_t = -3u_x$. Verify that $u = \cos\left((x-3t)^2\right)$ is a solution to this equation.

$$u_t = -3u_x$$

$$u = \cos((x-3t)^2)$$

$$u_t = -\sin((x-3t)^2) \cdot (2(x-3t)) \cdot (-3)$$

$$u_t = \underline{6(x-3t)\sin((x-3t)^2)}$$

$$u_x = -\sin((x-3t)^2) \cdot 2(x-3t)$$

$$-3u_x = -3\left[-\sin((x-3t)^2) \cdot 2(x-3t)\right]$$

$$= \underline{6(x-3t)\sin((x-3t)^2)}$$

$$6(x-3t)\sin((x-3t)^2) = 6(x-3t)\sin((x-3t)^2)$$

$$u_t = -3u_x$$

$\log(x)$

$\ln(x) + x$?

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3. For both parts of this question, let $f(x, y) = \log(8x^2 + 2y^2)$. *(Note that this is* log *base ten)*

(a) (6 points) Find the equation of the tangent plane to the graph $z = f(x, y)$ at the point $(1, 1, 1)$.

$$z = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$$

$$\boxed{z = 1 + \frac{16}{10 \ln 10}(x-1) + \frac{4}{10 \ln 10}(y-1)}$$

$$\log(x) \Rightarrow \frac{1}{\ln(10)} \cdot \frac{1}{x}$$

$$f_x(a,b) = \frac{1}{\ln(10)} \cdot \frac{1}{(8x^2+2y^2)} \cdot (16x) = \frac{16}{\ln(10)(10)}$$

$$f_y(a,b) = \frac{1}{\ln(10)} \cdot \frac{1}{(8x^2+2y^2)} \cdot (4y) = \frac{4}{\ln(10)(10)}$$

(b) (4 points) Is there any point on the graph $z = f(x, y)$ at which the tangent plane is horizontal? If yes, find one. If no, explain why.

horizontal tangent plane means $f_x = f_y = 0$

$$f_x = \frac{16x}{(8x^2+2y^2)\ln 10} = \frac{1}{\ln(10)} \cdot \frac{16x}{2(4x^2+y^2)} = \frac{1}{\ln(10)} \cdot \frac{8x}{4x^2+y^2} = 0$$

$$f_y = \frac{4y}{(8x^2+2y^2)\ln(10)} = 0$$

No, there is no point at which both f_x and f_y can be zero at the same time. A horizontal tangent plane means that the gradient must equal zero, as there is no movement / change in xy plane. This makes sense because a log (or ln) function can never be zero.

4. Consider the function $f(x, y) = e^{x^2+3y^2}$.

(a) (5 points) Find the gradient, ∇f.

$\nabla f = \langle f_x, f_y \rangle$

$\nabla f = \langle e^{x^2+3y^2}(2x), e^{x^2+3y^2}(6y) \rangle$

(b) (5 points) Let $\mathbf{u} = \langle 1, -1 \rangle$ Use your work from part (a) to calculate the directional derivative of $f(x, y)$ in the direction of $\mathbf{u}$ at the point $(2, 1)$.

$\langle 1, -1 \rangle$

$\vec{u} = \frac{1}{\sqrt{2}} \langle 1, -1 \rangle$

$D_u f(a, b) = \nabla f \cdot \vec{u}$

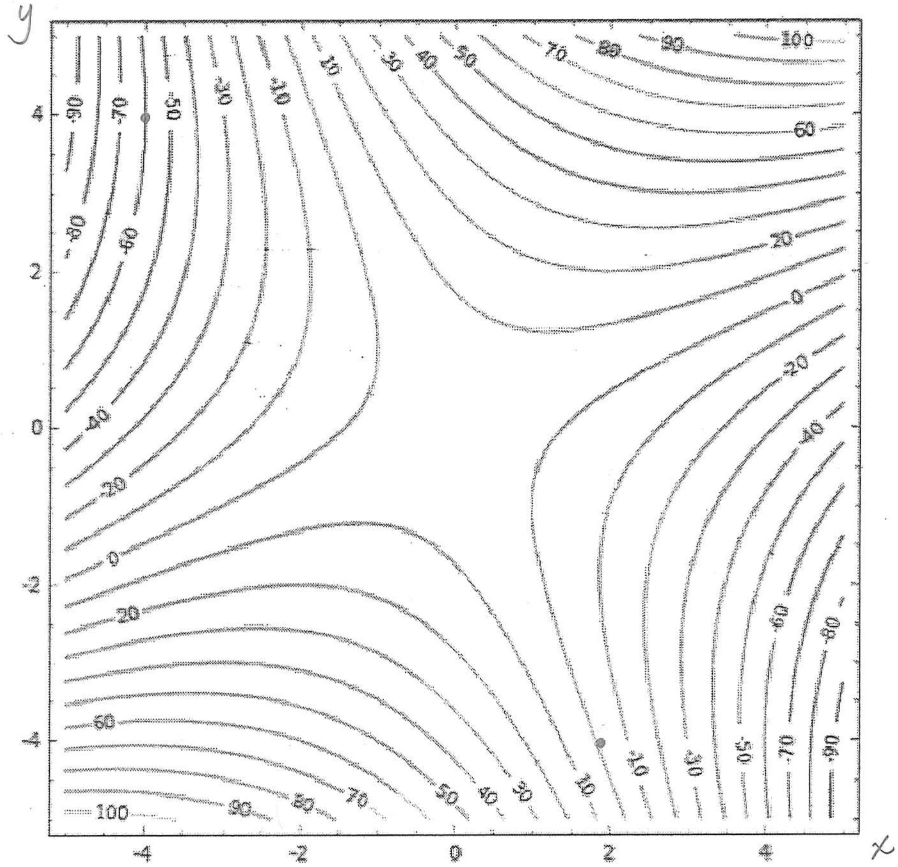
$(2,1) \quad = \langle e^{x^2+3y^2}(2x), e^{x^2+3y^2}(6y) \rangle \cdot \langle \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}} \rangle$

$= \langle e^{4+3}(4), e^{4+3}(6) \rangle \cdot \langle \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}} \rangle$

$= \langle 4e^7, 6e^7 \rangle \cdot \langle \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}} \rangle$

$= \frac{4e^7}{\sqrt{2}} - \frac{6e^7}{\sqrt{2}} = \boxed{\frac{4e^7 - 6e^7}{\sqrt{2}}} = \frac{-2e^7}{\sqrt{2}}$

5. Consider the contour map of a function $f(x, y)$ given below. You may assume that the horizontal axis is the x-axis, and the vertical axis is the y-axis.



$f_x = f(x,y)_x$

$\langle 1, 0\rangle$

$\frac{1}{\sqrt{1+0}} = 1$

(a) (5 points) Let $\mathbf{u} = \langle 1, 0\rangle$. What is the sign of $D_{\mathbf{u}}f(2, -4)$? Justify your answer.

$D_u f(2,-4) = \nabla f \cdot \vec{u} = \nabla f \cdot \langle 1, 0\rangle$

$= \langle f_x, f_y\rangle \cdot \langle 1, 0\rangle = f_x$

$D_u f(2,-4)$ is negative because as the value of z increases, the level curves shift left, becoming more negative.

$\langle \frac{1}{\sqrt{5}}, \frac{-2}{\sqrt{5}}\rangle$

(b) (5 points) Let $\mathbf{u} = \langle 1, 0\rangle$ again while $\mathbf{v} = \langle \frac{1}{\sqrt{5}}, \frac{-2}{\sqrt{5}}\rangle$. Which is larger, $D_{\mathbf{u}}f(-4, 4)$ or $D_{\mathbf{v}}f(-4, 4)$? Justify your answer.

$\frac{1}{\sqrt{\frac{1}{5}+\frac{4}{5}}} = 1$

$D_u f(-4,4) = \nabla f \cdot \langle 1, 0\rangle$ $\qquad$ $D_v = \nabla f \cdot \langle \frac{1}{\sqrt{5}}, \frac{-2}{\sqrt{5}}\rangle$

$= f_x$ $\qquad$ $= \frac{f_x}{\sqrt{5}} - \frac{f_y}{\sqrt{5}}$

$D_u f(-4,4)$ is larger

because $\frac{f_y}{\sqrt{5}}$ is an extremely large number, meaning $f_x/\sqrt{5} - f_y/\sqrt{5}$ will be a very small number.