Math 32A/1
Fall 2016
10/17/16
Time Limit: 50 Minutes

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Day \ T.A.	Alex	David	Kevin
Tuesday	1A	1C	1E
Thursday	1B	1D	1F

This exam contains 5 pages (including this cover page) and 4 problems. Check to see if any pages are missing. Enter your name and SID number on the top of this page, cross the box correspondiing to your discussion section, and put your initials on the top of every page, in case the pages become separated. Also, have your photo ID on the desk in front of you during the exam.

Use a pen to record your answers. Calculators or computers of any kind are not allowed. You are not allowed to consult any other materials of any kind, including books, notes and your neighbors. You may use the back of this sheet for your notes ("scratch paper"). If you need additional paper, let the proctors know.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit.** A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Problem	Points	Score
1	10	10
2	10	10
3	10	10
4	10	9
Total:	40	39

Of course, if you have a question about a particular problem, please raise your hand and one of the proctors will come and talk to you.

1. (10 points) Let $P = (4,3)$, $Q = (2,-4)$. $O = (0,0)$ denotes the origin.
   (a) Find the vector $\overrightarrow{PQ}$.
   (b) Find the unit vector in the direction of $\overrightarrow{OP}$. Simplify your answer.
   (c) Determine whether the angle between $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ is obtuse ($> \pi/2$) or acute ($< \pi/2$).
   (d) Find the components of the vector

$$4(2\hat{\imath} - 3\hat{\jmath}) - 2(3\hat{\imath} + \hat{\jmath}).$$

   (e) Determine whether the following three points are collinear, i.e. whether they lie on the same line:

$$R = (-1,2,5), \quad T = (2,4,3), \quad U = (7,1,4).$$

(a) $\overrightarrow{PQ} = \langle 2-4, (-4-3 \rangle = \langle -2, -7 \rangle$

$\overrightarrow{PQ} = \langle -2, -7 \rangle$

(b) $\vec{u} = \dfrac{\overrightarrow{OP}}{\|\overrightarrow{OP}\|}$ $\quad \overrightarrow{OP} = \langle 4, 3 \rangle$ $\quad \|\overrightarrow{OP}\| = \sqrt{4^2 + 3^2} = \sqrt{25} = 5$

$\vec{u} = \frac{1}{5}\langle 4, 3 \rangle = \left\langle \frac{4}{5}, \frac{3}{5} \right\rangle$ $\quad \vec{u} = \left\langle \frac{4}{5}, \frac{3}{5} \right\rangle$

(c) If $\overrightarrow{OP} \cdot \overrightarrow{OQ}$ is negative, the angle is obtuse, because

$\overrightarrow{OP} \cdot \overrightarrow{OQ} = \|\overrightarrow{OP}\| \|\overrightarrow{OQ}\| \cos\theta$

$\langle 4, 3 \rangle \cdot \langle 2, -4 \rangle = 8 - 12 = -4$

The angle is obtuse

Because $\overrightarrow{OP} \cdot \overrightarrow{OQ}$ is negative, the angle is obtuse

(d) $\langle 8, -12 \rangle - \langle 6, 2 \rangle = \langle 2, -14 \rangle$

$2\hat{\imath} - 14\hat{\jmath}$. $\quad 2\hat{\imath} - 14\hat{\jmath}$

(e) $\overrightarrow{RT} = \langle 3, 2, -2 \rangle$ $\quad \overrightarrow{RU} = \langle 8, -1, -1 \rangle$

$$\overrightarrow{RT} \times \overrightarrow{RU} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & 2 & -2 \\ 8 & -1 & -1 \end{vmatrix} = \hat{\imath}(-2-2) - \hat{\jmath}(-3-(-16)) + \hat{k}(-3-16) \neq \vec{0}$$

The points are not collinear

Points are not collinear

2. (10 points) (a) Let $P = (1,2,3)$, $Q = (6,5,4)$, $R = (2,3,4)$. Compute the area of the triangle with vertices P, Q, R. Simplify your answer.

(b) Calculate the components of

$$(2\hat{\imath} - 4\hat{\jmath}) \times (3\hat{k} + \hat{\jmath}).$$

(a) Area of triangle $= \dfrac{\|\vec{PQ} \times \vec{PR}\|}{2}$

7/7

$\vec{PQ} = \langle 5, 3, 1 \rangle \quad \vec{PR} = \langle 1, 1, 1 \rangle$

$\vec{PQ} \times \vec{PR} \to \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & 3 & 1 \\ 1 & 1 & 1 \end{vmatrix} = \hat{i}(3-1) - \hat{j}(5-1) + \hat{k}(5-3) = \langle 2, -4, 2 \rangle$

$\dfrac{\|\vec{PQ} \times \vec{PR}\|}{2} = \frac{1}{2}\sqrt{4+16+4} = \frac{1}{2}\sqrt{24} = \sqrt{6}$

$\sqrt{6}$ units^2

(b) $\langle 2, -4, 0 \rangle \times \langle 0, 1, 3 \rangle$

$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -4 & 0 \\ 0 & 1 & 3 \end{vmatrix} = \hat{i}(-12) - \hat{j}(6) + \hat{k}(2)$

$\langle -12, -6, 2 \rangle$

$-12\hat{i} - 6\hat{j} + 2\hat{k}$

$-12\hat{i} - 6\hat{j} + 2\hat{k}$

3/3

3. (10 points) (a) Find the equation of the plane which contains the two lines parametrized by

$$\vec{r_1}(t) = \langle t, 1, 2t \rangle, \qquad \vec{r_2}(t) = \langle -1, -t, -2 \rangle.$$

(b) Find the equation of the plane through $P = (3, 4, 5)$ parallel to $x + 3y - z = 0$.

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(a) $\vec{n} = \langle 1,0,2\rangle \times \langle 0,-1,0\rangle \rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 2 \\ 0 & -1 & 0 \end{vmatrix} = \hat{i}(0-(-2)) - \hat{j}(0-0) + \hat{k}(-1) =$
$\langle 2,0,-1\rangle$

$P = \vec{r}_1(0) = \langle 0,1,0\rangle$

$2(x-0) + 0(y-1) - 1(z-0) = 0$

$2x - z = 0 \qquad 2x = z$

$2x = z$

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(b) $\vec{n} = \langle 1,3,-1\rangle \quad P = (3,4,5)$

$1(x-3) + 3(y-4) - 1(z-5) = 0$

$x - 3 + 3y - 12 - z + 5 = 0$

$x + 3y - z = 10$

$x + 3y - z = 10$

4. (10 points) (a) Parametrize the intersection of the surfaces $x^2 - z^2 = 2$ and $xy = 3$ using $x = t$ as parameter. Make sure to specify the domain of your parametrization.

(b) Let $\vec{u}$ and $\vec{v}$ be non-zero vectors, and let λ be a scalar. Show that

$$\lambda(\vec{v}_{\|\vec{u}}) = (\lambda\vec{v})_{\|\vec{u}}.$$

(a) $y = \frac{3}{x}$ $z^2 = x^2 - 2$

$z = \pm\sqrt{x^2-2}$

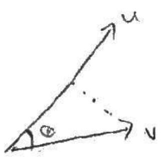
$\vec{r}_1(t) = \langle t, \frac{3}{t}, \sqrt{t^2-2} \rangle$, for $t \geq \sqrt{2}$

$\langle t, \frac{3}{t}, -\sqrt{t^2-2} \rangle$

(−1)

$\langle t, \frac{3}{t}, \sqrt{t^2-2} \rangle$,
$\langle t, \frac{3}{t}, -\sqrt{t^2-2} \rangle$ for $t \geq \sqrt{2}$

(b)



$\vec{p} = \vec{v}_{\|\vec{u}}$

$\cos\theta = \frac{\|\vec{p}\|}{\|\vec{v}\|}$ $\cos\theta = \frac{\vec{u}\cdot\vec{v}}{\|u\|\|v\|}$

$\frac{\vec{u}\cdot\vec{v}}{\|u\|\cancel{\|v\|}} = \frac{\|\vec{p}\|}{\cancel{\|\vec{v}\|}}$ $\vec{p} = \frac{\vec{u}\cdot\vec{v}}{\|u\|}\frac{\vec{u}}{\|u\|}$:

$\frac{\vec{u}\cdot\vec{v}}{\|u\|^2}\vec{u}$

$\lambda(\vec{v}_{\|\vec{u}}) = \lambda\left(\frac{\vec{u}\cdot\vec{v}}{\|u\|^2}\vec{u}\right)$

$(\lambda\vec{v})_{\|\vec{u}} = \frac{\vec{u}\cdot\lambda\vec{v}}{\|u\|^2}\vec{u} = \frac{\lambda(\vec{u}\cdot\vec{v})}{\|u\|^2}\vec{u} =$

$\lambda\left(\frac{\vec{u}\cdot\vec{v}}{\|u\|^2}\vec{u}\right) = \lambda(\vec{v}_{\|\vec{u}})$ ✓

$\vec{u}\cdot\lambda\vec{v} = \lambda(\vec{u}\cdot\vec{v})$

$\underline{\vec{u}\cdot\lambda\vec{v}} = \langle u_x, u_y, u_z\rangle \cdot \lambda\langle v_x, v_y, v_z\rangle =$
$\langle u_x, u_y, u_z\rangle \cdot \langle \lambda v_x, \lambda v_y, \lambda v_z\rangle =$

$\lambda u_x v_x + \lambda u_y v_y + \lambda u_z v_z =$
$\lambda(u_x v_x + u_y v_y + u_z v_z) = \underline{\lambda(\vec{u}\cdot\vec{v})}$

Because $\vec{v}_{\|\vec{u}} = \frac{\vec{u}\cdot\vec{v}}{\|u\|^2}\vec{u}$, and

$\vec{u}\cdot\lambda\vec{v} = \lambda(\vec{u}\cdot\vec{v})$,

$\lambda(\vec{v}_{\|\vec{u}}) = (\lambda\vec{v})_{\|\vec{u}}$