20F-MATH32A-2 Test 2

MATTHEW NIEVA

TOTAL POINTS

94 / 94

QUESTION 1

1 Instructions / Admonishment 0 / 0

✓ **- 0 pts** Correct

QUESTION 2

2 True of False 15 / 15

✓ **- 0 pts** Correct

- 7 pts Wrong answer for i. The answer is false.

- 8 pts Wrong answer for ii. The answer is false.

- 3 pts Correct answer for i but incorrect (or missing) justification. This is false because level curves refer to curve in the domain not the image. Common mistakes: say this is false because of possible complex values, or say this is false because level curve only contains one number in the range, not the entire range etc.

- 7 pts Correct answer for ii but incorrect justification. Some of you attempted integration but did it incorrectly, although it is possible to do this using integration. Typically for these sort of questions you use integrations to find such functions when they do exist, but use Clairaut's theorem when you want to show that it doesn't exist. Also there is no such thing as "a partial integral". Also Clairaut's theorem says f_xy=f_yx, NOT f_xx=f_yy.

- 2 pts Small messups in the justification of (i).

- 2 pts Wrong computation in (ii).

- 2 pts Other, see comment.

- 7 pts Some people wrote two explanations, one correct one incorrect.

QUESTION 3

3 Examples 15 / 15

✓ **+ 5 pts (a) Correct nontrivial example**

✓ **+ 5 pts (a) Correctly showed mixed partials disagree (i.e. conclusion of thm is false) OR that they are not continuous (i.e. hypotheses of thm are false).**

+ 5 pts (a) Clairaut's theorem did not apply for a trivial reason, e.g. the graph was not the graph of a function or was undefined/discontinuous at the point in question (Update: full credit given for this item)

+ 5 pts (a) Did not completely verify Clairaut's theorem did not apply, either made a mistake or did not fully justify a claim (e.g. claimed mixed partials disagreed or were discontinuous without showing this) OR showed mixed partials did not exist, but this was because original function was discontinuous or not differentiable. Either way must mention mixed partials. (Update: full credit given for this item)

✓ **+ 5 pts (b) Function has correct domain and range**

+ 2 pts (b) Function only has one of domain, range correct

+ 1 pts (b) Surface is not the graph of a function but otherwise sort of fits description

+ 2 pts Give correct example and state that mixed partial derivatives f_xy and f_yx dont not match without a proof

QUESTION 4

Limit and Continuity 15 pts

4.1 Region of continuity 8 / 8

✓ **+ 2 pts Continuity for points away from origin**

✓ **+ 4 pts Correctly show limit at origin is 0 (by using polar coordinate or by definition)**

✓ **+ 2 pts Discontinuity at origin because limit not equal to value of function.**

+ 1 pts Partial credit for continuity away from origin

+ 3 pts Show limit at origin is 0 with other method that is not sufficient (e.g. y=mx)

+ 1 pts Wrong conclusion for limit at the origin but show some work

+ **1 pts** Partial credit for discontinuity at origin

+ **3 pts** Showing limit in a correct way, but didn't derive the correct conclusion

+ **0 pts** Nothing correct

4.2 Limit at origin 7 / 7

✓ + **5 pts** **Multiplying by conjugate and simplify correctly**

✓ + **2 pts** **Correctly calculate limit = 4**

+ **4 pts** Use polar coordinate in the correct form

+ **3 pts** Correctly complete calculation with polar coordinate

+ **4 pts** Get correct result but with some method that is not sufficient for limit

+ **2 pts** Used some method that is not sufficient and didn't get correct result

+ **1 pts** Partial credit for computation

+ **0 pts** Nothing correct

QUESTION 5

Differentiability 15 pts

5.1 Discontinuity and differentiability at origin 9 / 9

✓ + **6 pts** **Check y=mx or polar coordinate for discontinuity**

✓ + **3 pts** **Not differentiable because of discontnuity**

+ **3 pts** Proved for discontinuity but not sufficient

+ **1 pts** Proved for non-differentiability but not sufficient

+ **0 pts** Nothing correct

5.2 Partial derivative 6 / 6

✓ + **2 pts** **Correctly calculate fx**

✓ + **2 pts** **Correctly calculate fy**

✓ + **2 pts** **Reasonable observation**

+ **1 pts** Partial credit for calculating fx

+ **1 pts** Partial credit for calculating fy

+ **1 pts** Partial credit for observation

+ **6 pts** Correct

QUESTION 6

6 Directional Derivative 15 / 15

✓ - **0 pts** **Correct**

- **2 pts** Did not find the correct largest slope

- **5 pts** Made mistake in the calculation of the gradient, but the process is correct; did not se a unit vector for the calculation of the directional derivative

- **5 pts** Use limit definition to compute the directions derivative but result was not correct

- **5 pts** Confuse the magnitude of the gradient with a vector

- **2 pts** Did not find the maximum rate of increase

- **10 pts** Find incorrect partial derivative

QUESTION 7

7 Chain Rule 15 / 15

✓ - **0 pts** **Correct**

- **5 pts** Computation error. Answer is 2/5 by chain rule.

- **15 pts** Not knowing how chain rule works.

QUESTION 8

8 Bonus Question 4 / 4

✓ + **4 pts** **Correct**

- **2 pts** Took correctly one partial derivative

- **4 pts** No action

TEST 2

MATH 32A @ UCLA (FALL 2020)

$\overset{x+y}{f_x = f_y}$

Assigned: December 02, 2020.

Instructions/Admonishment

1. SHOW ALL WORK.
   A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit.
2. Duration: 24 hours.
3. The following is my own work, without the aid of any other person.
   Signature: Matthew Nuera

Exercise 1 Matthew Nieva

i) Definition of a level curve: All points (x,y) where $f(x,y)=c$ within domain of $f(x,y)$. So, a level curve is a set of input points (x,y).

Definition of range: set of output values $f(x,y)=z$ within the domain of $f(x,y)$. So, an element of the range of $f(x,y)$ is an output value $z=f(a,b)$ of an input $(x,y)=(a,b)$. Since a level curve is defined as a set of input points (x,y), level curves will not contain elements of the range of $f(x,y)$ since elements of the range of $f(x,y)$ are output values, not input points

$\boxed{\therefore \text{FALSE}}$

ii) $f_x = f_y = e^x \sin(xy)$

$f_{xy} = xe^x \cos(xy)$

$f_{yx} = e^x \sin(xy) + ye^x \cos(xy)$

$f_{xy} \neq f_{yx}$ $\therefore$ Clairaut's Theorem doesn't apply

Clairaut's Theorem Preconditions:

f_{xy} & f_{yx} exist and are continuous

f_{xy} & f_{yx} are both continuous, using the conditions given, so a function with the partial

derivatives f_{xy} and f_{yx} must not be possible for Clairaut's Theorem to not apply.

$\therefore$ a function $f(x,y)$ where $f_x = f_y = e^x \sin(xy)$ with continuous second partial derivatives does not exist

$\boxed{\therefore \text{FALSE}}$

Exercise 2 Matthew Nieva

(i) Clairaut's Theorem Preconditions:
f_{xy} and f_{yx} exist and are continuous

Consider $f(x,y) = \begin{cases} \dfrac{x^3}{x^2+y^2} & ; (x,y) \neq (0,0) \\ 0 & ; (x,y) = (0,0) \end{cases}$

$$\lim_{(x,y)\to(0,0)} \frac{x^3}{x^2+y^2}$$

$$= \lim_{r\to 0} \frac{r^3\cos^3\theta}{r^2(\cos^2\theta + \sin^2\theta)} = \lim_{r\to 0} r\cos^3\theta$$

$\because -1 \leq \cos^3\theta \leq 1$

$$\lim_{r\to 0} -r \leq \lim_{r\to 0} r\cos^3\theta \leq \lim_{r\to 0} r$$

$$0 \leq \lim_{r\to 0} r\cos^3\theta \leq 0$$

$\therefore \lim_{(x,y)\to(0,0)} f(x,y) = 0 = f(0,0)$

So $f(x,y)$ is continuous on $\mathbb{R}^2$, so we can consider differentiability on all points (x,y)

$$f_x = \frac{3x^2(x^2+y^2) - 2x(x^3)}{(x^2+y^2)^2} = \frac{x^4+3x^2y^2}{(x^2+y^2)^2}$$

$$f_y = \frac{-2x^3y}{(x^2+y^2)^2}$$

$$f_{xy} = \frac{6x^2y(x^2+y^2)^2 - 4y(x^4+3x^2y^2)}{(x^2+y^2)^4}$$

$$f_{yx} = \frac{(x^2+y^2)^2(-6x^2y) - (-2x^3y)(4x[x^2+y^2])}{(x^2+y^2)^4}$$

(0,0) causes denominator to be zero

At x-y values of (0,0), the second partial derivatives of $f(x,y)$, f_{xy} and f_{yx}, are not continuous at (0,0) even though $f(x,y)$ is continuous at (0,0) $\therefore$ Clairaut's Theorem does not apply to $f(x,y)$, since it fails to meet the preconditions of Clairaut's Theorem

(ii) Domain: $\{(x,y): x>0\}$
Range: $\mathbb{R}$

Consider
$$f(x,y) = \frac{y\sqrt{x}}{x}$$

The $\sqrt{x}$ in the numerator restricts the domain to $x \geq 0$, and the x in the denominator further restricts the domain since $f(x,y)$ will not be defined if $x=0$. Taking that into account, $f(x,y)$ has a domain of $\{(x,y): x>0\}$

The range of the rational function has no restrictions and can be any number So Range of $f(x,y)$: $\mathbb{R}$

Exercise 3 Matthew Nieva

(i) $f(x,y) = \begin{cases} \dfrac{x^3+y^3}{x^2+y^2} ; & (x,y) \neq (0,0) \\ 1 ; & (x,y) = (0,0) \end{cases}$

We know that $f(x,y)$ is continuous at all points $(x,y) \neq (0,0)$ since $f(x,y)$ is a rational function. We must test continuity at $(0,0)$

$$\lim_{(x,y)\to(0,0)} \frac{x^3+y^3}{x^2+y^2}$$ since $(0,0)$ makes denominator 0

$$= \lim_{r\to 0} \frac{r^3(\cos^3\theta+\sin^3\theta)}{r^2(\cos^2\theta+\sin^2\theta)} \qquad \begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$$

$$= \lim_{r\to 0} r(\cos^3\theta+\sin^3\theta)$$

$$-1 \leq \cos^3\theta+\sin^3\theta \leq 1$$

$$\therefore \lim_{r\to 0}(-r) \leq \lim_{r\to 0} r(\cos^3\theta+\sin^3\theta) \leq \lim_{r\to 0} r$$

$$0 \leq \lim_{r\to 0} r(\cos^3\theta+\sin^2\theta) \leq 0$$

$$\therefore \lim_{(x,y)\to(0,0)} f(x,y) = \lim_{r\to 0} r(\cos^3\theta+\sin^3\theta) = 0$$

$$f(0,0) = 1$$

$$\lim_{(x,y)\to(0,0)} f(x,y) \neq f(0,0)$$

$\therefore f(x,y)$ is not continuous on $(0,0)$

$f(x,y)$ is continuous on the set of points.
$\{(x,y) \in \mathbb{R}^2 : x^2+y^2 \neq 0\}$
or all points except $(0,0)$

(ii)

$$= \lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{\sqrt{x^2+y^2+4}-2}\left(\frac{\sqrt{x^2+y^2+4}+2}{\sqrt{x^2+y^2+4}+2}\right)$$

$$= \lim_{(x,y)\to(0,0)} \frac{(x^2+y^2)(\sqrt{x^2+y^2+4}+2)}{(x^2+y^2)}$$

$$= \lim_{(x,y)\to(0,0)} \sqrt{x^2+y^2+4}+2$$

$$= \sqrt{4}+2 = \boxed{4}$$

$$\boxed{\lim_{(x,y)\to(0,0)}\left(\frac{x^2+y^2}{\sqrt{x^2+y^2+4}-2}\right) = 4}$$

Exercise 4 Matthew Nieva

$$f(x,y) = \begin{cases} \frac{-xy}{x^2+y^2} ; & (x,y) \neq (0,0) \\ 0 ; & (x,y) = (0,0) \end{cases}$$

(i) $f(0,0) = 0$

$$\lim_{(x,y)\to(0,0)} \frac{-xy}{x^2+y^2}$$

Let $y = mx$

$$\lim_{(x,mx)\to(0,0)} \frac{-m(x^2)}{x^2(1+m^2)}$$

$$= \lim_{(x,mx)\to(0,0)} \frac{-m}{1+m^2}$$

When $m=1$, limit is $-\frac{1}{2}$

When $m=0$, limit is 0

Along different paths, we get different limit values so $\lim_{(x,y)\to(0,0)} f(x,y)$ DNE

Since $f(0,0) \neq \lim_{(x,y)\to(0,0)} f(x,y)$, $f(x,y)$ is discontinuous at $(0,0)$

For a function to be differentiable at a point, it must be continuous at that point since differentiability implies continuity.

$f(x,y)$ is not continuous at $(0,0)$ so $f(x,y)$ is not differentiable at $(0,0)$

(ii)

$$f_x(0,0) = \lim_{h\to 0} \frac{f(0+h,0) - f(0,0)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{0}{h^2} - 0}{h}$$

$$= \lim_{h\to 0} \frac{0}{h} = 0 \qquad \boxed{f_x(0,0) = 0}$$

$$f_y(0,0) = \lim_{k\to 0} \frac{f(0,0+k) - f(0,0)}{k}$$

$$= \lim_{k\to 0} \frac{\frac{0}{k^2} - 0}{k}$$

$$= \lim_{k\to 0} \frac{0}{k} = 0 \qquad \boxed{f_y(0,0) = 0}$$

For a function to be differentiable at a point (a,b) $f_x(a,b)$ and $f_y(a,b)$ must both exist and be continuous at (a,b)

Since $f_x(0,0)$ and $f_y(0,0)$ exist BUT $f(x,y)$ is not differentiable at $(0,0)$, we can conclude that f_x or f_y or both f_x and f_y are not continuous at $(0,0)$

Exercise 5 Matthew Nieva

$f(x,y) = \ln(xy)$

(i) $f_x = \frac{1}{xy}(y) = \frac{y}{xy} = \frac{1}{x}$

$f_y = \frac{1}{xy}(x) = \frac{1}{y}$

$\nabla f = \langle \frac{1}{x}, \frac{1}{y} \rangle$

$\boxed{\nabla f(\frac{1}{2}, \frac{1}{3}) = \langle 2,3 \rangle}$

$\vec{v} = \langle 1,2 \rangle$ Let $\vec{u} = \frac{\vec{v}}{\|\vec{v}\|}$

$\vec{u} = \frac{\langle 1,2 \rangle}{\|\langle 1,2 \rangle\|} = \frac{\langle 1,2 \rangle}{\sqrt{1+4}} = \frac{1}{\sqrt{5}}\langle 1,2 \rangle$

$D_{\vec{u}} f(\frac{1}{2},\frac{1}{3}) = \nabla f(\frac{1}{2},\frac{1}{3}) \cdot \vec{u}$

$= \langle 2,3 \rangle \cdot \frac{1}{\sqrt{5}}\langle 1,2 \rangle$

$= \frac{1}{\sqrt{5}}(2+6) = \frac{8}{\sqrt{5}}$

$\boxed{D_{\vec{u}} f(\frac{1}{2},\frac{1}{3}) = \frac{8\sqrt{5}}{5}}$

(ii) $D_u f(p) = \|\nabla f_p\| \cos\theta$ as well

where θ is angle between ∇f_p and u

$-1 \leq \cos\theta \leq 1$

$\cos\theta$ has a max when $\theta = 0$, so ∇f_p and u would point in same direction of max $D_u f(p)$

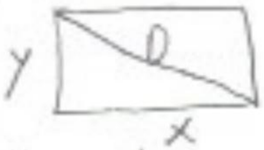
$\therefore \nabla f_p$ points in direction of the largest directional derivative.

Direction of largest directional derivative at $(\frac{1}{2},\frac{1}{3})$ is $\nabla f(\frac{1}{2},\frac{1}{3}) = \langle 2,3 \rangle$

Largest value: $\|\nabla f_p\| \cos 0$

$= \|\nabla f_p\| = \sqrt{2^2+3^2} = \sqrt{13}$

Exercise 6 Matthew Nieva



Let $D = \sqrt{x^2+y^2}$

Given: $\frac{dx}{dt} = 2$ $\frac{dy}{dt} = -1$ Find $\frac{dD}{dt}$

$$D = (x^2+y^2)^{\frac{1}{2}}$$

$$D_x = \frac{2x}{2\sqrt{x^2+y^2}} = \frac{dD}{dx}$$

$$D_y = \frac{2y}{2\sqrt{x^2+y^2}} = \frac{dD}{dy}$$

$$\frac{dD}{dt} = \frac{dD}{dx}\frac{dx}{dt} + \frac{dD}{dy}\frac{dy}{dt}$$

$$\left.\frac{dD}{dt}\right|_{\substack{x=3\\y=4}} = D_x(3,4)(2) + D_y(3,4)(-1)$$

$$= \frac{6(2)}{2\sqrt{25}} - \frac{8}{2\sqrt{25}} = \frac{4}{10} = \frac{2}{5}$$

$$\frac{dD}{dt} = \frac{2}{5}$$

When $x=3$ and $y=4$, the length of a diagonal of the rectangle is increasing at a rate of $\frac{2}{5}$

Exercise 7 Matthew Nieva

$$\frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2}, \quad k=1$$

Prove that $T(x,t) = Ae^{-a^2 t}\cos(ax)$ satisfies $\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2}$

$T(x,t) = Ae^{-a^2t}\cos(ax)$

$T_t = A\cos(ax)e^{-a^2t}(-a^2)$

$= -a^2 A\cos(ax)e^{-a^2t}$

$T_x = Ae^{-a^2t}(-\sin(ax))(a)$

$= -aAe^{-a^2t}\sin(ax)$

$T_{xx} = -a^2 Ae^{-a^2t}\cos(ax)$

$= -a^2 A\cos(ax)e^{-a^2t}$

$T_{xx} = T_t$

$\therefore$ $T(x,t)$ satisfies the heat equation with $k=1$ because

$$\boxed{\frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial x^2} \text{ when } k=1}$$