

1. (10 points) Consider $r(t) = (t+1, t^2, -t^3+1)$. Compute the curvature at $t=0$.

$$r'(t) = \langle 1, 2t, -3t^2 \rangle$$

$$r''(t) = \langle 0, 2, -6t \rangle$$

$$r'(t) \times r''(t) = \langle 1, 2t, -3t^2 \rangle \times \langle 0, 2, -6t \rangle$$

$$\begin{vmatrix} i & j & k \\ 1 & 2t & -3t^2 \\ 0 & 2 & -6t \end{vmatrix} = \begin{vmatrix} 2t & -3t^2 \\ 2 & -6t \end{vmatrix} i - \begin{vmatrix} 1 & -3t^2 \\ 0 & -6t \end{vmatrix} j + \begin{vmatrix} 1 & 2t \\ 0 & 2 \end{vmatrix} k$$

$$= \langle -12t^2 + 6t^2, -(-6t), 2 \rangle$$

$$= \langle -6t^2, 6t, 2 \rangle$$

$$\|r'(t) \times r''(t)\| = \sqrt{(-6t^2)^2 + (6t)^2 + 2^2} = \sqrt{36t^4 + 36t^2 + 4}$$

$$\|r'(t)\| = \sqrt{(1)^2 + (2t)^2 + (-3t^2)^2} = \sqrt{1 + 4t^2 + 9t^4}$$

$$\therefore K(t) = \frac{\|r'(t) \times r''(t)\|}{\|r'(t)\|^3} = \frac{\sqrt{36t^4 + 36t^2 + 4}}{\sqrt{1 + 4t^2 + 9t^4}^3}$$

$$K(0) = \frac{\sqrt{36(0)^4 + 36(0)^2 + 4}}{\sqrt{1 + 4(0)^2 + 9(0)^4}^3} = \frac{\sqrt{4}}{\sqrt{1}^3} = \frac{2}{1} = \boxed{2}$$

1 Problem 1 10 / 10

✓ + 10 pts Correct formula/method and scalar answer (full credit if no mistakes)

- 1 pts Minor computation mistake

- 2 pts Plugged wrong thing into correct formula

- 1 pts Didn't compute the curvature at $t=0$ but found a more general formula

+ 7 pts Used the wrong formula but did everything else correct

+ 5 pts Found the curvature of a planar curve given by the first two coordinates (this happened to be the right answer by coincidence...)

+ 5 pts Found magnitude of derivative of tangent vector (curve was not arclength parametrized, but this also happened to be the right answer by coincidence)

2. (20 points) Consider $\mathbf{r}(t) = \langle t+1, 2t, t^2 \rangle$. Decompose the acceleration vector into the sum of tangential and normal components at $t=1$ (find the unit tangent vector and unit normal vector as well).

$$\mathbf{v}(t) = \mathbf{r}'(t) = \langle 1, 2, 2t \rangle$$

$$\mathbf{a}(t) = \mathbf{r}''(t) = \langle 0, 0, 2 \rangle$$

$$\mathbf{v}(1) = \langle 1, 2, 2(1) \rangle = \langle 1, 2, 2 \rangle$$

$$\mathbf{a}(1) = \langle 0, 0, 2 \rangle$$

$$\mathbf{T}(1) = \frac{\mathbf{r}'(1)}{\|\mathbf{r}'(1)\|} = \frac{\langle 1, 2, 2 \rangle}{\sqrt{1^2+2^2+2^2}} = \frac{1}{3} \langle 1, 2, 2 \rangle$$

$$\begin{aligned} \therefore a_T &= \mathbf{a}(1) \cdot \mathbf{T}(1) = \langle 0, 0, 2 \rangle \cdot \frac{1}{3} \langle 1, 2, 2 \rangle \\ &= \frac{1}{3} [(0 \cdot 1) + (0 \cdot 2) + (2 \cdot 2)] = \boxed{\frac{4}{3}} \end{aligned}$$

$$a_N \cdot \mathbf{N}(1) = \mathbf{a}(1) - a_T \cdot \mathbf{T}(1)$$

$$= \langle 0, 0, 2 \rangle - \frac{4}{3} \cdot \frac{1}{3} \langle 1, 2, 2 \rangle$$

$$= \langle 0, 0, 2 \rangle - \frac{4}{9} \langle 1, 2, 2 \rangle = \langle 0, 0, 2 \rangle - \langle \frac{4}{9}, \frac{8}{9}, \frac{8}{9} \rangle = \langle -\frac{4}{9}, -\frac{8}{9}, \frac{10}{9} \rangle$$

$$a_N = \|\mathbf{a}_N \cdot \mathbf{N}(1)\| = \sqrt{\left(-\frac{4}{9}\right)^2 + \left(-\frac{8}{9}\right)^2 + \left(\frac{10}{9}\right)^2} = \sqrt{\frac{16}{81} + \frac{64}{81} + \frac{100}{81}} = \sqrt{\frac{180}{81}} = \sqrt{\frac{20}{9}} = \boxed{\frac{2}{3}\sqrt{5}}$$

$$\mathbf{N}(1) = \frac{\mathbf{a}_N \cdot \mathbf{N}(1)}{a_N} = \frac{\langle -\frac{4}{9}, -\frac{8}{9}, \frac{10}{9} \rangle}{\frac{2}{3}\sqrt{5}} = \left\langle \frac{1}{\frac{2}{3}\sqrt{5}} \cdot -\frac{4}{9}, \frac{1}{\frac{2}{3}\sqrt{5}} \cdot -\frac{8}{9}, \frac{1}{\frac{2}{3}\sqrt{5}} \cdot \frac{10}{9} \right\rangle = \left\langle \frac{-4}{6\sqrt{5}}, \frac{-8}{6\sqrt{5}}, \frac{10}{6\sqrt{5}} \right\rangle$$

$$= \left\langle \frac{-2}{3\sqrt{5}}, \frac{-4}{3\sqrt{5}}, \frac{5}{3\sqrt{5}} \right\rangle$$

$$= \left\langle \frac{-2}{3\sqrt{5}}, \frac{-4}{3\sqrt{5}}, \frac{\sqrt{5}}{3} \right\rangle$$

$$\therefore \mathbf{a}(1) = \frac{4}{3} \cdot \mathbf{T}(1) + \frac{2}{3}\sqrt{5} \cdot \mathbf{N}(1)$$

$$\mathbf{T}(1) = \left\langle \frac{1}{3}, \frac{2}{3}, \frac{2}{3} \right\rangle$$

$$\mathbf{N}(1) = \left\langle \frac{-2}{3\sqrt{5}}, \frac{-4}{3\sqrt{5}}, \frac{\sqrt{5}}{3} \right\rangle$$

2 Problem 2 20 / 20

✓ + 20 pts Correct decomposition including both unit vectors. (Supersedes partial credit below)

- 1 pts Found decomposition or unit vectors for arbitrary t and did not substitute $t = 1$

- 2 pts Minor mistake

+ 5 pts Correct method to find T

+ 5 pts Correct method to find a_T

+ 5 pts Correct method to find N

+ 5 pts Correct method to find a_N

3. (15 points) Consider the surface $xy^3 + ye^z + x^3z^3 = 2$. Find an equation of the tangent plane at $(1, 1, 0)$.

$$F(x, y, z) = xy^3 + ye^z + x^3z^3$$

$$\nabla F = \langle F_x, F_y, F_z \rangle = \langle y^3 + 3x^2z^3, 3xy^2 + e^z, ye^z + 3x^3z^2 \rangle$$

$$F_x = y^3 + 3x^2z^3$$

$$F_y = 3xy^2 + e^z$$

$$F_z = ye^z + 3x^3z^2$$

$$\nabla F = \langle (1)^3 + 3(1)^2(0)^3, 3(1)(1)^2 + e^0, (1)e^0 + 3(1)^3(0)^2 \rangle$$

(at $(1, 1, 0)$)

$$= \langle 1, 3+1, 1 \rangle$$

$$= \langle 1, 4, 1 \rangle \quad \text{: normal vector}$$

$$F_x(a, b, c)(x-a) + F_y(a, b, c)(y-b) + F_z(a, b, c)(z-c) = 0$$

$$\therefore 1(x-1) + 4(y-1) + 1(z-0) = 0$$

$$x-1+4y-4+z=0$$

$$\boxed{x+4y+z=5}$$

3 Problem 3 15 / 15

✓ - **0 pts** Correct

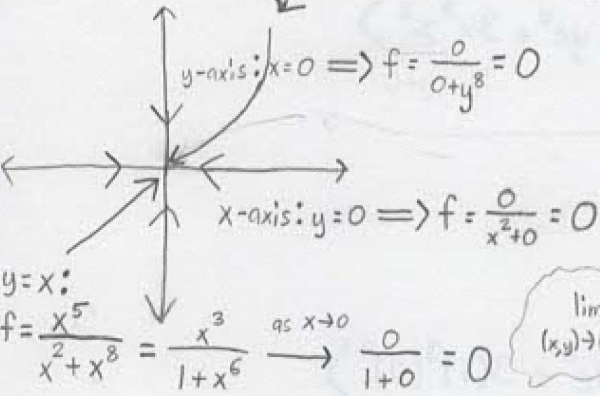
- **3 pts** wrong formula for tangent plane
- **2 pts** calculation error
- **1 pts** correct steps, final answer wrong
- **3 pts** serious calculation error

4. (a) (15 points) Determine if the limit exists. If limit exists, find the limit. If limit does not exist, explain why.

$$f = \frac{xy^4}{x^2+y^8}$$

$$\lim_{(x,y) \rightarrow (0,0)} \frac{xy^4}{x^2+y^8}$$

method 1



$$y = x^2 : f = \frac{x(x^2)^4}{x^2+(x^2)^8} = \frac{x^9}{x^2+x^{16}} = \frac{x^7}{1+x^{14}} \xrightarrow{\text{as } x \rightarrow 0} \frac{0}{1+0} = 0 \quad \text{method 2}$$

$$\lim_{x \rightarrow 0} f(x, m) = \lim_{x \rightarrow 0} \frac{x(mx)^4}{x^2+(mx)^8} = \lim_{x \rightarrow 0} \frac{x^5 m^4}{x^2+x^8 m^8} = \lim_{x \rightarrow 0} \frac{x^3 m^4}{1+x^6 m^8} = \frac{0}{1+0} = 0$$

$$x = r \cos \theta \\ y = r \sin \theta \quad \text{method 3}$$

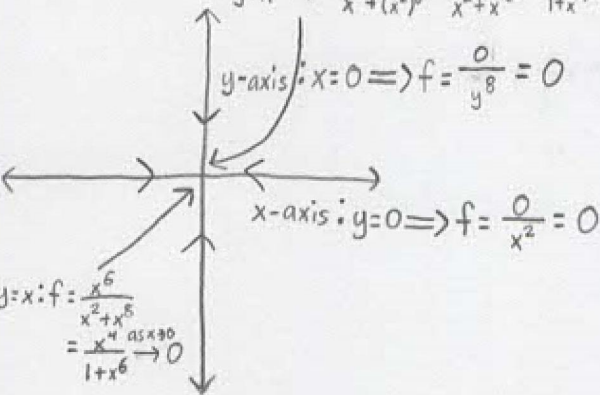
$$\lim_{(x,y) \rightarrow (0,0)} \frac{xy^4}{x^2+y^8} = \lim_{r \rightarrow 0} \frac{(r \cos \theta)(r^4 \sin^4 \theta)}{r^2 \cos^2 \theta + r^8 \sin^8 \theta} = \lim_{r \rightarrow 0} \frac{r^3 \cos \theta \sin^4 \theta}{\cos^2 \theta + r^6 \sin^8 \theta} = \frac{(0)^3 \cos \theta \sin^4 \theta}{\cos^2 \theta + (0)^6 \sin^8 \theta} = \frac{0}{\cos^2 \theta + 0} = 0$$

$$\boxed{\lim_{(x,y) \rightarrow (0,0)} \frac{xy^4}{x^2+y^8} = 0}$$

(b) (15 points) Determine if the limit exists. If limit exists, find the limit. If limit does not exist, explain why.

$$f = \frac{x^2 y^4}{x^2 + y^8}$$

$$y = x^2 : f = \frac{x^2(x^2)^4}{x^2+(x^2)^8} = \frac{x^{10}}{x^2+x^{16}} = \frac{x^8}{1+x^{14}} \xrightarrow{\text{as } x \rightarrow 0} \frac{0}{1+0} = 0 \quad \lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y^4}{x^2 + y^8}$$



$$0 \leq \left| \frac{x^2 y^4}{x^2 + y^8} \right| \leq \frac{|x^2 y^4|}{x^2} = |y^4|$$

(remove y^8)

$$\lim_{(x,y) \rightarrow (0,0)} 0 = 0 \quad > \text{match}$$

$$\lim_{(x,y) \rightarrow (0,0)} |y^4| = 0$$

$$\text{by Squeeze theorem, } \lim_{(x,y) \rightarrow (0,0)} \left| \frac{x^2 y^4}{x^2 + y^8} \right| = 0$$

$$\text{Thus } \lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y^4}{x^2 + y^8} = 0$$

4.1 (a) 5 / 15

+ 15 pts Full credit; correctly concludes that the limit does not exist.

For instance, we could look along the line $y = 0$ and get $\lim_{x \rightarrow 0} \frac{0}{x^2 + 0} = 0$

and along the curve $x = y^4$ and get

$$\lim_{y \rightarrow 0} \frac{y^8}{y^8 + y^8} = \frac{1}{2}$$

Since these path limits are not equal, the limit DNE.

[Of course, many other solutions are possible].

[Note: this rubric item is feedback about the problem as a whole and not intended to be combined with other rubric items.]

+ 5 pts Incorrect answer by misuse of polar coordinate limit. If we just plug in the standard polar coordinates, we can reduce to $f(r, \theta) = \frac{r^3 \cos \theta \sin^4 \theta}{\cos^2 \theta + r^6 \sin \theta}$

but this does not go to 0 just because the numerator goes to 0. (In particular, if $\cos^2 \theta \rightarrow 0$, then we can reach a situation where the limit is indeterminate. This is true of the path $x = y^4$ above). Note that showing that the numerator goes to 0 is itself nontrivial and to do it properly would involve the squeeze theorem.

When using polar coordinates, one must keep in mind that θ is not fixed; considering only fixed values of θ is equivalent to considering only lines to the origin in the plane.

This fits in the "incorrect method" category but the error is a little more subtle (thus a greater number of points).

[Note: this rubric item is feedback about the problem as a whole and not intended to be combined with other rubric items.]

+ 15 pts Correct answer based on modified polar coordinates. If we let $w = \pm y^4$, then the limit becomes $\lim_{(x,w) \rightarrow (0,0)} \frac{xw}{x^2 + w^2}$

Applying polar coordinates gives $f(r, \theta) = \cos \theta \sin \theta$ which is dependent on θ ; so lines through the origin do not have the same limits in this transformed situation which means the limit DNE.

[Note: this rubric item is feedback about the problem as a whole and not intended to be combined with other rubric items.]

+ 3 pts Incorrect answer based on misapplication of the squeeze theorem. If we have functions f, g, h such that $f \leq g \leq h$, and $\lim f = \lim h$, then this does not tell us whether or not g converges.

(For an example, let $f = -1$, $g = 0$, $h = 1$; then the limits of f and h are never equal, but we do not conclude that g doesn't have limits).

[Note: this rubric item is feedback about the problem as a whole and not intended to be combined with other rubric items.]

+ **3 pts** Incorrect answer based on checking a limited number of paths; in order for the limit to exist, we need the limit along any path to be equal, so checking just a finite number of them is not sufficient.

[Note: this rubric item is feedback about the problem as a whole and not intended to be combined with other rubric items.]

+ **8 pts** Concludes that the limit does not exist based on paths disagreeing [which is a correct method], but the paths computed were done incorrectly, and in fact the limit agrees on the considered paths.

[Note: this rubric item is feedback about the problem as a whole and not intended to be combined with other rubric items.]

+ **0 pts** Grading note: In general the point values are 15 for a correct solution, 8 for a correct method applied incorrectly, and 3 for an incorrect method (e.g. using a nonexhaustive set of paths to conclude that the limit exists, or misapplying polar coordinates). I've tried to set up the rubric items to give you feedback on your particular solution.

+ 5 Point adjustment

- Both of these methods (check only some paths and misuse of polar coordinates) are incorrect. So this still falls under the "incorrect methods" category, even if there are two of them.

1 lines are not the only paths, so this does not suffice to prove that the limit exists (and x^2 is not the only other path)

4. (a) (15 points) Determine if the limit exists. If limit exists, find the limit. If limit does not exist, explain why.

$$f = \frac{xy^4}{x^2+y^8}$$

$$\lim_{(x,y) \rightarrow (0,0)} \frac{xy^4}{x^2+y^8}$$

method 1

$$y\text{-axis: } x=0 \Rightarrow f = \frac{0}{0+y^8} = 0$$

$$x\text{-axis: } y=0 \Rightarrow f = \frac{0}{x^2+0} = 0$$

$$y=x^2: f = \frac{x(x^2)^4}{x^2+(x^2)^8} = \frac{x^9}{x^2+x^{16}} = \frac{x^7}{1+x^{14}} \xrightarrow{\text{as } x \rightarrow 0} \frac{0}{1+0} = 0 \quad \text{method 2}$$

$$\lim_{x \rightarrow 0} f(x, m) = \lim_{x \rightarrow 0} \frac{x(mx)^4}{x^2+(mx)^8} = \lim_{x \rightarrow 0} \frac{x^5 m^4}{x^2+x^8 m^8} = \lim_{x \rightarrow 0} \frac{x^3 m^4}{1+x^6 m^8} = \frac{0}{1+0} = 0$$

$$x = r \cos \theta \\ y = r \sin \theta \quad \text{method 3}$$

$$\lim_{(x,y) \rightarrow (0,0)} \frac{xy^4}{x^2+y^8} = \lim_{r \rightarrow 0} \frac{(r \cos \theta)(r^4 \sin^4 \theta)}{r^2 \cos^2 \theta + r^8 \sin^8 \theta} = \lim_{r \rightarrow 0} \frac{r^3 \cos \theta \sin^4 \theta}{\cos^2 \theta + r^6 \sin^8 \theta} = \frac{(0)^3 \cos \theta \sin^4 \theta}{\cos^2 \theta + (0)^6 \sin^8 \theta} = \frac{0}{\cos^2 \theta + 0} = 0$$

$$\boxed{\lim_{(x,y) \rightarrow (0,0)} \frac{xy^4}{x^2+y^8} = 0}$$

(b) (15 points) Determine if the limit exists. If limit exists, find the limit. If limit does not exist, explain why.

$$f = \frac{x^2 y^4}{x^2+y^8}$$

$$y=x^2: f = \frac{x^2(x^2)^4}{x^2+(x^2)^8} = \frac{x^{10}}{x^2+x^{16}} = \frac{x^8}{1+x^{14}} \xrightarrow{\text{as } x \rightarrow 0} \frac{0}{1+0} = 0 \quad \lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y^4}{x^2+y^8}$$

$$y\text{-axis: } x=0 \Rightarrow f = \frac{0}{y^8} = 0$$

$$x\text{-axis: } y=0 \Rightarrow f = \frac{0}{x^2} = 0$$

$$0 \leq \left| \frac{x^2 y^4}{x^2+y^8} \right| \leq \frac{|x^2 y^4|}{x^2} = |y^4|$$

(remove y^8)

$$\lim_{(x,y) \rightarrow (0,0)} 0 = 0 \quad > \text{match}$$

$$\lim_{(x,y) \rightarrow (0,0)} |y^4| = 0$$

$$\text{by Squeeze theorem, } \lim_{(x,y) \rightarrow (0,0)} \left| \frac{x^2 y^4}{x^2+y^8} \right| = 0$$

$$\text{Thus } \lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y^4}{x^2+y^8} = 0$$

$$y=x: f = \frac{x^6}{x^2+x^8} = \frac{x^4}{1+x^6} \xrightarrow{\text{as } x \rightarrow 0} 0$$

4.2 (b) 15 / 15

✓ + 15 pts Full credit; correctly concludes that the limit is 0 .

One solution is to use the fact that $x^2 + y^8 \geq 2|xy^4|$, and write $0 \leq \frac{x^2 y^4}{x^2 + y^8} \leq \frac{x^2 y^4}{2|xy^4|} = \frac{|x|}{2}$

and apply the squeeze theorem. Another is to note that $x^2 + y^8 \geq x^2$, so $0 \leq \frac{x^2 y^4}{x^2 + y^8} \leq \frac{x^2 y^4}{x^2} = y^4$

and the squeeze theorem applies again.

[Note: this rubric item is feedback about the problem as a whole and not intended to be combined with other rubric items.]

+ 14 pts Correct, except for error in the absolute values; it is not true that $\frac{x^2 y^4}{x^2 + y^8} \leq \frac{x}{2}$

(as this is false whenever x is negative and $y \neq 0$).

Note that $\frac{x^2}{|x|} = |x|$.

+ 13 pts Same as previous but forgot the factor of 2 as well - it's necessary.

+ 5 pts Incorrect answer by misuse of polar coordinate limit. If we just plug in the standard polar coordinates, we can reduce to $f(r, \theta) = \frac{r^4 \cos^2 \theta \sin^4 \theta}{\cos^2 \theta + r^6 \sin \theta}$

but this does not go to 0 just because the numerator goes to 0. (In particular, if $\cos^2 \theta \rightarrow 0$, then we can reach a situation where the limit is indeterminate. This is true of the path $x = y^4$ as in part (a)). Note that showing that the numerator goes to 0 is itself nontrivial and to do it properly would involve the squeeze theorem.

When using polar coordinates, one must keep in mind that θ is not fixed; considering only fixed values of θ is equivalent to considering only lines to the origin in the plane.

Note how this is only slightly different from the polar form of part (a) - it happens that this gives the right answer in this part, but this is mostly a coincidence.

[Note: this rubric item is feedback about the problem as a whole and not intended to be combined with other rubric items.]

+ 15 pts Correct answer based on modified polar coordinates. If we let $w = \pm y^4$, then the limit becomes $\lim_{(x,w) \rightarrow (0,0)} \frac{x^2 w}{x^2 + w^2}$

Applying polar coordinates gives $f(r, \theta) = r \cos \theta \sin \theta$ so we can say

$0 \leq |f| \leq r$ and apply the squeeze theorem again

[Note: this rubric item is feedback about the problem as a whole and not intended to be combined with other rubric items.]

+ **3 pts** Incorrect answer based on checking a limited number of paths; in order for the limit to exist, we need the limit along any path to be equal, so checking just a finite number of them is not sufficient.

[Note: this rubric item is feedback about the problem as a whole and not intended to be combined with other rubric items.]

+ **8 pts** Checks the limit along paths, gets different limits, and concludes that the full limit DNE. This means (at least) one of the paths was computed incorrectly

+ **0 pts** Grading note: In general the point values are 15 for a correct solution, 8 for a correct method applied incorrectly, and 3 for an incorrect method (e.g. using a nonexhaustive set of paths to conclude that the limit exists, or misapplying polar coordinates). I've tried to set up the rubric items to give you feedback on your particular solution.

5. (25 points) Consider a function

$$f(x, y) = \begin{cases} \frac{x^4 y}{x^4 + y^4} & (x, y) \neq (0, 0), \\ 0 & (x, y) = (0, 0). \end{cases}$$

Determine if partial derivatives $f_x(0, 0)$ and $f_y(0, 0)$ exist (find the value if exist). Then, determine if f is differentiable at $(0, 0)$.

$$f_x(0, 0) = 0$$

$$f_y(0, 0) = 0$$

$$f(0, y) = \begin{cases} 0 & y \neq 0 \\ 0 & y = 0 \end{cases}$$

$$\therefore \left(\frac{d}{dy} f(0, y) \right)_{y=0} = 0$$

$$f(x, 0) = \begin{cases} 0 & x \neq 0 \\ 0 & x = 0 \end{cases}$$

$$\therefore \left(\frac{d}{dx} f(x, 0) \right)_{x=0} = 0$$

$$\Rightarrow L(x, y) = 0$$

$$\begin{aligned} z &= f(0, 0) + f_x(0, 0)(x-0) + f_y(0, 0)(y-0) \\ &= 0 + 0(x) + 0(y) \\ &= 0 \end{aligned}$$

check:

$$f_x(0, 0) = \lim_{h \rightarrow 0} \frac{f(0+h, 0) - f(0, 0)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{f(h, 0) - 0}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h^4(0)}{h^4 + 0^4} = \lim_{h \rightarrow 0} \frac{0}{h} = 0$$

$$= \lim_{h \rightarrow 0} \frac{0}{h} = 0$$

$$f_y(0, 0) = \lim_{h \rightarrow 0} \frac{f(0, 0+h) - f(0, 0)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{f(0, h) - 0}{h}$$

$$= \lim_{h \rightarrow 0} \frac{0^4 h}{0^4 + h^4} = \lim_{h \rightarrow 0} \frac{0}{h} = 0$$

$$= \lim_{h \rightarrow 0} \frac{0}{h} = 0$$

$$\lim_{(x, y) \rightarrow (0, 0)} \frac{f(x, y) - L(x, y)}{\sqrt{(x-a)^2 + (y-b)^2}} = \frac{\frac{x^4 y}{x^4 + y^4} - 0}{\sqrt{x^2 + y^2}}$$

$$= \frac{x^4 y}{(x^4 + y^4)(\sqrt{x^2 + y^2})}$$

$$= \lim_{r \rightarrow 0} \frac{r^4 \cos^4 \theta \cdot r \sin \theta}{(r^4 \cos^4 \theta + r^4 \sin^4 \theta) \sqrt{r^2 \cos^2 \theta + r^2 \sin^2 \theta}}$$

$$= \frac{r^5 \cos^4 \theta \sin \theta}{r^4 (\cos^4 \theta + \sin^4 \theta) \cdot r \sqrt{\cos^2 \theta + \sin^2 \theta}}$$

$$= \frac{r^5 \cos^4 \theta \sin \theta}{r^5 (\cos^4 \theta + \sin^4 \theta)}$$

$$= \frac{\cos^4 \theta \sin \theta}{\cos^4 \theta + \sin^4 \theta}$$

$$= \frac{\cos^4 \theta \sin \theta}{\cos^4 \theta + \sin^4 \theta}$$

$$= \frac{\cos^4 \theta \sin \theta}{\cos^4 \theta + \sin^4 \theta}$$

limits along lines are different \therefore DNE

$$\lim_{(x, y) \rightarrow (0, 0)} \frac{f(x, y) - L(x, y)}{\sqrt{(x-a)^2 + (y-b)^2}} \neq 0 \therefore f \text{ is NOT differentiable at } (0, 0)$$

$$\begin{aligned} x &= r \cos \theta \\ y &= r \sin \theta \end{aligned}$$

5 Problem 5 25 / 25

✓ + 25 pts Correct

+ 12 pts Correct partial derivatives

+ 5 pts Compute partial derivative incorrectly (e.g. using the quotient rule)

+ 3 pts Correct definition of differentiability

- 3 pts Mistake

+ 0 pts No point